# OPTIMISATION OF A SEMI-EMPIRICAL MODEL FOR ACCURATE DETERMINATION OF EXCITATION ENERGIES AND ABSORPTION SPECTRA OF QUANTUM DOTS 

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#### Abstract

Quantum dots are nanomaterials that have several potential applications including the production of efficient solar cells. Accurate theoretical studies of excitation energies and absorption spectra of quantum dots are essential for harnessing such potentials. The existing high-level ab-initio methods for obtaining excitation energies and absorption spectra are computationally expensive for quantum dots. However, the semi-empirical methods, including the Intermediate Neglect of Differential Overlap for spectroscopy (INDO/s) model, are computationally cheap but are generally less accurate. Unlike some ground-state semi-empirical methods, INDO/s has not attracted significant attention to improving its level of accuracy because of some difficulties associated with optimising its parameters. Therefore, this research was aimed at developing an improved INDO/s model that will be computationally cheap and capable of producing accurate excitation energies and absorption spectra for quantum dots.

A semi-empirical Hamiltonian based on INDO/s was parameterised with benchmark excitation energies from Equation-Of-Motion Coupled-Cluster Singles Doubles (EOMCCSD) for $\mathrm{Si}, \mathrm{S}, \mathrm{Cd}$ and Zn diatomics at different interatomic separations. The Mean Absolute Errors (MAE) were calculated for different sets of parameters and the optimised set of parameters were those with the least MAEs. The optimised model was called optimised for excitation Intermediate Neglect of Differential Overlap (oeINDO). The oeINDO was validated by computing the MAEs of the oeINDO and INDO/s excitation energies and absorption spectra maxima for $\mathrm{Si}_{n}, \mathrm{~S}_{n}, \mathrm{Zn}_{n}, \mathrm{Cd}_{n},(\mathrm{ZnS})_{n}$ and $(\mathrm{CdS})_{n}(\mathrm{n}$ is the number of atoms) clusters. The validation was carried out relative to EOM-CCSD for small clusters ( $n<6$ ) and Time-Dependent Density Functional Theory (TDDFT) for large clusters ( $\mathrm{n} \geq 6$ ). All computation times were recorded. The oeINDO was then employed to predict the absorption spectra of $\mathrm{Si}, \mathrm{S}, \mathrm{Zn}, \mathrm{Cd}, \mathrm{ZnS}$, and CdS quantum dots, and the optimal size of CdS and ZnS quantum dots for solar cell applications.

The optimised parameters obtained for $\mathrm{Si}, \mathrm{S}, \mathrm{Zn}$ and Cd diatomics had MAEs 0.21, 0.19, 0.23 , and 0.29 eV , respectively. The oeINDO produced excitation energies with MAEs $0.18,0.56,0.25,0.22 \mathrm{eV}$ for small $\mathrm{Si}, \mathrm{S}, \mathrm{Zn}$, and Cd clusters, respectively, and MAEs $0.22,0.36,0.15,0.24,0.36$ and 0.23 eV , for large $\mathrm{Si}, \mathrm{S}, \mathrm{Zn}, \mathrm{Cd}, \mathrm{ZnS}$, and CdS clus-


ters, respectively. The unoptimised INDO/s however, produced excitation energies with MAEs $1.23,1.29,0.70$, and 1.23 eV for small $\mathrm{Si}, \mathrm{S}, \mathrm{Zn}, \mathrm{Cd}$ clusters, respectively, and MAEs $1.05,2.51,2.49,0.63,0.76$ and 1.04 eV for large $\mathrm{Si}, \mathrm{S}, \mathrm{Zn}, \mathrm{Cd}, \mathrm{ZnS}$, and CdS clusters, respectively. Also, the MAEs of oeINDO and INDO/s absorption spectra maxima relative to those from TDDFT were 0.41 eV and 1.49 eV , respectively. The results showed that oeINDO agreed reasonably well with the benchmarks and it was more accurate than INDO/s. The time of computing with oeINDO ( 0.08 minutes) was found to be less than a hundredth of the time utilised for EOM-CCSD (2946.51 minutes). The oeINDO predicted a red-shift in the quantum dots absorption spectra with an increase in dot size. It also predicted $\mathrm{Si}, \mathrm{Zn}$ and Cd dots to be metallic. The 1.2 nm and 1.4 nm spherical-like CdS and ZnS quantum dots, respectively, were found to be theoretically optimal for solar cell applications.

The improved INDO/s was computationally cheap and capable of producing more accurate excitation energies and absorption spectra for quantum dots.

Keywords: Cluster excitation energies, Cluster absorption spectra, Hamiltonian, Highlevel ab-initio Method, Nanomaterials

Word count: 500

## DEDICATION

This work is dedicated to the Glory of God.

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## CERTIFICATION

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## TABLE OF CONTENTS

ABSTRACT ..... i
DEDICATION ..... iii
ACKNOWLEDGEMENTS ..... iv
CERTIFICATION ..... V
TABLE OF CONTENTS ..... vi
LIST OF TABLES ..... xi
LIST OF FIGURES ..... xiv
LIST OF ABBREVIATIONS ..... xxii
Chapter 1: INTRODUCTION ..... 1
1.1 Introduction ..... 1
1.2 Statement of Problem ..... 7
1.3 Research Justification ..... 9
1.4 Aim and Objectives ..... 10
1.5 Outline of Thesis ..... 10
Chapter 2: THEORETICAL BACKGROUND AND LITERATURE REVIEW ..... 12
2.1 Nanomaterials ..... 12
2.1.1 Quantum Dot: Confinement effect and quantization of energy and state ..... 12
2.2 Theoretical spectroscopy ..... 18
2.2.1 Time Dependent Perturbation Theory: Transition Probability and Absorption Coefficient ..... 19
2.2.2 Polarizability and absorption coefficient ..... 22
2.3 Theoretical approaches for excitation energies of atomic clusters ..... 23
2.3.1 Wave function methods: Hartree Fock and Post Hatree Fock methods ..... 24
2.3.1.1 Configuration Interaction and Configuration Interac- tion Singles ..... 27
2.3.1.2 Couple Cluster (CC) and EOM-CCSD Method ..... 29
2.3.2 Density Functional Theory ..... 31
2.3.2.1 Time-Dependent Density Functional Theory (TDDFT) ..... 33
2.3.3 Semi-empirical approaches ..... 36
2.3.3.1 The Tight Binding Model ..... 37
2.3.3.2 Density Functional Tight Binding (DFTB) ..... 38
2.3.3.3 Empirical pseudopotential based model ..... 39
2.3.3.4 Huckel and Extended Huckel model ..... 41
2.3.3.5 Neglect of Differential Overlap (NDO) ..... 43
2.3.3.6 Complete Neglect of Differential Overlap (CNDO) ..... 44
2.3.3.7 Intermediate Neglect of Differential Overlap(INDO) ..... 46
2.3.3.8 Neglect of Diatomics of Differential Overlap (NDDO) ..... 47
2.4 INDO/s formalism and parametrization ..... 48
2.4.1 INDO/s formalism and parameterisation for organic molecules ..... 48
2.4.2 INDO/s formalism and parametrization for transition metals ..... 50
2.4.3 Determination of electronic and optical properties with INDO/s ..... 51
2.4.4 Applications of INDO/s Methods ..... 52
2.4.5 Limitations and improvement in INDO/s ..... 52
2.5 Quantum Mechanical Methods for Determination of Excitations Ener- gies and Absorption Spectra ..... 53
2.6 Development and Improvement in Semi-empirical Molecular Orbital The- ory ..... 55
2.7 Parameterization scheme in NDO semi-empirical methods ..... 58
2.7.1 Reference Data ..... 59
2.7.2 Parametrization procedure ..... 59
2.8 Theoretical studies of quantum dots ..... 60
2.9 Software Packages for quantum molecular structure calculations ..... 62
Chapter 3: MATERIALS AND METHOD ..... 64
3.1 Calculation of benchmark excited state energies and UV-Vis absorption spectra from ab-initio methods ..... 66
3.1.1 Calculation of excited state energies for dimer geometries ..... 66
3.2 Parameterisation of the ZINDO/s (INDO/s) Hamiltonian model ..... 66
3.3 Validation of the oeINDO ..... 70
3.3.1 Calculation of excited state energies and Ultraviolet-Visible (UV- Vis) absorption spectra for complex structures ..... 70
3.3.2 Geometry Optimisation ..... 71
3.4 Study of large atomic clusters and quantum dots using the oeINDO model ..... 72
Chapter 4: RESULTS and DISCUSSION ..... 73
4.1 Results of Parameterization ..... 73
4.2 Validation of the oeINDO model ..... 86
4.2.1 Equilibrium Structures ..... 86
4.2.2 Transferability of oeINDO silicon parameters ..... 87
4.2.3 Transferability of oeINDO Zinc Parameters ..... 106
4.2.4 Transferability of oeINDO Cadmium parameters ..... 120
4.2.5 Transferability of oeINDO Sulphur Parameters ..... 134
4.2.6 Transferability of oeINDO to ZnS clusters ..... 142
4.2.7 Transferability of oeINDO to CdS clusters ..... 152
4.2.8 Transferability oeINDO parameters to $\mathrm{Cd}_{x} \mathrm{Zn}_{y} \mathrm{~S}_{19}$ clusters ..... 167
4.3 Study of large clusters and quantum dots using the oeINDO model ..... 172
4.4 Computation Time Expended for Excitation Energies and Absorption Spectra Calculations Using Different Methods ..... 187
Chapter 5: CONCLUSION AND RECOMMENDATIONS ..... 189
5.1 Summary and Conclusion ..... 189
5.2 Contribution to Knowledge ..... 191
5.3 Recommendations ..... 192
REFERENCES ..... 213
APPENDICES ..... 214
Chapter A:Materials Used in the Research Work ..... 214
A. 1 Hardware ..... 214
A. 2 Software algorithms and Packages ..... 214
A.2.1 ORCA 4.0 ..... 215
A.2.2 Amoeba Optimiser Algorithm ..... 215
A.2.3 MOPAC7 ..... 215
A.2.4 QuantumATK ..... 216
A.2.5 Gnuplot 4.6 ..... 216
A.2.6 Gabedit 2.5.0 ..... 216
Chapter B: Optimized atomic structures ..... 217
B. 1 Zinc clusters ..... 217
B.1.1 Cartesian coordinates of equilibrium zinc clusters in Angstrom ..... 218
B. 2 Cadmium Clusters ..... 219
B.2.1 Cartesian coordinates of equilibrium cadmium clusters in Angstrom ..... 220
B. 3 Zinc Quantum Dots ..... 221
B.3.1 Cartesian coordinates of equilibrium zinc quantum dots in Angstrom ..... 222
B. 4 Sulphur Quantum Dots ..... 226
B.4.1 Cartesian coordinates (Angstroms) of equilibrium sulphur quan- tum dots ..... 227
B.5.1 Cartesian coordinates (Angstroms) of equilibrium silicon quan- tum dots ..... 230
B. 5 Silicon Quantum Dots ..... 231
B.6.1 Cartesian coordinates (Angstroms) of equilibrium ZnS quantum dots ..... 233
B. 6 ZnS Quantum Dots ..... 234
B.7.1 Cartesian coordinates (Angstroms) of equilibrium CdS quantum dots ..... 237
B. 7 CdS Quantum Dots ..... 238
Chapter C: Vertical excitation energies from different methods for various atomic clusters ..... 240
C. 1 Excited State Energies (eV) for $\mathrm{Si}_{3}$ ..... 240
C. 2 Excited State Energies (eV) for $\mathrm{Zn}_{4}$ ..... 241
C. 3 Excited State Energies (eV) for $\mathrm{Zn}_{6}$ ..... 242
C. 4 Excited State Energies (eV) for $\mathrm{Zn}_{8}$ ..... 243
C. 5 Excited State Energies (eV) for $\mathrm{Zn}_{16}$ ..... 244
C. 6 Excited State Energies (eV) for $\mathrm{Zn}_{24}$ ..... 245
C. 7 Excited State Energies (eV) for $\mathrm{Cd}_{4}$ ..... 246
C. 8 Excited State Energies (eV) for $\mathrm{Cd}_{6}$ ..... 247
C. 9 Excited State Energies (eV) for $\mathrm{Cd}_{8}$ ..... 248
C. 10 Excited State Energies (eV) for $\mathrm{Cd}_{16}$ ..... 249
C. 11 Excited State Energies (eV) for $\mathrm{S}_{3}$ ..... 250
C. 12 Excited State Energies (eV) for $\mathrm{S}_{5}$ ..... 251
C. 13 Excited State Energies (eV) for $\mathrm{S}_{5}$ ..... 252
C. 14 Excited State Energies (eV) for $\mathrm{S}_{6}$ ..... 253
C. 15 Excited State Energies (eV) for $\mathrm{S}_{10}$ ..... 254
C. 16 Excited State Energies (eV) for $\mathrm{S}_{20}$ ..... 255
C. 17 Excited State Energies (eV) for $(\mathrm{ZnS})_{2}$ ..... 256
C. 18 Excited State Energies (eV) for $(\mathrm{ZnS})_{3}$ ..... 257
C. 19 Excited State Energies (eV) for (ZnS) 4 ..... 258
C. 20 Excited State Energies (eV) for $(\mathrm{ZnS})_{10}$ ..... 259
C. 21 Excited State Energies (eV) for $(\mathrm{ZnS})_{19}$ ..... 260
C. 22 Excited State Energies (eV) for $(\mathrm{CdS})_{2}$ ..... 261
C. 23 Excited State Energies (eV) for (CdS) ${ }_{3}$ ..... 262
C. 24 Excited State Energies (eV) for $(\mathrm{CdS})_{10}$ ..... 263
C. 25 Excited State Energies (eV) for (CdS) ${ }_{19}$ ..... 264
Chapter D: Published Paper from the Thesis ..... 265

## LIST OF TABLES

4.1 MAEs of excitation energies (eV) from ZINDO/s and oeINDO rela- tive to EOM-CCSD energies for various diatomics ..... 79
4.2 ZINDO/s and oeINDO parameters for $\mathrm{Si}_{2}$ ..... 81
4.3 oeINDO and ZINDO/s parameters for $\mathrm{S}_{2}$ ..... 82
4.4 oeINDO and ZINDO/s parameters for $\mathrm{Zn}_{2}$ ..... 83
4.5
oeINDO and ZINDO/s parameters for $\mathrm{Cd}_{2}$ ..... 84
4.6
First Eight Lowest Excited State Energies (eV) for $\mathrm{Si}_{3}$ ..... 88
4.7 MAEs in eV of semi-empirical methods, CIS(D) and TDDFT from ab-initio methods for Si equilibrium geometries ..... 90
4.8 First Eight Lowest Excited State Energies (eV) for $\mathrm{Si}_{4}$ ..... 91
4.9 First Eight Lowest Excited State Energies (eV) for $\mathrm{Si}_{5}$ ..... 94
4.10 First Eight Lowest Excited State Energies (eV) for $\mathrm{Si}_{19}$ ..... 96
4.11 First Eight Lowest Excited State Energies (eV) for $\mathrm{Si}_{40}$ ..... 97
4.12 MAEs (eV) of excitations from various methods as compared to EOM-CCSD excitations for different (non-equilibrium and equilib- rium) Si clusters ..... 104
4.13 First Eight Lowest Excited State Energies (eV) for $\mathrm{Zn}_{3}$ ..... 107
4.14 First Eight Lowest Excited State Energies (eV) for $\mathrm{Cd}_{3}$ ..... 121
4.15 First eight (8) lowest vertical excitation energies (eV) for large sili- con clusters ..... 168
4.16 First eight (8) vertical excitation energies (eV) for Zn quantum dots of different sizes ..... 170
4.17 Vertical excitation energies for Cadmium quantum dots of different sizes ..... 173
4.18 Lowest eight (8) vertical excitation energies(eV) for sulphur quan- tum dots of different sizes ..... 175Computational time (in minutes) expended in carrying out excita-tion energies and absorption spectra for some atomic clusters usingdifferent methods186
C. 1 First Eight Lowest Excited State Energies (eV) for $\mathrm{Si}_{3}$ ..... 240
C. 2 First Eight Lowest Excited State Energies (eV) for $\mathrm{Zn}_{4}$ ..... 241
C. 3 First Eight Lowest Excited State Energies (eV) for $\mathrm{Zn}_{6}$ ..... 242
C. 4 First Eight Lowest Excited State Energies (eV) for $\mathrm{Zn}_{8}$ ..... 243
C. 5 First Eight Lowest Excited State Energies (eV) for $\mathrm{Zn}_{16}$ ..... 244
C. 6 First Eight Lowest Excited State Energies (eV) for $\mathrm{Zn}_{24}$ ..... 245
C. 7 First Eight Lowest Excited State Energies (eV) for $\mathrm{Cd}_{4}$ ..... 246
C. 8 First Eight Lowest Excited State Energies (eV) for $\mathrm{Cd}_{6}$ ..... 247
C. 9 First Eight Lowest Excited State Energies (eV) for $\mathrm{Cd}_{8}$ ..... 248
C. 10 First Eight Lowest Excited State Energies (eV) for $\mathrm{Cd}_{16}$ ..... 249
C. 11 First Eight Lowest Excited State Energies (eV) for $\mathrm{S}_{3}$ ..... 250
C. 12 First Eight Lowest Excited State Energies (eV) for $\mathrm{S}_{5}$ ..... 251
C. 13 First Eight Lowest Excited State Energies (eV) for $\mathrm{S}_{5}$ ..... 252
C. 14 First Eight Lowest Excited State Energies (eV) for $\mathrm{S}_{6}$ ..... 253
C. 15 First Eight Lowest Excited State Energies(eV) for $\mathrm{S}_{10}$ ..... 254
C. 16 First Eight Lowest Excited State Energies (eV) for $\mathrm{S}_{20}$ ..... 255
C. 17 First Eight Lowest Excited State Energies (eV) for $(\mathrm{ZnS})_{2}$ cluster ..... 256
C. 18 First Eight Lowest Excited State Energies (eV) for $(\mathrm{ZnS})_{3}$ cluster ..... 257
C. 19 First Eight Lowest Excited State Energies (eV) for $(\mathrm{ZnS})_{4}$ cluster ..... 258
C. 20 First Eight Lowest Excited State Energies (eV) for ( ZnS$)_{10}$ cluster ..... 259
C. 21 First Eight Lowest Excited State Energies (eV) for $(\mathrm{ZnS})_{19}$ cluster ..... 260
C. 22 First Eight Lowest Excited State Energies(eV) for $(\mathrm{CdS})_{2}$ cluster ..... 261
C. 23 First Eight Lowest Excited State Energies (eV) for (CdS) $)_{3}$ cluster ..... 262
C. 24 First Eight Lowest Excited State Energies (eV) for (CdS $)_{10}$ cluster . 263
C. 25 First Eight Lowest Excited State Energies (eV) for (CdS $)_{19}$ cluster . 264

## LIST OF FIGURES

1.1 (a) Electronic excitation and de-excitation (b) Absorption spectrum -absorbance of a system against wavelength of incident photon (Har-
vey, 2011).
2.1 A one-dimensional infinite potential square well. The regions I and III are barriers with infinite potentials while the region II is a well with potential equal to zero
2.2 Electronic density of states of systems with $3,2,1$, and 0 degrees of freedom. These systems with $3,2,1$, and 0 degrees of freedom are referred to as bulk (3D), quantum well or sheet (2D), quantum wire (1D) and quantum dot, respectively (Kuno, 2005)17
4.1 A scatter plot of $\mathrm{Si}_{2}$ excitation energies obtained using ZINDO/s and oeINDO against those from EOM-CCSD (the benchmark). The blue circles are the oeINDO excitation energies while the red plus signs are the ZINDO/s excitation energies. The straight green line represent the benchmark.
4.2 Scatter plot of $S_{2}$ excitation energies obtained using ZINDO/s and oeINDO against those from EOM-CCSD (the benchmark). The blue circles are the oeINDO excitation energies while the red plus signs are the ZINDO/s excitation energies. The straight green line represents the benchmark75
4.3 Scatter plot of $\mathrm{Cd}_{2}$ excitation energies obtained using ZINDO/s and oeINDO against those from EOM-CCSD (the benchmark). The blue circles are the oeINDO excitation energies while the red plus signs are the ZINDO/s excitation energies. The straight green line represents the benchmark.

4.4 | Scatter plot of $\mathrm{Zn}_{2}$ excitation energies obtained using $\mathrm{ZINDO} / \mathrm{s}$ and |
| :--- |
| oeINDO against those from EOM-CCSD (the benchmark). The blue |
| circles are the oeINDO excitation energies while the red plus signs |
| are the ZINDO/s excitation energies. The straight green line repre- |
| sents the benchmark. . . . . . . . . . . . . . . . . . . . . . . . . . 77 |

4.5 Absorption spectra for $\mathrm{Si}_{3}$ equilibrium geometry obtained from dif- ferent methods. The intensities have been scaled so that the highest intensity is equal to unity. ..... 89
4.6 Absorption spectra of $\mathrm{Si}_{4}$ equilibrium geometry obtained from dif- ferent methods. The intensities have been scaled so that the highest intensity is equal to unity ..... 92
4.7 Absorption spectra for $\mathrm{Si}_{5}$ equilibrium geometry obtained from dif- ferent methods.. The intensities have been scaled so that the highest intensity is equal to unity ..... 95
4.8 Absorption spectra for $\mathrm{Si}_{19}$ equilibrium geometry obtained from dif- ferent methods.. The intensities have been scaled so that the highest intensity is equal to unity. ..... 98
4.9 Absorption spectra for $\mathrm{Si}_{40}$ equilibrium geometry obtained from dif- ferent methods.. The intensities have been scaled so that the highest intensity is equal to unity. ..... 99
4.10
Highest peaks against number of units $n$ in equilibrium $\mathrm{Si}_{n}$ structures. 101
4.11 Semi-empirical energies against TDDFT energies for $\operatorname{Si}_{n}(n=3,4,5,19,40)$ equilibrium. structures. ..... 102
4.12 Scatter plot of different (non-equilibrium and equilibrium) $\mathrm{Si}_{n}$ ( $n=$ $5,4,3)$ clusters. ..... 103
4.13 Comparing excitation energies obtained using different methods with those calculated at EOM-CCSD/TZVPP level for different $\mathrm{Zn}_{n}(n=$ $3,4)$ equilibrium and non-equilibrium structures. ..... 108
4.14 Excitation energies obtained using semi-empirical methods (oeINDO(blue circles) and ZINDO/s (red plus signs) against TDDFT energiesfor $\mathrm{Zn}_{n}(n=3,4,6,8,16,24)$ equilibrium structures . . . . . . . . 109
4.15 Excitation energies obtained using semi-empirical methods against CIS(D) energies for $\mathrm{Zn}_{n}(n=3,4,6,8,16,24)$ equilibrium structures. ..... 110
4.16 Absorption spectra for $\mathrm{Zn}_{3}$ equilibrium geometry obtained using dif- ferent methods.The intensities have been scaled so that the highest intensity is equal to unity ..... 112
4.17 Absorption spectra for $\mathrm{Zn}_{4}$ equilibrium geometry obtained using dif- ferent methods.The intensities have been scaled so that the highest intensity is equal to unity ..... 113
4.18 Absorption spectra for $\mathrm{Zn}_{6}$ equilibrium geometry obtained using dif- ferent methods.The intensities have been scaled so that the highest intensity is equal to unity ..... 114
4.19 Absorption spectra for $\mathrm{Zn}_{8}$ equilibrium geometry obtained using dif- ferent methods.The intensities have been scaled so that the highest intensity is equal to unity. ..... 115
4.20 Absorption spectra for $\mathrm{Zn}_{16}$ equilibrium geometry obtained using different methods.The intensities have been scaled so that the highest intensity is equal to unity ..... 117
4.21 Absorption spectra for $\mathrm{Zn}_{24}$ equilibrium geometry obtained using different methods.The intensities have been scaled so that the highest intensity is equal to unity ..... 118
4.22
Highest peaks against number of units $n$ in equilibrium $\mathrm{Zn}_{n}$ structures ..... 119
4.23 Semi-empirical methods against TDDFT energies for $\mathrm{Cd}_{n}(n=3,4$,$6,8,16,20,25,30)$ equilibrium structures. The straight green linerepresents energies obtained using TDDFT.122
4.24 Absorption spectra for $\mathrm{Cd}_{3}$ equilibrium geometry obtained from dif- ferent methods.The intensities have been scaled so that the highest intensity is equal to unity. ..... 124
4.25 Absorption spectra for $\mathrm{Cd}_{4}$ equilibrium geometry obtained from dif- ferent methods.The intensities have been scaled so that the highest intensity is equal to unity. ..... 125
4.26 Absorption spectra for $\mathrm{Cd}_{6}$ equilibrium geometry obtained from dif- ferent methods.The intensities have been scaled so that the highest intensity is equal to unity ..... 126
4.274.28Absorption spectra of $\mathrm{Cd}_{16}$ equilibrium geometry obtained from dif-ferent methods.The intensities have been scaled so that the highestintensity is equal to unity129
4.29 Absorption spectra for $\mathrm{Cd}_{20}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the high- est intensity is equal to unity ..... 130
4.30 Absorption spectra for $\mathrm{Cd}_{25}$ equilibrium geometry obtained from different methods. The intensities have been scaled so that the high- est intensity is equal to unity. ..... 131
4.31
Absorption spectra for $\mathrm{Cd}_{30}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the high- est intensity is equal to unity. ..... 132
4.32
Highest peaks against number of units $n$ in equilibrium $\mathrm{Cd}_{n}$ structures ..... 133
4.33
Semi-empirical methods against TDDFT energies for $S_{n}(n=5,6,10,20)$ equilibrium structures ..... 135
4.34
Absorption spectra for $S_{3}$ equilibrium geometry obtained from dif- ferent methods.The intensities have been scaled so that the highest intensity is equal to unity ..... 136
4.35
Absorption spectra for $\mathrm{S}_{5}$ equilibrium geometry obtained from dif- ferent methods.The intensities have been scaled so that the highest intensity is equal to unity ..... 137
4.36 Absorption spectra for $S_{6}$ equilibrium geometry obtained from dif- ferent methods.The intensities have been scaled so that the highest intensity is equal to unity ..... 138
4.37 ..... 1404.384.394.404.414.424.43
4.444.45
4.46
intensity is equal to unity. ..... 150
Absorption spectra for $\mathrm{Zn}_{19} \mathrm{~S}_{19}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest
148 intensity is equal to unity. ..... 148
147 intensity is equal to unity
146
intensity is equal to unity.
Absorption spectra for $(\mathrm{ZnS})_{2}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest
144
$2,3,4,10,19)$ equilibrium structures.
141 intensity is equal to unity
Absorption spectra for $S_{20}$ equilibrium geometry obtained from dif- ferent methods.The intensities have been scaled so that the highestScatter plot of $\mathrm{Zn}-\mathrm{S}$ diatomics excitation energies from ZINDO/sand oeINDO against those from EOM-CCSD (the benchmark). Theblue circles are the oeINDO excitation energies while the red plussigns are the ZINDO/s excitation energies. The straight green linerepresents the benchmark.143Absorption spectra for $(\mathrm{ZnS})_{3}$ equilibrium geometry obtained fromdifferent methods.The intensities have been scaled so that the highestAbsorption spectra for $(\mathrm{ZnS})_{4}$ equilibrium geometry obtained fromdifferent methods.The intensities have been scaled so that the highest
Absorption spectra for $\mathrm{Zn}_{10} \mathrm{~S}_{10}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity. ..... 149
Highest peaks against number of units $n$ in equilibrium $(\mathrm{ZnS})_{n}$ struc- tures ..... 151

Scatter plot of Cd-S diatomics excitation energies from ZINDO/s and oeINDO against those from EOM-CCSD (the benchmark). The blue circles are the oeINDO excitation energies while the red plus signs are the ZINDO/s excitation energies. The straight green line represents the benchmark.153
4.48 Semi-empirical methods against TDDFT energies for (CdS) $)_{n}(n=$ $2,3,4,10$ ) equilibrium structures154
4.49 Absorption spectra for $(\mathrm{CdS})_{2}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.156
4.50 Absorption spectra for $(\mathrm{CdS})_{3}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.157
4.51 Absorption spectra for $(\mathrm{CdS})_{4}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity158

Absorption spectra for $(\mathrm{CdS})_{10}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity159
Absorption spectra for $(\mathrm{CdS})_{19}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity ..... 160
4.54 Highest peaks against number of units $n$ in equilibrium $(\mathrm{CdS})_{n}$ structures. The oeINDO curve and not the ZINDO/s curve qualitatively reproduces the TDDFT one.161

Plot comparing excitation energies from semi-empirical methods (ZINDO/s and oeINDO) with those from TDDFT for $\mathrm{Cd}_{5} \mathrm{Zn}_{14} \mathrm{~S}_{19}, \mathrm{Cd}_{10} \mathrm{Zn}_{9} \mathrm{~S}_{19}$ and $\mathrm{Cd}_{15} \mathrm{Zn}_{4} \mathrm{~S}_{19}$.163
Absorption spectra for $\mathrm{Cd}_{5} \mathrm{Zn}_{14} \mathrm{~S}_{19}$ equilibrium structure obtained from different methods. The intensities have been scaled so that the highest intensity is equal to unity. ..... 164
4.57 Absorption spectra for $\mathrm{Cd}_{10} \mathrm{Zn}_{9} \mathrm{~S}_{19}$ equilibrium structure obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity. ..... 165
4.584.59Absorption spectra of $\mathrm{Si}_{n}(n=124,147,172,779)$ equilibrium ge-ometries obtained from oeINDO.The intensities have been scaled sothat the highest intensity is equal to unity.169
4.60 Absorption spectra of Zn quantum dots of different sizes (diameters) obtained from oeINDO.The intensities have been scaled so that the highest intensity is equal to unity. ..... 171
4.61 Absorption spectra of Cd quantum dots of different sizes (diameters) obtained from oeINDO.The intensities have been scaled so that the highest intensity is equal to unity. ..... 174
4.624.63Absorption spectra of ZnS quantum dots of different sizes (diame-ters) obtained from oeINDO.The intensities have been scaled so thatthe highest intensity is equal to unity.179
4.64 Plot of electronic gap for $(\mathrm{CdS})_{n}$ and $(\mathrm{ZnS})_{n}$ quantum dots for dif- ferent number of units, $n$ ..... 180
4.65
Absorption spectra of CdS quantum dots of different sizes (diame- ters) obtained from oeINDO.The intensities have been scaled so that the highest intensity is equal to unity. ..... 1824.66
Absorption spectra of CdZnS quantum dots obtained from oeINDO.The intensities have been scaled so that the highest intensity is equal to unity. ..... 185
B. 1 Equilibrium structures of $\mathrm{Zn}_{n}(n=3,4,6,8,16,24)$. The structures were obtained from geometry optimizations using B3LYP/DEF2TZVPP for small clusters and B3LYP/DEF2-SVP for moderate-sized clusters.217
B. 2 Equilibrium structures of $\mathrm{Cd}_{n}(n=3,4,6,8,16)$. The structures were obtained from geometry optimization using B3LYP/DEF2-TZVPP for small clusters and B3LYP/DEF2-SVP for moderate-sized clusters. 219
B. 3 Equilibrium structures of Zn quantum dots of sizes 1.0, 1.2, 1.4, 1.6, 1.8 and 2.0 nm . The structures were obtained from geometry optimization using PM7 as implemented in MOPAC.221

B. $4 \quad$ Equilibrium structures of S quantum dots of sizes $1.0,1.2,1.4,1.8$
and 2.0 nm . The structures were obtained from geometry optimiza
tion using PM7 as implemented in MOPAC. ..... 226
B. $5 \quad$ Equilibrium structures of Si quantum dots of sizes 1.6, 1.8, 2.0 and 3.0 nm . The structures were obtained from geometry optimization using PM7 as implemented in MOPAC.231
B. 6 Equilibrium structures of ZnS quantum dots of sizes 1.0, 1.2, 1.4, 1.6, 1.8, 2.0 and 2.2 nm . The structures were obtained from geometry optimization using PM7 as implemented in MOPAC.234
B. 7 Equilibrium structures of CdS quantum dots of sizes 1.0, 1.2, 1.4, 1.8 and 2.2 nm . The structures were obtained from geometry optimization using PM7 as implemented in MOPAC.238

## LIST OF ABBREVIATIONS

| ALDA | Adiabatic Local-Density Approximation |
| :---: | :---: |
| AM1 | Austin Model 1 |
| B3LYP | Becke, 3-parameter, LeeYangParr |
| BSE | Bethe Selpeter Equation |
| CASPT2 | Complete Active Space Perturbation Theory |
| CC | Coupled Cluster |
| CCSD | Coupled Cluster |
| CCSDT | Coupled Cluster |
| CIS | Configuration Interaction Singles |
| CISD | Configuration Interaction Singles Doubles |
| CNDO | Complete Neglect of Differential Overlap |
| CNDO/s | Complete Neglect of Differential Overlap for Spectroscopy |
| CASSCF | Complete Active Space Self-Consistent Field |
| DEF2-TZVPP | Valence triple-zeta with two sets of polarization functions |
| DFT | Density Functional Theory |
| DFTB | Density Functional Tight Binding |
| DOS | Density of States |
| EELS | Electron Energy Loss Spectroscopy |
| EMA | Effective Mass Approximation |
| EOM-CCSD | Equation-of-Motion Coupled Cluster Singles Doubles |
| EPM | Empirical Pseudopotential Method |
| GGA | Generalized Gradient Approximation |


| HF | Hartree-Fock |
| :--- | :--- |
| HOMO | Highest Occupied Molecular Orbital |
| INDO | Intermediate Neglect of Differential Overlap |
| INDO/s | Intermediate Neglect of Differential Overlap for Spectroscopy |
| LCAO | Linear Combination of Atomic Orbitals |
| LDA | Local Density Approximation |
| LUMO | Lowest Unoccupied Molecular Orbital |
| MOPAC | Molecular Orbital PACkage |
| MCSCF | Multi-configuration Self-consistent Field |
| oeINDO | Optimized for excitation Intermediate Neglect of Differential Overlap |
| PM3 | Parametric Method 3 |
| PM6 | Parametric Method 6 |
| PM7 | Parametric Method 7 |
| QMC | Quantum Monte Carlo |
| TDDFT | Time Dependent Density Functional Theory |
| UV-Vis | Ultra-violet-Visible |
| ZDO | Zero Differential Overlap |
| ZINDO/s | Zerner Intermediate Neglect of Differential Overlap for Spectroscopy |

## Chapter 1

## INTRODUCTION

### 1.1 Introduction

Novel materials are crucial ingredients for the emerging and next-generation technologies that could impact human life positively. For instance, materials with combined features like cost-effectiveness, lightweight, environmental friendliness, high stability, abundance and energy-effectiveness are needed for photovoltaic devices, electric car batteries, portable electronic devices, high speed and large memory computers, medical devices for diagnosis and treatment of diseased cells, artificial intelligence etc. Nanotechnology offers a lot of possibilities to design new materials. It involves the manipulation and study of materials at the nanometer dimensions where unique phenomena enable novel application. Different nanosystems, such as quantum dots, have been discovered with several applications cutting across disciplines including physics, chemistry, biomedical sciences and engineering.

Quantum dot is a tiny man-made material with a size in the range of $1-100 \mathrm{~nm}$ (about ten-thousandth of the thickness of a hair strand). It is composed of hundreds to thousands of atoms. Quantum dot was discovered in the early 1980s independently by two researchers. Alexei Ekimov, a Russian Physicist, discovered a quantum dot in a glass matrix while Brus Louis in a separate work in the Bells Laboratory, discovered it in a colloidal solution (Brus, 1984, Ekimov et al., 1985). The nano-scale system (quantum dot) is called a zero-dimensional system because it restricts the motion of its particles (electrons and holes) in all spatial directions. Unlike its bulk counterpart, the quantum dot behaves like an atom or molecules with discrete energy levels and states and so, it
is sometimes called an 'artificial atom'. It has remarkably tunable properties such as size-dependent absorption and emission spectra and energy levels. A red-shift (decrease in energy) is observed in the quantum dot spectra and electronic gap as its size increases and a blue-shift (increase in energy) as its size decreases. Another fascinating feature of the quantum dot is the multiexciton generation. This is the ability of the dot to generate more than one excitons per incoming high-energy photon. In contrast, the bulk material generates only one exciton per incoming high-energy photon and loses its excess photon energy as heat. In addition, due to its high extinction coefficient, quantum dot shows descent stability (Soloviev et al., 2001, Nozik, 2008, Rühle et al., 2010, Suri et al., 2013). These unique and exciting features of quantum dots facilitate their wide applications.

Quantum dots are promising materials for cost-effective, portable, environmentally friendly and power-efficient solar cells. Their tunable electronic gap and multiexciton generation features could lead to an optimal harvest of sunlight and the generation of more electrons. Thus, a maximum theoretical conversion efficiency of 66\%, twice that achievable by the conventional solar cells, is possible with quantum dot solar cell (Nozik, 2002, Zhou, 2015). Moreover, the dots tiny size and stability feature could lead to the production of inexpensive, portable and stable solar cells.

Quantum dots are novel materials in biomedical sciences. Their reduced size and tunability characteristics made them useful for the study of single molecules and thus, making them potential material for bio-imaging, diagnosis and treatment of tumours, drug delivery, bio-sensing etc. (Bae et al., 2011, Hubbell and Chilkoti, 2012, Nazir et al., 2014).

Nanodots are materials for more energy-efficient and brighter and purer colour television (TV) displays in the electronic industry. Unlike the conventional displays (organic light-emitting diode displays), which require a backlight to emit their colours, quantum dot TV displays are self-emissive, thus making them more cost-effective. The Sony electronic company in 2013, incorporated quantum dots in their TV displays and recently, the Samsung and LG electronic company researchers are working hard to incorporate dots in their TV displays (Bourzac, 2013).

Studies have shown that nano-sized materials like quantum dots exhibit quantum en-
tanglement, which makes them relevant for the next generation high speed and high computing-power computers (quantum computers) (Predojević, 2016). More efficient, miniaturized and cost-effective detectors, lasers, sensors can be produced with quantum dots. The future high power density (lighter, cheaper, and more efficient) and stable lithium-ion batteries can be realized by incorporating nano-sized materials in the batteries (Zhou, 2015). The list of the potential applications of quantum dots is 'endless' cutting across various disciplines.

Spectroscopy and excitation energies are important tools for the investigation of the properties of low-dimensional materials, such as quantum dots. Spectroscopy is a technique that is concerned with the study of the response of a system when perturbed with a probe (electron, neutron, or electromagnetic radiation). The plot of the system response against the probe wavelength is called a spectrum. Spectroscopy grants one access and insight into the microscopic nature of the system, which avail one a great deal of information about the properties of the system (Gatti, 2007, Marques et al., 2012, Delerue and Lannoo, 2013). Spectroscopy can provide information about the system structure, electronic gap, dielectric, susceptibility, fluorescence, molecular dynamics, geometry, chemical composition and other properties of a material. Most of the properties of molecular systems known today were obtained from spectroscopic studies.

Generally, in spectroscopy, a probe impinges on an electronic system and the system responds by moving from one state to another state (transition). Information about the electronic transition can be accessed from the determination of the energies of the probe and the outgoing particles (photon, electron, etc). Transition in electronic systems, $W$ can be expressed using the Fermi Golden Rule given as

$$
\begin{equation*}
\left.W=2 \pi\left|\left\langle\psi_{f}\right| H_{\text {int }}\right| \psi_{i}\right\rangle\left.\right|^{2} \delta\left(\varepsilon_{f}-\varepsilon_{i}-\hbar \omega\right), \tag{1.1}
\end{equation*}
$$

where $H_{\text {int }}$ is the perturbation and $\hbar \omega$ is the photon energy. $\psi_{i}$ and $\psi_{f}$ are the initial and the final orbitals (wavefunctions), respectively. The $\varepsilon_{i}$ and $\varepsilon_{f}$ are the initial and final orbital energies, respectively. The Fermi Golden Rule expression is a fundamental formula for all spectroscopy techniques.

Numerous spectroscopy techniques have been developed and they are categorized according to the probe employed, the outgoing particles and the conservation or nonconservation of the total number of particles. The different categories are namely, photoemission, inverse photo-emission, photo-absorption and electron loss spectroscopy. In the photoemission spectroscopy technique, the system absorbs a photon (probe) and its electron is excited above the vacuum level. The photo-emission spectrum is specified by the distribution of the photoelectron kinetic energy. Conversely, in the inverse photoemission spectroscopy, the system absorbs electrons (addition of electrons) and emits photons. In both techniques, the total number of electrons is not conserved. In the case of photo-emission, electrons are removed from the system while for inverse photoemission, they are added to the system. For photo-absorption spectroscopy, the system absorbs a photon, which results in electron excitations from a lower energy state to a higher energy state. However, in electron energy loss spectroscopy, electrons are excited in the system, when it is perturbed by an electron. In both photo-absorption and electron energy loss spectroscopy, the total number of electrons is conserved.

The absorption and emission spectroscopy are the first set of techniques developed and are still very relevant to study the properties of systems like nano-particles (Marques et al., 2012). In absorption spectroscopy, different transitions (excitations) are possible depending on the energy range of the electromagnetic radiation absorbed by the electronic system. If the system absorbs radiation in the ultraviolet-visible (UV-VIS) region, an electronic transition (excitation) from a lower energy state to a higher energy state is observed. Absorption in the infra-red region of the electromagnetic radiation spectrum results in the vibrational transition while absorption in the microwave region results in rotational transition.

In UV-VIS absorption spectroscopy, if the photon energy, $E=h \nu$, matches a difference in electron energy levels, $\Delta E=E_{2}-E_{1}$ of the system, the system absorbs the photon and its electron is excited from the lower energy level $E_{1}$ to higher energy level $E_{2}$ (Fig.1.1). Absorption spectra are usually plots of the fraction of the unabsorbed photon (absorbance or absorption coefficient) against the incidence photon energy (Fig. 1.1).


Figure 1.1: (a) Electronic excitation and de-excitation (b) Absorption spectrum absorbance of a system against wavelength of incident photon (Harvey, 2011).

Both theoretical and experimental approaches have been employed for the determination of the electronic excitations and absorption spectra of materials. Although experiment is the most accurate and reliable approach, it sometimes poses some challenges, especially in large atomic clusters and nanostructure materials (Delerue and Lannoo, 2013). On one hand, the cost of infrastructure used in carrying out experiments is high and this could limit their acquisition and usage. On the other hand, experiments could be time-consuming and sometimes involve a high degree of complexity in their procedures. These make it difficult for individual researchers and research groups to carry out cutting-edge research within a short period of time. Consequently, material researchers have sometimes relied on efficient theoretical approaches to the study properties of materials. This approach serves to confirm and complement experimental results. They are useful for making predictions for experimentalists and sometimes help to describe some processes that are intractable in experiments (Gatti, 2007, Barone, 2011, Voityuk, 2013). One of the commonly used theoretical approaches in the study of the properties of materials are those based on quantum mechanical models.

Quantum mechanical modelling of ground-state, excited state and spectra of atomic systems continues to be an active area of research. The modelling method is based on seeking a solution to the Schrödinger equation to obtain state functions and their corresponding energies, which are useful in describing the atomic systems. Quantum mechanical models are basically divided into the ab-initio and semi-empirical model.

The ab-initio methods, also referred to as first principle methods, are methods that do not incorporate adjustable parameters in their model but calculate every term (including all integrals) in the model exactly. Several ab-initio methods based on a different level of theories have been developed. Some of the theories are the density functional theory, Hartree Fock theory, configuration interaction theory, perturbation theory, coupledcluster theory, etc. (Jurs et al., 1990). Density functional theory and the Hartree Fock method have been successfully applied for ground-state calculations. Within a large basis set, they give good results, which are comparable to experiments. They have been employed for the calculations of system structural properties, bond properties, the heat of formation, etc. However, these methods fail in prediction of the electronic gaps, excitation energies and spectra properties of systems. Conversely, ab-initio methods like
equation-of-motion coupled-cluster singles doubles (EOM-CCSD), configuration interactions (CI), configuration interaction singles (CIS), complete active-space second-order perturbation theory (CASPT2), time-dependent density functional theory (TDDFT), etc. have been developed and applied successfully for the calculation of excitation energies and spectroscopy. However, these methods are limited by system size. They are compute-intensive and sometimes prohibitive for large atomic clusters and nanostructure materials. Due to the limitations of these methods, semi-empirical methods have been widely adopted for studying large clusters and nanostructure materials.

Semi-empirical methods are methods built from ab-initio models but some terms in the models, which make calculations expensive are omitted or replaced with simple expressions or adjustable parameters. In most cases, the one-center and two-center (e.g exchange and coulomb interaction) integrals are replaced with adjustable parameters or simple expressions while the higher-center integrals, which are assumed to have insignificant effects, are omitted. The values of adjustable parameters are then obtained from fits to experiments or high-level ab-initio methods. If the calibration of the parameters were accurately carried out, the obtained optimised parameters are capable of correcting errors introduced into the semi-empirical model due to the drastic approximations to the ab-initio model. Another drastic approximation in the semi-empirical methods is the expansion of their wave functions with minimal basis sets (the valence orbitals only) and the use of only valence electrons. Several semi-empirical methods have been developed, which include Empirical Pseudopotential Method (EPM), Effective Mass Approximation (EMA), Huckel model, extended Huckel model, intermediate neglect of differential overlap (INDO) based methods, neglect of diatomic of differential overlap (NDDO) based methods, and complete neglect of differential overlap (CNDO) based methods, Tight binding, Density Functional Tight Binding (DFTB), etc. Most of these semi-empirical methods were developed for ground-state calculations but a few including INDO/s have been developed uniquely for excitations and spectra calculations.

### 1.2 Statement of Problem

Theoretical modelling and determination of excitation energies and absorption spectra play important roles in the study and development of novel materials like quantum dots (Delerue and Lannoo, 2013, Voityuk, 2013). The recent powerful computing resources and high-level algorithm, have made it possible for the theoretical approaches to reproduce experimental results with good accuracy and less effort. Moreover, while the results from theoretical approaches normally complement the information obtained by experiment, it can in some cases enable prediction of hitherto unobserved chemical phenomena.

Both ab-initio and semi-empirical theoretical approaches have been developed for excitation energy and absorption spectra calculations. However, despite the available powerful computing resources, high-level and accurate ab-initio methods like GW/BSE, EOMCCSD, CASPT2, Quantum Monte Carlo (QMC) are restricted to small systems (Delerue and Lannoo, 2013). As the system grows bigger, these methods become computationally expensive and sometimes prohibitive. In particular, they are prohibitive for large atomic clusters and quantum dots. Alternative methods to the high-level methods are the TDDFT and configuration interaction singles with perturbative doubles (CIS(D)). They could handle moderate-sized systems ( $\leq 100$ atoms) but are compute-intensive for larger clusters and quantum dots (Voityuk, 2013, Gieseking et al., 2016).

Due to the limitation of the accurate ab-initio methods, researchers have adopted semiempirical methods for calculations in large clusters and quantum dots. Although semiempirical methods are computationally cheap, they are generally less accurate and less transferable compared to the high-level ab-initio methods. In particular, INDO/s, a Hartree Fock based semi-empirical method, has been uniquely developed and widely applied for excitations and spectra calculations. However, due to drastic approximations and poor parametrization, INDO/s like any other semi-empirical approach, is generally less accurate and less transferable. (Jelski and George, 1999, Bredow and Jug, 2005, Thiel, 2014, Husch et al., 2018).

Reports have shown that, with the recent powerful computing resources and reliable data, improvement of the semi-empirical models could produce new models capable
of giving results comparable to those from experiments or high-level methods. While the NDDO, a ground-state semi-empirical model, has received significant improvement since its development in 1977 (Silva-Junior and Thiel, 2010, Stewart, 2013, Dral et al., 2016), INDO/s has not been significantly improved upon. The INDO/s parameters for several elements, especially the transition metal ones, are considered unreliable due to insufficient and inaccurate training data sets used during its parameterisation (Voityuk, 2013).

There is a need to develop computationally cheap theoretical methods for the accurate calculation of excitation energies and absorption spectra of large systems such as quantum dots.

### 1.3 Research Justification

Quantum dot is a novel material with a lot of untapped potentials. It is a promising candidate for a cost-effective, stable and more efficient solar cell. Reports have shown that quantum dot solar cells could convert sunlight to electricity with an efficiency greater than $65 \%$ (Nozik, 2002, Service, 2008). The quantum dot solar cells appeared as new technology on the National Renewable Energy Laboratory (NREL) chart in 2010 with a starting conversion efficiency of less than $4 \%$ and by 2019, the conversion efficiency has reached $16.6 \%$ (NREL, 2019). Evidently, with continuous and intense research into the quantum dot solar cell, an efficiency above $65 \%$ may be achieved in a short period of time. Quantum dot is a useful material for energy-efficient, cost-effective and excellent colour quality large displays (e.g TV display). However, a major setback is the use of cadmium-based and non-self-emissive phosphor quantum dots in the displays. Currently, researchers are working hard for new dot materials, that are heavy metal (cadmium) free and self-emissive (Brazis, 2017, Won et al., 2019). Quantum dots are potential materials for the next-generation quantum computers (fast and high storage powerful computing system) because they exhibit a phenomenon called quantum entanglement (an ability to localize single electron and give single-photon pulses) (Imamoglu, 2003, Predojević, 2016). Neuroscientists and biomedical scientists have discovered the novelty in quantum dots and are exploring them for possible applications in vivo and in vitro imaging, bio-
sensing, diagnosis and therapy of cancer and drug delivery (Saadeh et al., 2014, Abbasi et al., 2016). The list of potential applications of quantum dots is 'inexhaustible', cutting across various disciplines.

To harness the numerous potentials of quantum dots, access to and studies of their microscopic nature are required. To achieve these, accurate theoretical investigation of excitation energies and absorption spectra of these nanosystems are crucial (Varsano, 2006, Gatti, 2007, Delerue and Lannoo, 2013).

The existing high-level ab-initio methods like EOM-CCSD, CASPT2, GW/BSE for obtaining accurate excitation energies and absorption spectra are computationally expensive for quantum dots (Jin and Yang, 2019). For instance, the EOM-CCSD is known to compare very well with the experiment but is restricted to a few atom-systems ( $\leq 10$ atoms). The semi-empirical methods, however, are computationally cheap but are generally less accurate and less transferable as compared to the high-level methods (Jelski and George, 1999, Bredow and Jug, 2005, Thiel, 2014, Husch et al., 2018).

Thus, there is a need to develop a computationally cheap, reliable and accurate semiempirical approach for the computation of excitation energies and absorption spectra of large atomic clusters and quantum dots.

### 1.4 Aim and Objectives

The aim of the research was to develop a computationally cheap method capable of yielding accurate excited state energies and ultra-violet-visible (UV-Vis) absorption spectra of quantum dots.

The objectives are to:

1. parameterise a semi-empirical Hamiltonian model with diatomic data from a highlevel ab-initio method and to validate it.
2. predict excited state energies and UV-Vis absorption spectra of large atomic clusters and quantum dots using the newly obtained Hamiltonian model.
3. predict a theoretical optimal quantum dot size/shape for solar cell applications.

### 1.5 Outline of Thesis

The thesis is organized as follows. The next chapter presents the literature review, which covers the basics of nanomaterials and theoretical spectroscopy. It also contains the review of different approaches for the calculation of electronic excitations and absorption spectra of different systems including quantum dots. Chapter three gives detail of the tools used and steps followed in carrying out the present work. In chapter four, the results of the work are presented in graphical and tabular form and discussed in details. Conclusion and recommendations were presented in chapter five.

## Chapter 2

## THEORETICAL BACKGROUND AND LITERATURE REVIEW

### 2.1 Nanomaterials

Nanomaterials are materials with length scale in the range 1-100 nm, which lies between bulk materials and molecules. Within this length scale, the quantum mechanical laws come to play and the materials experienced an effect called the quantum confinement. This effect is responsible for the unique and exciting features exhibited by the nanomaterials. In the nanomaterials, particles (electrons and holes) are restricted along the direction the confinement effect is experienced. The effect gives rise to the quantization of the energy levels and states along the direction of the confinement. A material that restricts its particles along one direction is a two-dimensional material referred to as quantum well. When particles are restricted along two directions, a one-dimensional material called nanowire or nanotube is obtained. The restriction of particles in a material along all direction gives rise to a zero-dimensional material called a quantum dot. Quantum dots are sometimes referred to as 'artificial atoms' because they possess atomic-like optical and electronic properties (discrete energy levels and states). Quantum dot is one of the most prominent among the nanomaterials with a lot of potential of applications that cut across various disciplines (Kuno, 2005).

### 2.1.1 Quantum Dot: Confinement effect and quantization of energy and state

Quantum dot was discovered in the early '80s by Brus and Ekimov (Ekimov and Onushchenko, 1982, Rossetti et al., 1983). Since then, it has attracted the attention of researchers be-
cause of its amazing features, which are different from those of its bulk counterpart. Due to the confinement effect (Yoffe, 1993), quantum dot exhibits atomic-like properties - its energy levels and states are quantized. So, a quantum dot is sometimes referred to as an 'artificial atoms'. In addition, the properties of quantum dot can be tuned by changing its size or shape. For instance, the electronic gap of the dot is red-shifted as its size increases and blue-shifted as its size decreases.

An important effect in low-dimensional materials like quantum dot is the quantum confinement effect. It is an effect observed in a material, when the physical size of the material is less than or equal to its bulk exciton Bohr radius (the length of the electronhole pair bound) . The exciton Bohr radius, $a_{e x}$ is expressed as (Yoffe, 1993)

$$
\begin{equation*}
a_{e x}=\frac{4 \pi \varepsilon \hbar^{2}}{m_{e f f} e^{2}}, \tag{2.1}
\end{equation*}
$$

where $\frac{1}{m_{e f f}}=\frac{1}{m_{e}}+\frac{1}{m_{h}}$ and $m_{e f f}$ is the effective mass, $\varepsilon$ is the dielectric constant of the material, $\hbar$ is the Planck constant, e is the electronic charge, $m_{e}$ is the mass of an electron and $m_{h}$ is the mass of hole. Quantum confinement effect in material results in the quantization of the energies and states of the material. A qualitative description of the confinement effect will be explored using a particle in the box model (idealized quantum dot) and effective mass approximation. Considering an electron confined along the x -axis in an infinite square well, the potential, $V(x)$ in the different regions of the well (2.1) is defined as follows:

$$
V(x)= \begin{cases}\infty, & \text { if } x \leq 0  \tag{2.2}\\ 0, & \text { if } 0<x<L \\ \infty, & \text { if } x \geq L\end{cases}
$$



Figure 2.1: A one-dimensional infinite potential square well. The regions I and III are barriers with infinite potentials while the region II is a well with potential equal to zero.

The one-dimensional Schrödinger equation in equation 2.3 is solved for electron in each of the regions;

$$
\begin{equation*}
-\frac{\hbar^{2}}{2 m} \nabla^{2} \psi(x)+V(x) \psi(x)=E \psi(x) \tag{2.3}
\end{equation*}
$$

In the barrier regions, $I$ and $I I I$,

$$
\begin{gathered}
V(x)=\infty \\
\psi_{I}(x)=\psi_{I I}(x)=0
\end{gathered}
$$

Inside the well, region $I I, \mathrm{~V}(\mathrm{x})=0$ and the Schrödinger equation becomes

$$
\begin{equation*}
\left(\nabla^{2}+k^{2}\right) \psi(x)=0, \tag{2.4}
\end{equation*}
$$

where $k^{2}=\frac{2 m E}{\hbar^{2}}$. The general solution is

$$
\begin{equation*}
\psi(x)=A \cos (k x)+B \sin (k x) . \tag{2.5}
\end{equation*}
$$

Applying the boundary conditions, $\psi_{I}(0)=\psi_{I I}(0)$ at $x=0$ and $\psi_{I}(L)=\psi_{I I}(L)$ at $x=L$. Then, the particular solution becomes

$$
\begin{equation*}
\psi_{n(x)}=\sqrt{\frac{2}{L}} \sin \left(\frac{n \pi x}{L}\right) \tag{2.6}
\end{equation*}
$$

and

$$
\begin{equation*}
E_{n}=\frac{n^{2} \pi^{2} \hbar^{2}}{2 m L^{2}}, \tag{2.7}
\end{equation*}
$$

where $n=1,2,3, \ldots m$ is index of the discretized energy levels and states.

For confinement along all three directions (ideal quantum dot),

$$
\begin{gather*}
\psi_{n(x, y, z)}=\sqrt{\frac{8}{L_{x} L_{y} L_{z}}} \sin \left(\frac{n_{x} \pi x}{L_{x}}\right) \sin \left(\frac{n_{y} \pi y}{L_{y}}\right) \sin \left(\frac{n_{z} \pi z}{L_{z}}\right)  \tag{2.8}\\
E_{n_{x} n_{y} n_{z}}=\frac{n_{x}^{2} \pi^{2} \hbar^{2}}{2 m L_{x}^{2}}+\frac{n_{y}^{2} \pi^{2} \hbar^{2}}{2 m L_{y}^{2}}+\frac{n_{z}^{2} \pi^{2} \hbar^{2}}{2 m L_{z}^{2}} \tag{2.9}
\end{gather*}
$$

where $n_{x}, n_{y}, n_{z}=1,2,3, \ldots$

The energy and state of the levels (Equations (2.8) and (2.9) are quantized since the quantum numbers, $n_{i}$ are quantized. The lowest energy, $E_{111}$ corresponds to the first sorbital energy of an atom. The first excited states ( $E_{211}, E_{121}, E_{112}$ ), which are degenerate (the different energy levels are described by the same state function) correspond to the 3 degenerate p-orbital energies in an atom. These show that the quantum dot behaves like an atom. In addition, the energy levels increase as the size of the quantum dot decreases. The density of state of quantum dot states is a spike function expressed as:

$$
\begin{equation*}
\operatorname{DOS}(E)=2 \delta\left(E-E_{c}\right) . \tag{2.10}
\end{equation*}
$$

Equation (2.10) and figure (2.2) show that the states of quantum dot exist only at discrete energies in contrast to the bulk system, which has a continuous energy state (Kuno, 2005, Delerue and Lannoo, 2013).


Figure 2.2: Electronic density of states of systems with 3, 2, 1, and 0 degrees of freedom. These systems with $3,2,1$, and 0 degrees of freedom are referred to as bulk (3D), quantum well or sheet (2D), quantum wire (1D) and quantum dot, respectively (Kuno, 2005)

### 2.2 Theoretical spectroscopy

Spectroscopy is an essential tool for the study of electronic and optical properties of materials. It is a powerful tool that gives one access to the microscopic nature of the materials (Varsano, 2006, Delerue and Lannoo, 2013), from which information about the microscopic states, structures, bonds, conformation, chemical properties of the materials can be obtained. Within the spectroscopic measurements, a probe (e.g electron, electromagnetic, neutron) is impinged on a material. The response of the material to this probe is measured and plotted as a function of the probe frequency or wavelength. This plot is referred to as a spectrum. In particular, in absorption spectroscopy, the probe is an electromagnetic radiation and the system responds to the probe by moving from a lower energy state to a higher energy state.

Electromagnetic radiation is a wave composed of an electric and a magnetic field component whose directions are perpendicular to each other and mutually perpendicular to the direction of the propagation of the wave. If the electric field is along, the $x$-axis, $E_{x}$ and the magnetic field is along the y -axis, $H_{y}$ then,

$$
\begin{gather*}
E_{x}=E_{0} \sin (\omega t-k z)  \tag{2.1}\\
H_{y}=H_{0} \sin (\omega t-k z)  \tag{2.2}\\
\omega=2 \pi \nu \\
k=\frac{2 \pi}{\lambda}
\end{gather*}
$$

, where $E_{0}$ and $H_{0}$ are the electric field and magnetic amplitude respectively. $\lambda$ and $v$ are the wavelength and frequency of the wave. The relationship between the energy, E, $\lambda$ and $v$ is expressed given by

$$
\begin{equation*}
E=h v=\frac{h c}{\lambda} \tag{2.3}
\end{equation*}
$$

where $h$ and $c$ are the Planck constant and the speed of light, respectively. The radiations range from short wavelength gamma to the long wavelength, radio wave. However, for transition in a material, the most useful parts of the spectrum are the x-ray, ultraviolet, visible, infrared and microwave. When a material absorbs electromagnetic waves, three
important transitions are possible namely, electronic excitation or transition, vibrational transition and rotational transition (Marques et al., 2012).

The interest in this study is the electronic transitions or excitations. Electronic transitions are observed when a material absorbs electromagnetic radiation in the ultraviolet-visible region of the spectrum. This transition occurs when the energy difference of the states involved in the transition matches an energy ultraviolet-visible electromagnetic radiation range, defined by equation (2.3).

### 2.2.1 Time Dependent Perturbation Theory: Transition Probability and Absorption Coefficient

When system is given a small perturbation, the total Hamiltonian of the system can be expressed as

$$
\begin{equation*}
H=H_{0}+W(t), \tag{2.4}
\end{equation*}
$$

where $H_{0}$ is the Hamiltonian of the unperturbed system and $W(t)$ is the small timedependent perturbation. The full time-dependent Schrödinger equation becomes

$$
\begin{equation*}
i \hbar \frac{\partial \Psi(r, t)}{\partial t}=H \Psi(r, t)=\left[H_{0}+W(t)\right] \Psi(r, t) . \tag{2.5}
\end{equation*}
$$

The solution to equation (2.5) is written as

$$
\begin{equation*}
\Psi(r, t)=\sum_{n} C_{n}(t) \Phi_{n}^{(0)}(r, t), \tag{2.6}
\end{equation*}
$$

where $C_{n}$ is the linear combination constant. The $\Phi_{n}^{(0)}(r, t)$ is the solution to the timedependent Schrödinger equation for the Zeroth- order Hamiltonian (unperturbed Hamiltonian), which is expressed as

$$
\begin{equation*}
\Phi_{n}^{(0)}(r, t)=\exp \left(\frac{-i E_{n}^{(0)} t}{\hbar}\right) \phi_{n}^{(0)}(r, t) . \tag{2.7}
\end{equation*}
$$

Substituting 2.6 into 2.5 gives

$$
\begin{equation*}
i \hbar \sum_{n} \frac{\partial C_{n}(t) \Phi_{n}^{(0)}(r, t)}{\partial t}=\sum_{n} C_{n}(t) H_{0} \Phi_{n}^{(0)}(r, t)+\sum_{n} C_{n}(t) W(t) \Phi_{n}^{(0)}(r, t) . \tag{2.8}
\end{equation*}
$$

But, $H_{0}(r, t) \Phi_{n}^{(0)}(r, t)=E_{n}^{(0)} \Phi_{n}^{(0)}(r, t)$. Hence, equation (2.8) becomes

$$
\begin{equation*}
i \hbar \sum_{n} \frac{\partial C_{n}(t) \Phi_{n}^{(0)}(r, t)}{\partial t}=\sum_{n} C_{n}(t) E_{n}^{(0)} \Phi_{n}^{(0)}(r, t)+\sum_{n} C_{n}(t) W(t) \Phi_{n}^{(0)}(r, t) \tag{2.9}
\end{equation*}
$$

Multiplying equation (2.9) through with $\Phi_{m}^{(0)} *$, integrating with respect to r and orthonormalising of $\phi_{n}^{(0)}$ gives

$$
\begin{equation*}
i \hbar \frac{\partial C_{n}(t)}{\partial t}=\sum_{n} C_{n}(t) \exp \left\{i \frac{E_{m}^{(0)}-E_{n}^{(0)}}{\hbar}\right\} W_{m n}(t) \tag{2.10}
\end{equation*}
$$

where $W_{m n}(t)=\left\langle\Phi_{m}^{(0)}\right| W(t)\left|\Phi_{n}^{(0)}\right\rangle$.
For $\left|C_{m}\right| \ll\left|C_{n}\right|,\left|C_{m}\right| \approx 1$. Hence, equation (2.10) becomes

$$
\begin{equation*}
i \hbar \frac{\partial C_{n}(t)}{\partial t}=\exp \left(i \omega_{m n} t\right) W_{m n}(t) \tag{2.11}
\end{equation*}
$$

where $\omega_{m n}=\frac{E_{m}^{(0)}-E_{n}^{(0)}}{\hbar}$. Re-arranging equation (2.11) gives

$$
\begin{equation*}
C_{m}(t)=\frac{1}{i \hbar} \int_{t_{0}}^{t} d \tau \exp \left(i \omega_{m n} \tau\right) W_{m n}(t) \tag{2.12}
\end{equation*}
$$

$\left|C_{m}(t)\right|^{2}$ is the probability that the system at time, t moves from an initial state $\phi_{n}$ to final state $\phi_{m}$. If a system is perturbed by weak electromagnetic radiation, the electric field component is responsible for the interaction and is expressed as

$$
\begin{equation*}
\vec{E}=E_{0} \hat{z} \cos (\omega t+k x) \tag{2.13}
\end{equation*}
$$

where $\omega=2 \pi f$ and $k=\frac{2 \pi}{\lambda}$. The field is assumed to be polarized along the z -axis and the E-M propagated along x -axis. The potential of the electromagnetic field is expressed as

$$
\begin{equation*}
V=-\sum_{i} Q_{i} z_{i} E_{z}=-\sum_{i} z_{i} Q_{i} E_{0} \cos \left(\omega t+k x_{i}\right) . \tag{2.14}
\end{equation*}
$$

Assuming $k x_{i}=0$, then,

$$
\begin{equation*}
V(z, t)=W(z, t)=-E_{0} \sum_{i} z_{i} Q_{i} \cos (\omega t) . \tag{2.15}
\end{equation*}
$$

The transition probability due to weak interaction of electromagnetic wave with a system is

$$
\begin{equation*}
C_{m}(t)=-\frac{1}{i \hbar} E_{0}\left\langle\phi_{m}\right| \sum_{i} z_{i} Q_{i}\left|\phi_{n}\right\rangle \int_{t_{0}}^{t} d \tau \exp \left(i \omega_{m n} \tau\right) \cos (\omega \tau) . \tag{2.16}
\end{equation*}
$$

If electronic transition dipole is defined as

$$
\begin{equation*}
d_{m n}=\left\langle\phi_{m}\right| \sum_{i} z_{i} Q_{i}\left|\phi_{n}\right\rangle, \tag{2.17}
\end{equation*}
$$

Then, equation (2.16) can be expressed as (Atkins and Friedman, 2011)

$$
\begin{equation*}
C_{m}(t)=-\frac{1}{i \hbar} E_{0} d_{m n} \int_{t_{0}}^{t} d \tau \exp \left(i \omega_{m n} \tau\right) \cos (\omega \tau) \tag{2.18}
\end{equation*}
$$

$\left|C_{m}(t)\right|^{2}$ can be reduced to (Atkins and Friedman, 2011);

$$
\begin{equation*}
\left|C_{m}(t)\right|^{2}=\frac{2 t \pi E_{0}}{\hbar^{2}}\left|d_{m n}\right|^{2} \delta\left(\omega_{n m}-\omega\right) \tag{2.19}
\end{equation*}
$$

Transition probability rate:

$$
\begin{equation*}
P_{n \rightarrow m}=\frac{2 \pi E_{0}^{2}}{\hbar^{2}}\left|d_{m n}\right|^{2} \delta\left(\omega_{n m}-\omega\right) \tag{2.20}
\end{equation*}
$$

Equation (2.20) is the well known Golden rule formula (Griffiths, 1995, Schiff, 1995). Radiation field energy loss per unit time due to the absorption of a single photon is expressed as

$$
\begin{equation*}
-\frac{\partial E}{\partial t}=\sum_{n m} \hbar P_{n \rightarrow m} \tag{2.21}
\end{equation*}
$$

and

$$
E=\frac{\varepsilon_{0} \vec{E}_{0}}{2}
$$

The optical absorption coefficient is proportional to (Struve, 1989)

$$
\begin{equation*}
\varepsilon(\omega)=-\frac{\frac{\partial E}{\partial t}}{\omega \vec{E}}=\frac{4 \pi^{2}}{\hbar c} \omega\left(1-e^{-\beta \hbar \omega}\right) \sum_{n m} P_{n}\left|d_{n m}\right|^{2} \delta\left(\omega_{n m}-\omega\right) \tag{2.22}
\end{equation*}
$$

Integrated absorption coefficient, $A$ is expressed in terms of molar extinction coefficient, $\varepsilon(v)$ as (Atkins and Friedman, 2011)

$$
\begin{equation*}
A=\int_{v_{n}}^{v_{m}} \varepsilon(v) d v \tag{2.23}
\end{equation*}
$$

where $v, v_{n}$ and $v_{m}$ are the frequencies of the photon, state $n$ and state $m$, respectively. The dimensionless oscillator strength, $f_{n m}$, which is used to quantify the intensity of absorption is express as (Struve, 1989, Atkins and Friedman, 2011):

$$
\begin{equation*}
f_{n m}=\frac{4 \pi m_{e} v_{n m}}{3 \hbar e^{2}}\left|d_{n m}\right|^{2} \tag{2.24}
\end{equation*}
$$

The relation between $f_{n m}$ and $A$ can be obtained as (Atkins and Friedman, 2011)

$$
\begin{equation*}
f_{n m}=\frac{4 \varepsilon_{0} m_{e} c}{N_{A} e^{2}} \int_{v_{n}}^{v_{m}} \varepsilon(v) d v \tag{2.25}
\end{equation*}
$$

In practice (Atkins and Friedman, 2011),

$$
\begin{equation*}
f_{n m}=6.26 \times 10^{-19} \int_{v_{n}}^{v_{m}} \varepsilon(v) d v=6.26 \times 10^{-19} A \tag{2.26}
\end{equation*}
$$

A transition is forbidden if oscillator strength, $f_{n m}$ is zero and allowed otherwise.

### 2.2.2 Polarizability and absorption coefficient

In the response theory, when a weak time-dependent electric field $V_{\text {ext }}$ interacts with a system of electrons, the relationship between the change in electron density, $\delta n\left(r^{\prime}, \omega\right)$ (dynamic polarizability), dynamic susceptibility, $\chi\left(r, r^{\prime}, \omega\right)$ and the $V_{e x t}(r, \omega)$ is given by

$$
\begin{equation*}
\delta n\left(r^{\prime}, \omega\right)=\int d r \chi\left(r, r^{\prime}, \omega\right) V_{e x t}(r, \omega) \tag{2.27}
\end{equation*}
$$

It can be shown that the polarizability, $P$, the complex function of the susceptibility, $\chi$ and complex function of the dielectric constant, $\varepsilon$ are related by

$$
\begin{equation*}
P=1+\chi+i \chi^{\prime}=\varepsilon_{1}+i \varepsilon_{2} \tag{2.28}
\end{equation*}
$$

Equating the imaginary parts of equation (2.28), one obtains

$$
\begin{equation*}
\chi^{\prime}=\varepsilon_{2} . \tag{2.29}
\end{equation*}
$$

The absorption coefficient is written given by

$$
\begin{equation*}
\alpha(\omega)=\frac{\omega \varepsilon_{2}}{n_{1} c}, \tag{2.30}
\end{equation*}
$$

where $n_{1}$ is the real part of the complex refractive index, $\omega$ is the circular frequency, and the c is the speed of light. The quantity $\varepsilon_{2}$ is defined as

$$
\begin{equation*}
\varepsilon_{2}(\omega) \propto \sum_{i, j}\left\langle\psi_{j}\right| \mathbf{e} \cdot \mathbf{P}\left|\psi_{i}\right\rangle \delta\left(E_{j}-E_{i}-\hbar \omega\right), \tag{2.31}
\end{equation*}
$$

where $\omega$ is the frequency of the electromagnetic and $\mathbf{e}$ is polarization. The $\psi_{i}, \psi_{j}, \mathrm{E}_{i}$ and $\mathrm{E}_{j}$ are state of the occupied orbital, state of the unoccupied orbital, energy of the occupied orbital and energy of the unoccupied orbital (Marques et al., 2012).

### 2.3 Theoretical approaches for excitation energies of atomic clusters

Excitation energies and their corresponding states are important tools in absorption spectrum calculation (Gatti, 2007). From the earlier discussion on absorption spectroscopy, it is evident that, the accuracy of the absorption spectrum of an electronic system depends on the accurate determination of its orbital energies and their corresponding states. These quantities can be obtained by solving exactly the many-body Schrödinger equation, which is expressed as (Martin, 2020)

$$
\begin{equation*}
H \Psi(r, R)=E \Psi(r, R) \tag{2.1}
\end{equation*}
$$

where the many-body Hamiltonian, $H$ is defined as

$$
\begin{equation*}
H=-\frac{1}{2} \sum_{i} \nabla^{2}-\frac{1}{2} \sum_{I} \nabla^{2}-\sum_{I} \sum_{i} \frac{1}{\left|r_{i}-R_{I}\right|}+\sum_{i} \sum_{j} \frac{1}{\left|r_{i}-r_{j}\right|}+\sum_{I} \sum_{J} \frac{1}{\left|R_{I}-R_{J}\right|} . \tag{2.2}
\end{equation*}
$$

The first term on the R.H.S of equation (2.2) is the total kinetic energy of the electrons. The second term is the total kinetic energy of the nuclei, third term defines the nuclearelectron interactions, fourth term is the electron-electron interactions and the fifth is the nucleus-nucleus repulsion interactions. $\Psi(r, R)$ is the many body wave function or the eigenfunction and $E$ the eigenvalue (total energy). $r$ and $R$ are the position vectors of the electron and the nucleus, respectively. The solution to the many-body equation (equation (2.1)) for a system of atoms is intractable and this led to the introduction of the Born Oppenheimer approximation (Born and Oppenheimer, 1927). In the Born approximation, it is assumed that the nuclei are stationary with reference to the motion of electrons and thus, kinetic energy of the nuclei are neglected and the nuclei-nuclei repulsions are treated as constants. With these approximations, the Hamiltonian in equation (2.2) reduces to (Martin, 2020)

$$
\begin{equation*}
H_{e}=-\frac{1}{2} \sum_{i} \nabla^{2}-\sum_{I} \sum_{i} \frac{1}{\left|r_{i}-R_{I}\right|}+\sum_{i} \sum_{j} \frac{1}{\left|r_{i}-r_{j}\right|}, \tag{2.3}
\end{equation*}
$$

where $H_{e}$ is the electronic Hamiltonian. Thus, the electronic eigenvalue problem is

$$
\begin{equation*}
H_{e} \Psi(r, R)=E_{e} \Psi(r, R) \tag{2.4}
\end{equation*}
$$

Also, the total energy, E for a particular nuclei configuration, is written as

$$
\begin{equation*}
E=E_{e}+\sum_{I} \sum_{J} \frac{1}{\left|R_{I}-R_{J}\right|}, \tag{2.5}
\end{equation*}
$$

where $E_{e}$ is the electronic eigenvalues (orbital energies). Unfortunately, obtaining the solution to the electronic eigenvalue problem (equation (2.4)) is still a difficult task because of the electron-electron interaction term in the electronic Hamiltonian and the many-body wave-function. Hence, further approximations were introduced, which led to the development of different approximate methods. The key and fundamental among these approximate methods are the wave function methods and Density Functional methods.

### 2.3.1 Wave function methods: Hartree Fock and Post Hatree Fock methods

The Hartree Fock (HF) is a single particle approximation method, which assumes each electron in a system is moving in the mean-field of the other electrons in the system. The HF wave-function for N electrons is expressed as a Slater determinant in order to ensure that the antisymmetric principle for fermions(e.g electrons) is obeyed. The HF wave-function, $\Psi$ is then expressed as

$$
\Psi=\frac{1}{\sqrt{N!}}\left|\begin{array}{ccccc}
\chi_{1}\left(x_{1}\right) & \chi_{1}\left(x_{2}\right) & \chi_{1}\left(x_{3}\right) & \ldots & \chi_{1}\left(x_{N}\right)  \tag{2.6}\\
\chi_{2}\left(x_{1}\right) & \chi_{2}\left(x_{2}\right) & \chi_{2}\left(x_{3}\right) & \ldots & \chi_{2}\left(x_{N}\right) \\
\chi_{3}\left(x_{1}\right) & \chi_{3}\left(x_{2}\right) & \chi_{3}\left(x_{3}\right) & \ldots & \chi_{3}\left(x_{N}\right) \\
\chi_{4}\left(x_{1}\right) & \chi_{4}\left(x_{2}\right) & \chi_{4}\left(x_{3}\right) & \ldots & \chi_{4}\left(x_{N}\right) \\
\cdot & & & & \\
\cdot & & & & \\
\cdot & & & & \\
\chi_{N}\left(x_{1}\right) & \chi_{N}\left(x_{2}\right) & \chi_{N}\left(x_{3}\right) & \ldots & \chi_{N}\left(x_{N}\right)
\end{array}\right|
$$

Compactly, $\Psi$ can be written as

$$
\begin{equation*}
\Psi=\left|\chi_{1}\left(x_{1}\right) \chi_{2}\left(x_{2}\right) \chi_{3}\left(x_{3}\right) \ldots \chi_{N}\left(x_{N}\right)\right\rangle, \tag{2.7}
\end{equation*}
$$

where $\chi(x)$ are spin orbitals and $x$ represent both the electron space and spin coordinates. The spin orbitals also called the molecular orbitals are expressed as a linear combination of atomic orbitals, given by

$$
\begin{equation*}
\chi_{i}\left(x_{i}\right)=\sum_{m} C_{i, m} \phi_{m}\left(x_{i}\right), \tag{2.8}
\end{equation*}
$$

where $\phi_{m}$ is the atomic orbitals and $C_{i m}$ are the molecular orbital coefficient. If the wave-function is normalized, the expectation value of the energy (Hartree Fock energy) is

$$
\begin{equation*}
E=\langle\Psi| H|\Psi\rangle=\sum_{i} H_{i i}+\frac{1}{2} \sum_{i j}\left(J_{i j}-K_{i j}\right) \tag{2.9}
\end{equation*}
$$

where $H_{i i}$ is defined by

$$
\begin{equation*}
H_{i i}=\langle\chi(1)|-\frac{1}{2} \nabla^{2}-\sum_{R} \frac{1}{\left|r_{i}-R\right|}|\chi(1)\rangle, \tag{2.10}
\end{equation*}
$$

$J_{i j}$ is the coulomb interaction between two electrons with spin orbitals $\chi_{i}$ and $\chi_{j}$ is expressed as

$$
\begin{equation*}
J_{i j}=\left\langle\chi_{i}(1) \chi_{j}(2)\right| \frac{1}{\left|r_{1}-r_{2}\right|}\left|\chi_{i}(1) \chi_{j}(2)\right\rangle \tag{2.11}
\end{equation*}
$$

and $K_{i j}$, the exchange interaction is expressed as

$$
\begin{equation*}
K_{i j}=\left\langle\chi_{i}(1) \chi_{j}(2)\right| \frac{1}{\left|r_{1}-r_{2}\right|}\left|\chi_{i}(2) \chi_{j}(1)\right\rangle . \tag{2.12}
\end{equation*}
$$

The variational principle is applied to equation 2.3.1 in order to obtain the best wave function that minimizes the HF Energy, E. The minimization of energy, E with respect to spin orbital give rise to an eigenvalue equation known as the Hartree Fock equation. The equation is expressed as (Magnasco, 2009, Szabo and Ostlund, 2012)

$$
\begin{equation*}
F(i) \chi_{i}(1)=\varepsilon_{i} \chi_{i}(1) \tag{2.13}
\end{equation*}
$$

where $F(i)$, the one-particle operator called the Fock operator, is defined by

$$
\begin{equation*}
F(i)=-\frac{1}{2} \nabla_{i}^{2}-\sum_{R} \frac{1}{\left|r_{i}-R\right|}+\sum_{i \neq j}\left\langle\chi_{j}(2)\right| \frac{1}{\left|r_{1}-r_{2}\right|}\left|\chi_{i}(2)\right\rangle-\sum_{i \neq j}\left\langle\chi_{j}(2)\right| \frac{1}{\left|r_{1}-r_{2}\right|}\left|\chi_{i}(2)\right\rangle, \tag{2.14}
\end{equation*}
$$

$\varepsilon_{i}$ and $\chi_{i}(1)$ are the orbital energies and orbitals, respectively. Hartree Fock equation is a self-consistent field theory (SCF) since it is non-linear and needed to be solved iteratively. It solution gives rise to orbitals and their corresponding energies.

The energy obtained from Hartree Fock equation is an upper bound of the exact energy because of some missing correlation (an effect whereby electrons try to avoid each) in the wave function (the single Slater determinant) used. In the Hartree Fock formalism, the parallel spin electrons are well correlated while anti-parallel spin electrons are uncorrelated. The correlation energy is define as

$$
\begin{equation*}
E_{\mathrm{corr}}=E_{\mathrm{exact}}-E_{H F}, \tag{2.15}
\end{equation*}
$$

where $E_{\text {exact }}$ and $E_{H F}$ are the exact and Hartree Fock energies, respectively. The correlation energy is always negative since HF energy is greater than the Exact energy. Although, Hartree Fock has been applied successfully for some ground-state calculations including modelling and calculation of equilibrium structures, bonds, dissociation
energies etc, However, due to the non-inclusion of some correlations, HF overestimates electronic gaps and excitation energies (the difference between the energy of the occupied and unoccupied orbital) (Anisimov, 2010, Szabo and Ostlund, 2012).

A variety of post-Hartree Fock methods like configuration interaction (CI), multi-configuration self-consistent field, couple cluster sets, etc have been developed to improve the accuracy of Hartree Fock total energy, orbitals, orbital energies and excitation energies. These methods seek to recover some correlation energies that are not included in the Hartree Fock method.

### 2.3.1.1 Configuration Interaction and Configuration Interaction Singles

In the configuration interaction approach, the many-body wave function is expressed as follows:

$$
\begin{equation*}
\Psi=l_{0} \Psi_{0}+\sum_{i, a} l_{i}^{a} \Psi_{i}^{a}+\sum_{i<j, a<b} l_{i j}^{a b} \Psi_{i j}^{a b}+\sum_{i<j<k, a<b<c} l_{i j k}^{a b c} \Psi_{i j k}^{a b c}+\ldots \tag{2.16}
\end{equation*}
$$

where $\Psi_{0}$ is obtained from a solution to the Hartree Fock equation. $\Psi_{i}^{a}, \Psi_{i j}^{a b}, \Psi_{i j k}^{a b c}$ etc are slater determinants of different configuration obtained from $\Psi_{0}$ by replacing one or more of its occupied spin-orbital by the unoccupied. $l^{\prime} s$ are the slater determinant coefficients, which are determined from the variational principle. The Full configuration interaction is an accurate method for electronic excitations. However, it is computationally expensive and restricted to a small system of atoms. In practice, the full CI is truncated to obtain different approximate methods namely configuration interaction singles (only single excitation determinants are included in the basis sets +HF slater determinant), configuration interaction doubles (only double excitations wave functions are included in the basis sets +HF slater determinant), configuration interaction singles and doubles ( both single and double excitations wave functions are included +HF slater determinant), etc (Sherrill and Schaefer III, 1999, Szabo and Ostlund, 2012).

In the configuration interaction singles (CIS), only the ground state (HF slater determinant) and the single substitution configurations slater determinants are utilized in the
basis sets. The CIS wave function is expressed as (Szabo and Ostlund, 2012)

$$
\begin{equation*}
\Psi_{C I S}=l_{0} \Psi_{0}+\sum_{i, a} l_{i}^{a} \Psi_{i}^{a} \tag{2.17}
\end{equation*}
$$

and the CIS eigenvalue problem is

$$
\begin{equation*}
H \Psi_{C I S}=E_{C I S} \Psi_{C I S} \tag{2.18}
\end{equation*}
$$

The ground and excited state energies obtained at the CIS level are upper bounds to the exact energies since $\Psi_{C I S}$ is obtained using a variational approach. According to the Brillouin theorem, $\left\langle\Psi_{0}\right| H\left|\Psi_{i}^{a}\right\rangle=0$. That is, there are no interactions between single excitations and the ground state Hartree Fock. So, the CIS ground state energy is not different from that of Hartree Fock and so, it is size-extensive (that is, the energy of the many-body system is proportional to the number of particles in the system) because Hartree Fock is also size-extensive. But there is also $\left\langle\Psi_{i}^{a}\right| H\left|\Psi_{i}^{a}\right\rangle \neq 0$. The CIS method is computationally less expensive as compare to other truncated CI methods and could handle moderately large atomic systems. Unfortunately, It overestimates excitation energies by an error that ranges from 0.5 eV to 2.0 eV when compared to experiment. It also gives only qualitative transition dipole moment. Thus, calculations performed with this method including INDO/CIS, produce qualitative spectra intensities when compare with experiments or high-level methods like EOM-CCSD (Dreuw and Head-Gordon, 2005).

Generally, the truncated CI methods except methods with doubly-excitation configurations, do not treat electron correlation properly. Even though CISD recovers some correlation, it still overestimates excitations. Also, the truncated CI methods except for CIS, are not size-extensive and are also computationally intensive as the system grows bigger. For instance, the configuration interaction singles doubles (CISD) scales $N^{6}$ computationally (Sherrill and Schaefer III, 1999).

The multi-configuration self-consistent field (MCSCF) is another post-Hartree Fock method which is based on truncated CI. The wave function, in this case, is expressed as in equation (2.16), but both the coefficient $l_{k}$ and the atomic orbital coefficients are optimised to minimize the total energy. Examples of MCSCF are the complete active space selfconsistent field (CASSCF), CASPT2, etc. The MCSCF is cumbersome and computa-
tionally expensive. Optimization of the active and virtual orbitals is tedious and not straightforward. Hence, this method is restricted to small atomic clusters (Szalay et al., 2011).

### 2.3.1.2 Couple Cluster (CC) and EOM-CCSD Method

A good alternative to the approximate full CI is couple cluster (CC) method. It describes the full CI ( the 'exact' method) wave function as

$$
\begin{equation*}
\Psi_{c c}=\exp (T) \Psi_{0} \tag{2.19}
\end{equation*}
$$

where $\Psi_{0}$ is the reference wavefuntion and $T$ is the cluster operator. $T$ is defined by

$$
\begin{equation*}
T=\sum_{n=1} T_{n}, \tag{2.20}
\end{equation*}
$$

where $T_{n}$ ( $n$ is the number of electrons) is the $n t h$ excitation operator. The $T_{1}$ (single excitation operator) is written as

$$
\begin{equation*}
T_{1} \Psi_{0}=\sum_{a, i} t_{a}^{i} \Psi_{a}^{i} \tag{2.21}
\end{equation*}
$$

and the $T_{1}$ (double excitation operator) as

$$
\begin{equation*}
T_{2} \Psi_{0}=\sum_{a>b, i>j} t_{a b}^{i j} \Psi_{a b}^{i j}, \tag{2.22}
\end{equation*}
$$

where $t_{a}^{i}$ and $t_{a b}^{i j}$ are linear combination coefficients. Truncating $T$ in Eq. (2.20), the following approximate coupled cluster methods could be obtained:

1. Couple cluster single (CCS) $\left(\mathrm{T}=T_{1}\right)$

$$
\Psi_{c c}=\left(1+T_{1}+\frac{T_{1}^{2}}{2!}+\frac{T_{1}^{3}}{3!}+\ldots\right) \Psi_{0}
$$

2. Couple cluster doubles (CCD) $\left(\mathrm{T}=T_{2}\right)$

$$
\Psi_{c c}=\left(1+T_{2}+\frac{T_{2}^{2}}{2!}+\frac{T_{2}^{3}}{3!}+\ldots\right) \Psi_{0}
$$

3. Couple cluster singles and doubles (CCSD) $\left(T=T_{1}+T_{2}\right)$

$$
\Psi_{c c}=\left(1+T_{1}+\frac{T_{1}^{2}}{2!}+\frac{T_{1}^{3}}{3!}+\ldots\right)\left(1+T_{2}+\frac{T_{2}^{2}}{2!}+\frac{T_{2}^{3}}{3!}+\ldots\right) \Psi_{0}
$$

4. Couple cluster singles doubles and triples (CCSDT) $\left(T=T_{1}+T_{2}+T_{3}\right)$

$$
\Psi_{c c}=\left(1+T_{1}+\frac{T_{1}^{2}}{2!}+\frac{T_{1}^{3}}{3!}+\ldots\right)\left(1+T_{2}+\frac{T_{2}^{2}}{2!}+\frac{T_{2}^{3}}{3!}+\ldots\right)\left(1+T_{3}+\frac{T_{3}^{2}}{2!}+\frac{T_{3}^{3}}{3!}+\ldots\right) \Psi_{0}
$$

Total energy is obtained by evaluating (Bartlett, 2012)

$$
\begin{equation*}
E_{c c}=\left\langle\Psi_{0}\right| H\left|e^{T} \Psi_{0}\right\rangle \tag{2.23}
\end{equation*}
$$

Generally, CC methods are size extensive (energy scales linearly with size of system) and accurate (Dutta et al., 2018, Bartlett, 2012). The CCSDT is a very accurate method, which is referred to as 'gold standard' in computational chemistry (Rezac and Hobza, 2013). Unfortunately, CC methods including CCSDT are computational intensive and limited to small-sized systems. For instance, $\operatorname{CCSD}$ scales as $N^{6}, \operatorname{CCSD}(\mathrm{~T})$ as $N^{7}$ and CCSDT as $N^{8}$ (Christiansen et al., 1995, Ochsenfeld et al., 2007, Evarestov, 2007).

A variant of the CC method employed for the excitation state calculations is the equation-of-motion coupled-cluster singles and doubles (EOM-CCSD) method. In EOM-CC, the excited state wave function $\Psi_{e x}$ is created from the couple cluster wave function $\Psi_{c c}$ by using the excitation operator, $B_{k}$. Thus,

$$
\begin{equation*}
\Psi_{e x}=B_{k} \Psi_{c c}=B_{k} e^{T} \Psi_{0} \tag{2.24}
\end{equation*}
$$

$B_{k}$ is the sum of singles $B_{0}$, doubles $B_{1}$, triples $B_{2}$ etc excitation operator terms.

$$
\begin{gather*}
B_{k}=B_{0}+B_{1}+B_{2}+\ldots  \tag{2.25}\\
B_{0}=b_{0}  \tag{2.26}\\
B_{1}=\sum_{a, i} b_{a}^{i} S_{a}^{i} \tag{2.27}
\end{gather*}
$$

$$
\begin{equation*}
B_{2}=\frac{1}{4} \sum_{a>b, i>j} b_{a b}^{i j} S_{a b}^{i j} \cdots \tag{2.28}
\end{equation*}
$$

The Schrödinger equation for the excited state is written as

$$
\begin{equation*}
H B_{k} e^{T} \Psi_{0}=E_{e x c} B_{k} e^{T} \Psi_{0} \tag{2.29}
\end{equation*}
$$

It can be shown that

$$
\begin{equation*}
\left[\bar{H}, B_{k}\right] \Psi_{0}=\triangle E_{e x c} \Psi_{0} \tag{2.30}
\end{equation*}
$$

where $\bar{H}=e^{-T} H e^{T}$ and $\Delta E_{\text {exc }}$ is the excitation energy (Stanton and Bartlett, 1993, Watts, 2008).

EOM-CCSD excitation energies are known to be comparable with experiments. It is a benchmark method for both ab-initio and semi-empirical methods. However, it is computationally intensive and prohibitive for large atomic clusters. It scales equivalently to CCSD and requires large computation memory for its computations. (Caricato et al., 2011, Bennie et al., 2017).

### 2.3.2 Density Functional Theory

The density functional theory (DFT) is an electronic structure method that utilizes the electron density as its major variable. While the wave function of an N-electron system, ignoring the spin variable, has 3 N dimensions, its electronic density $\mathrm{n}(\overrightarrow{( } r))$ has 3-dimensions. For a particular electronic system, more integrals are evaluated within the wave function theory than with the density functional theory. Thus, DFT is much less computationally intensive than the wave function theory methods.

In 1964, Hohenberg and Kohn (Hohenberg and Kohn, 1964) proposed the basic theorem of the DFT method, although the idea was earlier conceived by Thomas and Fermi. The theorems are as follows (Burke et al., 2007):

1. The external potential of a system of interacting electrons is determined uniquely by the electron density which in turn, determines the Hamiltonian, the wave function and other observables of the system.
2. The lowest energy can be obtained by optimising the electron density using the
variational principle.
3. The functional $\mathrm{F}[\mathrm{n}]=$ exact kinetic energy functional $(\mathrm{T}[\mathrm{n}])+$ exact electron-electron interaction $\left(\mathrm{V}_{e e}[\mathrm{n}]\right)$ is the same for all-electronic systems. That is, $\mathrm{F}[\mathrm{n}]$ is a universal functional.

The ground-state energy is given by

$$
\begin{gather*}
E=\min _{n}\left(F[n]+V_{e x t}\right)  \tag{2.31}\\
F[n]=\min _{\Psi \rightarrow n}\langle\Psi| T+V_{e e}|\Psi\rangle, \tag{2.32}
\end{gather*}
$$

where $\Psi$ is the ground-state wave function that minimizes $\mathrm{F}[\mathrm{n}]$. Kohn and Sham proposed an elegant approximation for the functional, F[n] (Kohn and Sham, 1965). They assumed the electronic system subjected to the Kohn-Sham potential $\left(V_{k s}\right)$ to be noninteracting but has the same density as the real system (interacting system). This approximation results to the single-particle Schrödinger equation (Burke et al., 2007, Koch and Holthausen, 2015),

$$
\begin{equation*}
\left(-\frac{1}{2} \nabla^{2}+V_{k s}[n(r)]\right) \phi_{i}(r)=\varepsilon_{i} \phi_{i}(r), \tag{2.33}
\end{equation*}
$$

where $n(r)=\sum_{i}\left|\phi_{i}(r)\right|^{2}$ and $V_{k s}=V_{e x t}+V_{H}+V x c$. The $\phi_{i}(r)$ is the Kohn-Sham orbital. The external potential, $V_{\text {ext }}$ is defined by

$$
\begin{equation*}
V_{e x t}=\int d r \frac{Z}{|r-R|} . \tag{2.34}
\end{equation*}
$$

The $V_{H}$ is the Hartree or coulomb potential expressed as

$$
\begin{equation*}
V_{H}=\int d r^{\prime} \frac{n\left(r^{\prime}\right)}{\left|r-r^{\prime}\right|} . \tag{2.35}
\end{equation*}
$$

The $V_{x c}$ is the exchange correlation, which is composed of the correction to the kinetic energy and the non-classical electron-electron interaction. It is expressed as

$$
\begin{equation*}
V_{x c}=\frac{\delta E_{x c}}{\delta n(r)} \tag{2.36}
\end{equation*}
$$

where $E_{x c}$ is the exchange-correlation energy. The Kohn-Sham equation will give exact
ground-state energies provided the exchange-correlation potential for which analytical solution is not available, is well defined. In other words, the accuracy of the KohnSham equation depends on the exchange-correlation functional since every other term in the equations are defined exactly. Different approximate methods have been developed to calculate exchange-correlation functionals. Prominent among these methods are the local density approximation (LDA) (Miehlich et al., 1989) and generalized-gradient approximation (GGA) (Perdew et al., 1996). In the LDA approximation, the system is assumed to be a homogeneous uniform electron gas and the exchange-correlation energy expressed by (Burke et al., 2007)

$$
\begin{equation*}
E_{x c}^{L D A}[n]=\int d r n(r) \varepsilon_{x c}(n(r)), \tag{2.37}
\end{equation*}
$$

where $\varepsilon_{x c}$ is the called the energy density, which is the sum of the individual exchange and correlation contribution. It can be calculated exactly using the Quantum Monte Carlo method. Although LDA works well for homogeneous systems and systems with slow varying densities, it fails for inhomogeneous systems (Burke et al., 2007). The GGA functional was developed in order to extend the exchange-correlation functional to inhomogeneous systems. In GGA formalism, the $\varepsilon_{x c}$ varies with density. The gradient of density parameters could be obtained by fit to experiments.

$$
\begin{equation*}
E_{x c}^{G G A}[n]=\int d r n(r) \varepsilon_{x c}(n(r), \nabla(n(r))) . \tag{2.38}
\end{equation*}
$$

Generally, DFT has been applied successfully for ground-state calculations. However, they are known to poorly predict the optical properties of a system. They underestimate excitation energies and electronic gaps. The time-dependent density functional theory (TDDFT), a post-DFT method, has been developed and employed for excitation energies and spectra calculations.

### 2.3.2.1 Time-Dependent Density Functional Theory (TDDFT)

TDDFT is an extension of DFT. Runge and Gross generalized Kohn and Hohenberg theorems to time-dependent DFT (Runge and Gross, 1984). The theorem states that
there is a one-one correspondence between the time-dependent density $n(r, t)$ and the time-dependent external potential $V(r, t)$. That is, $V(r, t)$ is uniquely defined by $n(r, t)$. Thus, time-dependent Hamiltonian and other time-dependent observables are defined by $n(r, t)$. The expectation values of observables are unique functionals of $n(r, t)$ and the initial state (the ground state). This leads to generalizing the Kohn-Sham non-interacting ground state equation to a time-dependent Kohn-Sham equation (Marques et al., 2006, Burke et al., 2007):

$$
\begin{equation*}
\left[-\frac{1}{2} \nabla^{2}+V_{k s}[n(r, t)]\right] \phi_{i}(r, t)=i \frac{\partial}{\partial t} \phi_{i}(r, t) . \tag{2.39}
\end{equation*}
$$

$n(r, t)$ is the time-dependent density expressed as

$$
\begin{equation*}
n(r, t)=\sum_{i=1}^{N}\left|\phi_{i}(r, t)\right|^{2} \tag{2.40}
\end{equation*}
$$

where $\phi_{i}(r, t)$ is the time-dependent orbital. The $V_{k s}(r, t)$ is the time-dependent KohnSham effective potential expressed as

$$
\begin{equation*}
V_{k s}(r, t)=V_{e x t}(r, t)+V_{H}(r, t)+V_{x c}(r, t), \tag{2.41}
\end{equation*}
$$

$V_{e x t}(r, t), V_{H}(r, t)$, and $V_{x c}(r, t)$ are the time-dependent external potential, time-dependent Coulomb potential and time-dependent exchange-correlation, respectively. The timedependent exchange-correlation is defined by

$$
\begin{equation*}
V_{x c}=\frac{\partial A}{\partial n(r, \tau)} \ln _{n(r, t)}, \tag{2.42}
\end{equation*}
$$

where $A$ is the action, which is analogous to total energy. $\tau$ is the Keldish pseudo time. Just as in the case of $V_{x c}$ for ground-state, the $V_{x c}(x, t)$ has no exact form. The simplest approximate form of $V_{x c}(x, t)$ is the adiabatic local density approximation (ALDA) written as

$$
\begin{equation*}
V_{x c}^{A L D A}(r, t)=V_{x c}^{L D A}(n(r, t)) . \tag{2.43}
\end{equation*}
$$

The TDDFT within the ALDA does well for slowly varying densities and gives good predicts for low-lying excited states.

Excitation energies could be extracted from TDDFT through linear response calculations
(Marques et al., 2012, Maitra, 2016). When a system is perturbed by a weak electric field, $V_{\text {ext }}(r, \omega)$, the change in density of the system, $\delta n(r, \omega)$ can be expressed as:

$$
\begin{equation*}
\delta n(r, t)=\int d t^{\prime} \int d r^{\prime} \chi[n]\left(r, r^{\prime}, t-t^{\prime}\right) \delta V_{e x t}\left(r, t^{\prime}\right) \tag{2.44}
\end{equation*}
$$

Where $\chi$ is the density-density response function that measures the change in density of the system due to the external potential from the weak electric field. Equivalently, the change in density can be obtained for the time-dependent Kohn-Sham effective potential:

$$
\begin{equation*}
\delta n(r, t)=\int d t^{\prime} \int d r^{\prime} \chi[n]\left(r, r^{\prime}, t-t^{\prime}\right) \delta V_{k s}\left(r, t^{\prime}\right) \tag{2.45}
\end{equation*}
$$

Equating (2.44) and (2.45) to have

$$
\begin{align*}
\delta n(r, t)= & \int d t^{\prime} \int d r^{\prime} \chi[n]\left(r, r^{\prime}, t-t^{\prime}\right) \delta V_{e x t}\left(r, t^{\prime}\right)=  \tag{2.46}\\
& \int d t^{\prime} \int d r^{\prime} \chi[n]\left(r, r^{\prime}, t-t^{\prime}\right) \delta V_{k s}\left(r, t^{\prime}\right)
\end{align*}
$$

and

$$
\begin{equation*}
\frac{\delta V_{k s}\left(r, t^{\prime}\right)}{\delta V_{e x t}\left(r, t^{\prime}\right)}=\frac{\delta V_{e x t}\left(r, t^{\prime}\right)}{\delta V_{e x t}\left(r, t^{\prime}\right)}+\frac{\delta V_{H}\left(r, t^{\prime}\right)}{\delta V_{e x t}\left(r, t^{\prime}\right)}+\frac{\delta V_{x c}\left(r, t^{\prime}\right)}{\delta V_{e x t}\left(r, t^{\prime}\right)} . \tag{2.47}
\end{equation*}
$$

Introducing equation (2.47) into equation (2.46), the TDDFT linear response equation called the Dyson-like equation is obtained and is given as

$$
\begin{align*}
\chi\left(r, r^{\prime}, \omega\right)= & \chi_{k s}\left(r, r^{\prime}, \omega\right)+\int d r_{1} \int d r_{2} \chi_{k s}(r, r 1, \omega)\left\{\frac{1}{\left|r_{1}-r_{2}\right|}+\right.  \tag{2.48}\\
& \left.f_{x c}\left(r_{1}, r_{2}, \omega\right)\right\} \chi_{k s}\left(r_{2}, r^{\prime}, \omega\right),
\end{align*}
$$

where the exchange-correlation kernel, $f_{x c}$ is written as

$$
f_{x c}\left(r_{1}, r_{2}, \omega\right)=\frac{\delta V_{x c}(r, \omega)}{\delta n(r, t)}
$$

and the Kohn-Sham response function, $\chi_{k s}\left(r, r^{\prime}, \omega\right)$ as

$$
\begin{equation*}
\chi_{k s}\left(r, r^{\prime}, \omega\right) \propto \sum_{i} \sum_{j} \frac{\left|\phi_{i}\right|^{2}\left|\phi_{j}\right|^{2}}{\omega-\left(\varepsilon_{i}-\varepsilon_{j}\right)} . \tag{2.49}
\end{equation*}
$$

$\phi$ and $\varepsilon$ are the orbital and orbital energy from the time-independent Kohn-Sham equa-
tion. The summations are over all occupied and unoccupied Kohn-Sham orbital and orbital energies. The poles of $\chi_{k s}\left(r, r^{\prime}, \omega\right)$ correspond to Kohn-Sham excitations while the poles of $\chi\left(r, r^{\prime}, \omega\right)$ correspond to the true excitation energies.

Casida provided another means of solving equation (2.48). He recasted the equation as an eigenvalue problem, which is expressed as follows (Casida, 1995, 2009, Maitra, 2016):

$$
\begin{equation*}
\sum_{p}\left[\delta_{p p^{\prime}} \xi_{p}+2 \sqrt{\omega_{p} \omega_{p}^{\prime}}\langle p|\left\{\frac{1}{\left|r_{1}-r_{2}\right|}+f_{x c}\left(r_{1}, r_{2}, \omega\right)\right\}|p\rangle\right] \mu_{p}=\xi \mu_{p} \tag{2.50}
\end{equation*}
$$

where $\xi=\omega^{2}$ and

$$
\begin{align*}
& \langle p|\left\{\frac{1}{\left|r_{1}-r_{2}\right|}+f_{x c}\left(r_{1}, r_{2}, \omega\right)\right\}|p\rangle=\int d r \int d r^{\prime} \phi_{i}(r)^{*} \phi_{j}(r)\left\{\frac{1}{\left|r_{1}-r_{2}\right|}+\right.  \tag{2.51}\\
& \left.f_{x c}\left(r_{1}, r_{2}, \omega\right)\right\} \phi_{i}\left(r^{\prime}\right)^{*} \phi_{j}\left(r^{\prime}\right) .
\end{align*}
$$

$\omega_{p}=\varepsilon_{i}-\varepsilon_{j}$ is the excitation energy and $\mu_{p}$ is the eigenvector from which the oscillator strength is calculated. $\varepsilon_{i}$ and $\varepsilon_{j}$ are the Kohn-Sham unoccupied and occupied orbital energies, respectively. $\phi_{i}$ and $\phi_{j}$ are the Kohn-Sham unoccupied and occupied orbital respectively (Marques et al., 2006, Burke et al., 2007, Maitra, 2016). TDDFT is known to handle moderate-sized systems but is computationally expensive for large atomic clusters like quantum dots (Gabay et al., 2017).

### 2.3.3 Semi-empirical approaches

Semi-empirical methods are obtained from drastic approximation to first principle wave function theory or density functional theory. They involve omitting or replacing with parametric expressions some terms (most times integrals) in the methods that make computation intensive. Thereafter, the parameters are calibrated with experiments or high-level ab-initio results. If the calibration is carried out correctly with reliable data, it is possible to correct for the errors introduced into semi-empirical approaches due to the approximations. Although the semi-empirical methods could handle large atomic clusters and nano-systems with less computational cost and effort, they are often less accurate and less transferable than the high-level first principle methods. Different semi-
empirical methods have been developed. An overview of the Huckel, extended Huckel, Neglect of differential overlap (NDO), empirical pseudopotential method (EPM) and density functional tight binding (DFTB) semi-empirical methods will be given.

### 2.3.3.1 The Tight Binding Model

The Tight Binding Model (Harrison, 1989) involves solving the eigenvalue equation,

$$
\begin{equation*}
H \psi_{i}(r)=\varepsilon_{i} \psi_{i} \tag{2.52}
\end{equation*}
$$

The $i^{\text {th }}$ molecular orbital, $\psi_{i}(r)$ is expressed as a linear combination of the valence atomic orbitals and is expressed as

$$
\begin{equation*}
\psi_{i}(r)=\sum_{n} \sum_{\alpha} C_{i \alpha}^{n} \phi_{\alpha}^{n} \tag{2.53}
\end{equation*}
$$

where n and $\alpha$ are atoms and their corresponding orbitals, respectively. C is the basis set expansion coefficient. Substitute equation 2.53 into equation 2.52 , multiply by orbital $\phi_{\beta}^{*}$ and integrate over all space to have

$$
\begin{gather*}
\sum_{n} \sum_{\alpha} C_{i \alpha}^{n}\left\langle\phi_{\beta}\right| H\left|\phi_{\alpha}\right\rangle=\varepsilon_{i} \sum_{n} \sum_{\alpha} C_{i \alpha}^{n}\left\langle\phi_{\beta} \mid \phi_{\alpha}\right\rangle  \tag{2.54}\\
\sum_{n} \sum_{\alpha} C_{i \alpha}^{n} H_{\beta \alpha}=\varepsilon_{i} \sum_{n} \sum_{\alpha} C_{i \alpha}^{n} S_{\beta \alpha}, \tag{2.55}
\end{gather*}
$$

where $S_{\beta \alpha}$ is the overlap matrix, which is expressed as a unit matrix since the orbitals are assumed to be orthogonal. $H_{\beta \alpha}$ is the Hamiltonian matrix whose diagonal matrix elements, $H_{\beta \beta}$ and $H_{\alpha \alpha}$ are the free atomic orbital energies. The off-diagonal elements are obtained using the two center Slater Koster approximations (Slater and Koster, 1954) while the adjustable parameters therein, are obtained from fits to experimental or $a b$ initio data. Within the Slater Koster approximations, the off-diagonal Hamiltonian matrix elements are reduced to the following for sp basis set (Harrison, 2012):

$$
H_{\beta \alpha}=H_{s s \sigma}, H_{s p \sigma}, H_{p p \sigma}, H_{p p \pi}
$$

. The off-diagonal Hamiltonian elements obtained by Harrison are defined by (Harrison, 2012)

$$
\begin{equation*}
H_{a b c}=-\frac{n_{a b c}}{d^{2}}, \tag{2.56}
\end{equation*}
$$

where $\mathrm{a}, \mathrm{b}=\mathrm{s}$ or $\mathrm{p}, \mathrm{c}=\sigma$ or $\pi$ and $n_{a b c}$ is an adjustable parameter that can be obtained from a fit. The variable $d$ is the separation between atomic orbitals. Other adjustable expressions for Hamiltonian matrix elements have been proposed (Papaconstantopoulos and Mehl, 2003, Shi and Papaconstantopoulos, 2004). After fixing the Hamiltonian and overlap matrix elements, the eigenvalue equation can be solved to obtain the orbital energies, $\varepsilon_{i}$ and the orbital coefficients, C. However, calculation of absorption spectra with the tight-binding is not straightforward because wave functions are not explicitly defined.

### 2.3.3.2 Density Functional Tight Binding (DFTB)

DFTB is a variant of tight binding, which is obtained from a drastic approximation of DFT. Its theoretical framework is given in the following (Seifert, 2007, Koskinen and Mäkinen, 2009, Seifert and Joswig, 2012). The total energy of a system within the DFT scheme is written as

$$
\begin{equation*}
E[n]=\sum_{i}^{o c c}\left\langle\psi_{i}\right|-\frac{1}{2} \nabla^{2}+\int d^{3} r V_{e x t} n(r)\left|\psi_{i}\right\rangle+\frac{1}{2} \int d^{3} r \frac{n n^{\prime}}{\left|r-r^{\prime}\right|}+E_{x c}+E_{n u c} . \tag{2.5}
\end{equation*}
$$

If the true density that minimizes $\mathrm{E}[\mathrm{n}]$ is given as $n=n_{0}+\delta n$, expanding $\mathrm{E}[\mathrm{n}]$ up to second order in the density fluctuation, $\delta n$ :

$$
\begin{align*}
E\left[n_{0}+\delta n\right]= & \sum_{i}^{o c c}\left\langle\psi_{i}\right|-\frac{1}{2} \nabla^{2}+V_{e f f}\left[n_{0}\right]\left|\psi_{i}\right\rangle+\frac{1}{2} \int d^{3} r \int d^{3} r^{\prime} \frac{\delta^{2} E_{x c}\left[n_{0}\right]}{\delta n \delta n^{\prime}}+ \\
& \frac{1}{\left|r-r^{\prime}\right|} \delta n \delta n^{\prime}-\frac{1}{2} \int d^{3} r V_{H}\left[n_{0}\right] n_{0}(r)-E_{x c}\left[n_{0}\right]-E_{n u c}  \tag{2.58}\\
& -\int d^{3} r V_{x c} E_{x c}\left[n_{0}\right] n_{0}(r)
\end{align*}
$$

The band structure energy is expressed as

$$
\begin{equation*}
E_{b s}=\sum_{i}^{o c c}\left\langle\psi_{i}\right|-\frac{1}{2} \nabla^{2}+V_{e f f}\left[n_{0}\right]\left|\psi_{i}\right\rangle=\sum_{i}^{o c c}\left\langle\psi_{i}\right| H_{0}\left[n_{0}\right]\left|\psi_{i}\right\rangle, \tag{2.59}
\end{equation*}
$$

where

$$
\begin{equation*}
V_{e f f}=V_{e x t}+V_{x c}+V_{H} . \tag{2.60}
\end{equation*}
$$

The energy from charge fluctuation, $E_{\text {coulomb }}$ :

$$
\begin{equation*}
E_{\text {coulomb }}=\frac{1}{2} \int d^{3} r \int d^{3} r^{\prime}\left(\frac{\delta^{2} E_{x c}\left[n_{0}\right]}{\delta n \delta n^{\prime}}+\frac{1}{\left|r-r^{\prime}\right|}\right) \delta n \delta n^{\prime} . \tag{2.61}
\end{equation*}
$$

The repulsive energy $\left(E_{\text {rep }}\right)$ is written as

$$
\begin{equation*}
E_{r e p}=-\frac{1}{2} \int d^{3} r V_{H}\left[n_{0}\right] n_{0}(r)-E_{x c}\left[n_{0}\right]-E_{n u c}-\int d^{3} r V_{x c} E_{x c}\left[n_{0}\right] n_{0}(r) \tag{2.62}
\end{equation*}
$$

Hence, equation (2.58) becomes

$$
\begin{equation*}
E\left[n_{0}+\delta n\right]=E_{b s}+E_{\text {coulomb }}+E_{\text {rep }} . \tag{2.63}
\end{equation*}
$$

The following approximations are made for terms in equation (2.63):

$$
\begin{equation*}
E_{\text {rep }}=\sum_{Y>Z} V_{\text {rep }}^{Y Z}(R), \tag{2.64}
\end{equation*}
$$

where $V_{\text {rep }}^{Y Z}(R)$ represents the repulsive function for each atom pair separated by distance R and can be obtained from fit to accurate theoretical data.

$$
\begin{equation*}
E_{\text {coulomb }}=\sum_{Y Z} \gamma_{Y Z}\left(R_{Y Z}\right) \Delta q_{a} \Delta q_{b} \tag{2.65}
\end{equation*}
$$

Minimizing $E\left[n_{0}+\delta n\right]$ with the variational principle approach, one obtains an equivalent single particle Kohn-Sham equation. The wave function $\psi_{i}$ is expressed in the basis of the atomic valence orbitals and given by

$$
\begin{equation*}
\psi_{i}(r)=\sum_{a=1}^{n} C_{a i} \phi_{a}(r) . \tag{2.66}
\end{equation*}
$$

Just as observed in the DFT, the DFTB underestimates electronic gap (Marutaphan and Wongchoosuk, 2017). Also, it is not straightforward to calculate absorption spectra within the DFTB formalism because its wavefunction is not explicitly defined.

### 2.3.3.3 Empirical pseudopotential based model

Zunger and Co-worker developed a model based empirical pseudopotential method(EPM) for nanostructure calculations. The method is described as follows (Harrison and Valavanis, 2016, Wang and Zunger, 1994): Solution to the eigenvalue problem, equation (2.67) is sought for.

$$
\begin{equation*}
H \psi_{n, k}(r)=\varepsilon_{n} \psi_{n, k}(r) . \tag{2.67}
\end{equation*}
$$

The wave function, $\psi_{n, k}(r)$ is expanded in plane wave basis sets.

$$
\begin{equation*}
\psi_{n, k}(r)=\sum_{G} C_{G} \exp (i|\mathbf{G}+\mathbf{k}| \cdot \mathbf{r}) \tag{2.68}
\end{equation*}
$$

The Hamiltonian, $H$ is written as

$$
\begin{equation*}
H=-\frac{1}{2} \nabla^{2}+V(r) \tag{2.69}
\end{equation*}
$$

Substituting equations (2.68) and (2.69) into equation (2.67), multiply by $\exp \left(-i \mid \mathbf{G}^{\boldsymbol{\prime}}+\right.$ $\mathbf{k} \mid \cdot \mathbf{r})$ and integrating over all space, gives

$$
\begin{equation*}
\sum_{G} H_{\mathbf{G}, \mathbf{G}} C_{\mathbf{G}}=\varepsilon_{n} \sum_{G} S_{\mathbf{G},{ }_{\mathbf{G}}} C_{\mathbf{G}} \tag{2.70}
\end{equation*}
$$

where

$$
\begin{gather*}
H_{\mathbf{G}, \mathbf{G}}=|\mathbf{G}+\mathbf{k}| \delta_{\mathbf{G}}, \mathbf{G}  \tag{2.71}\\
S_{\mathbf{G}}, \mathbf{G}=\delta_{\mathbf{G}}, \mathbf{G},  \tag{2.72}\\
V=\sum_{t} \exp (i \mathbf{q} \cdot \mathbf{t}) V_{f}(q),  \tag{2.73}\\
\mathbf{q}=\mathbf{G},-\mathbf{G} . \tag{2.74}
\end{gather*}
$$

$V_{f}(q)$ is an adjustable parameter. It is chosen at a few discrete q points for a bulk semiconductor (Cohen and Bergstresser, 1966) while for nanostructures, $V_{f}(q)$ is continuous and dense(Wang and Zunger, 1994). The $V_{f}(q)$ for nanostructure material can be expressed as

$$
\begin{equation*}
V_{f}(q)=\frac{a_{1}\left(q^{2}-a_{2}\right)}{a_{3} \exp \left(a_{4} q^{2}\right)-1} \tag{2.75}
\end{equation*}
$$

or

$$
\begin{equation*}
V_{f}(q)=a_{i} \exp \left(-c_{i}\left(q-b_{i}\right)^{2}\right) \tag{2.76}
\end{equation*}
$$

where $a_{n} n=1,2,3,4$ and $a_{i}, b_{i}, c_{i}$ are empirical parameters obtained from fits to experimental or ab-initio data. Although computation cost due to different center-integrals has been drastically reduced, one is faced with the problem of diagonalizing the Hamiltonian matrix resulting from the dense plane waves employed in the nanostructure computations. The huge Hamiltonian matrix requires large computational memory to store them(Harrison and Valavanis, 2016). Zunger and co-workers proposed a method called the Folded spectrum method (FDM) to solve the huge matrix problem. Their method seeks to solve the matrix equation around the electronic gap of the nanostructure. (Wang and Zunger, 1994, 1996, Harrison and Valavanis, 2016).

### 2.3.3.4 Huckel and Extended Huckel model

The Huckel model is an approximate molecular orbital theory developed in 1931 (Hückel, 1931). Its molecular orbitals are expanded with basis sets of only pi orbitals. Thus, the model is restricted to only pi molecules e.g Benzene. The model is described as follows (Andrew, 2001, Magnasco, 2013). The model seeks to solve the eigenvalue equation,

$$
\begin{equation*}
H \psi_{i}(r)=\varepsilon_{i} \psi_{i}(r) \tag{2.77}
\end{equation*}
$$

The molecular orbital is expressed in terms of pi atomic orbitals only as follows:

$$
\begin{equation*}
\psi_{i}=\sum_{a} C_{a, \pi}^{i} \phi_{a, \pi}, \tag{2.78}
\end{equation*}
$$

where $\psi_{i}$ is the molecular orbital, C is the expansion coefficient, and $\phi_{a, \pi}$ is the pi orbital centered on atom a. The eigenvalue problem can be reduced to the secular equation,

$$
\begin{equation*}
\sum_{a} H_{a b, \pi} C_{a, \pi}^{i}=\varepsilon_{i} \sum_{a} S_{a b, \pi} C_{a, \pi}^{i} . \tag{2.79}
\end{equation*}
$$

The Hamiltonian matrix element, $H_{a b, \pi}$ is defined as

$$
\begin{equation*}
H_{a b, \pi}=\left\langle\phi_{b, \pi}\right| H\left|\phi_{a, \pi}\right\rangle . \tag{2.80}
\end{equation*}
$$

The diagonals element of the Hamiltonian matrix are defined by

$$
\begin{equation*}
H_{a a, \pi}=H_{b b, \pi}=\alpha . \tag{2.81}
\end{equation*}
$$

The off-diagonal Hamiltonian matrix elements, $a \neq b$ (resonance integral) are expressed for nearest neighbor interaction as

$$
\begin{equation*}
H_{a b, \pi}=\beta . \tag{2.82}
\end{equation*}
$$

Otherwise,

$$
\begin{equation*}
H_{a b, \pi}=0 . \tag{2.83}
\end{equation*}
$$

The overlap matrix element:

$$
\begin{equation*}
S_{a b, \pi}=\left\langle\phi_{b, \pi} \phi_{a, \pi}\right\rangle=I, \tag{2.84}
\end{equation*}
$$

where I is a unit matrix. The extended Huckel method was proposed by Hoffman (Hoffman, 1964) to extend the application of the Huckel model to other systems other than pi molecules only. In this model, the molecular orbital is written in the basis of the valence orbitals as

$$
\begin{equation*}
\psi_{i}=\sum_{a} \sum_{k} C_{a, k}^{i} \phi_{a, k} . \tag{2.85}
\end{equation*}
$$

The valence atomic orbital, $\phi_{k}$ can be expressed in terms of a Gaussian function as

$$
\begin{equation*}
\phi_{k}=d_{k} \exp \left(-\gamma_{k} r^{2}\right) . \tag{2.86}
\end{equation*}
$$

The Fock secular equation to be solved is expressed as

$$
\begin{equation*}
F C=S C E . \tag{2.87}
\end{equation*}
$$

F is a Fock matrix with its elements, $F_{\nu \mu}$, which can be expressed in diagonal and offdiagonal form. The diagonal matrix element,

$$
\begin{equation*}
F_{V v}=I_{v v} \tag{2.88}
\end{equation*}
$$

where $I_{v v}$ is the ionization energy for orbital $v$.
The off-diagonal matrix element,

$$
\begin{equation*}
F_{v \mu}=K S_{v \mu} \frac{I_{\mu \mu}+v v}{2} \tag{2.89}
\end{equation*}
$$

where K is an empirical parameter and $S_{v \mu}$ is the overlap matrix element. Other expressions for off-diagonal Fock matrix elements, $F_{\nu \mu}$ have been published (Wolfsberg and Helmholz, 1952, Hoffmann and Lipscomb, 1962).

With the Fock matrix and overlap matrix fixed, the eigenvalue problem can be solved to obtain the molecular energies, the transition energies and transition orbitals. The Huckel and extended Huckel models are rarely used for any meaningful calculation because they give only qualitative results (Wolfsberg and Helmholz, 1952, Hoffmann and Lipscomb, 1962, Magnasco, 2013).

### 2.3.3.5 Neglect of Differential Overlap (NDO)

The Neglect of Differential Overlap (NDO) methods are the most widely applied and successful semi-empirical methods (Zerner, 1991, Bredow and Jug, 2005, Christensen et al., 2016). They are approximate methods to the Hartree Fock formalism, which start from the Roothaan equation (a simplified and matrix form of the Hartree Fock equation). Within this formalism, one seeks to solve the secular equation,

$$
\begin{equation*}
F C=S C E, \tag{2.90}
\end{equation*}
$$

where $F$ is a Fock matrix with Fock elements $F_{a b} . C$ is the molecular basis coefficient vector with vector elements $c_{k a}, E$ is a set of orbital energies $\left\{\varepsilon_{a}\right\}$ and $S$ is the overlap matrix with matrix elements $S_{a b}$. Using Roothan HF formalism,

$$
\begin{equation*}
F_{a b}=H_{a b}+\sum_{c=1}^{n} \sum_{d=1}^{n}\left[P_{a b}\langle a b \mid c d\rangle+\frac{1}{2}\langle a c \mid b d\rangle\right], \tag{2.91}
\end{equation*}
$$

where

$$
\begin{equation*}
H_{a b}=\langle a|-\frac{1}{2} \nabla^{2}-\sum_{A}^{N} \frac{Z}{\left|r-R_{A}\right|}|b\rangle . \tag{2.92}
\end{equation*}
$$

The density matrix, $P_{a b}$ is written as

$$
\begin{equation*}
P_{a b}=2 \sum_{i=1}^{N / 2} c_{a i} c_{b i}, \tag{2.93}
\end{equation*}
$$

where $a, b, c$, and $d$ refer to atomic orbitals $\phi_{a}, \phi_{b}, \phi_{c}$, and $\phi_{d}$ respectively. The variables $n$ and $N$ are the numbers of atomic orbitals and electrons, respectively. Within the NDO approximation, the basic approximations applied to the Roothaan equation are the zero differential overlap approximations (ZDO). These approximations are as follows (Andrew, 2001):

1. $\phi_{a} \phi_{b} d^{3} r=0$
2. The overlap matrix is the unit matrix: $S_{a b}=\delta_{a b}$
3. The different centre integrals $\langle a b \mid c d\rangle=\langle a a \mid d d\rangle \delta_{a b} \delta_{c d}$. Consequently, all the three- and four-centre integrals are omitted.

Applying the ZDO approximations to the Roothaan secular equation, the equation reduces to (Andrew, 2001, Segal, 2012)

$$
\begin{equation*}
F C=C E, \tag{2.94}
\end{equation*}
$$

with the diagonal matrix elements define by

$$
\begin{equation*}
F_{a a}=H_{a a}+\sum_{c=1}^{n} P_{c c}\langle a a \mid c c\rangle-\frac{1}{2} P_{a a}\langle a a \mid a a\rangle \tag{2.95}
\end{equation*}
$$

and the off-diagonal matrix elements by

$$
\begin{equation*}
F_{a b}=H_{a b}-\frac{1}{2} P_{a b}\langle a a \mid b b\rangle . \tag{2.96}
\end{equation*}
$$

### 2.3.3.6 Complete Neglect of Differential Overlap (CNDO)

The ZDO approximation was first implemented by Pople and Segal (1965) in the complete neglect of differential overlap. The Fock matrix elements of the CNDO are expressed as follows (Pople and Segal, 1965, Andrew, 2001, Magnasco, 2013):

For diagonal matrix elements,

$$
\begin{equation*}
F_{a a}=H_{a a}+\left(P_{Y Y}-\frac{1}{2} P_{a a}\right) \gamma_{a a}+\sum_{Y \neq Z} P_{Z Z} \gamma_{a b}, \tag{2.97}
\end{equation*}
$$

where

$$
\begin{gather*}
H_{a a}=U_{a a}+\sum_{Y \neq Z} V_{Y Z},  \tag{2.98}\\
U_{a a}=\langle a|-\frac{1}{2} \nabla^{2}-\frac{Z A_{Y}}{\left|r-R_{Y}\right|}|a\rangle, \tag{2.99}
\end{gather*}
$$

and

$$
\begin{equation*}
V_{Y Z}=\langle a|-\sum_{Z \neq Y}^{N}-\frac{Z A_{Z}}{\left|r-R_{Z}\right|}|a\rangle . \tag{2.100}
\end{equation*}
$$

For off-diagonal matrix elements ( $a \neq b$, both $a$ and $b$ are on the same atom, say $Y$ ),

$$
\begin{equation*}
F_{a b}=-\frac{1}{2} P_{a b} \gamma_{Y Y} \tag{2.101}
\end{equation*}
$$

For off-diagonal matrix elements ( $a \neq b, a$ and $b$ different atoms , say $Y$ and $Z$ ),

$$
\begin{equation*}
F_{a b}=\beta_{Y Z} S_{a b}-\frac{1}{2} P_{a b} \gamma_{Y Z} \tag{2.102}
\end{equation*}
$$

The total electron density, $P_{Y Y}$ on an atom is define as

$$
\begin{equation*}
P_{Y Y}=\sum_{d o n Y}^{Y} P_{d d} . \tag{2.103}
\end{equation*}
$$

The two-electron repulsion integral with both orbitals $a$ and $d$ on atom $Y$, is

$$
\begin{equation*}
\gamma_{Y Y}=\langle a a \mid d d\rangle=\left\langle s_{Y} s_{Y} \mid s_{Y} s_{Y}\right\rangle . \tag{2.104}
\end{equation*}
$$

The two-electron repulsion integral with orbitals $a$ and $d$ on different atoms, $Y$ an $Z$ is

$$
\begin{equation*}
\gamma_{Y Z}=\langle a a \mid d d\rangle=\left\langle s_{Y} s_{Y} \mid s_{Z} s_{Z}\right\rangle . \tag{2.105}
\end{equation*}
$$

In order to deal with the problem of orbital invariance experienced in the Huckel and extended Huckel models, the electron interactions are approximated as interactions be-
tween the s-orbitals. $s_{Y}$ and $s_{Z}$ are s-orbitals on atoms $Y$ and $Z$, respectively (Zerner, 1991). $U_{a a}$ is the one-centre electron integral obtained from the experimental ionization potential. $V_{Y Z}$ is the attraction interaction between electrons on atom $Y$ and nuclei of atom $Z$. It is calculated over s-orbitals representing the valence orbitals. $\beta$ is the resonance integral responsible for bonding and is obtained from a fit to ab-initio results (Zerner, 1991, Andrew, 2001). Different CNDO methods have been developed which include CNDO/1, CNDO/2 and CNDO/s. In CNDO also known as CNDO/1, equilibrium bond distances are underestimated and energies are overestimated due a penetration effect of the valence electrons. The CNDO/2 was developed in 1966 as an improvement over $\mathrm{CNDO} / 1$. It was developed in order to remove the electron penetration effects found in CNDO/1, by adjusting the expression for $V_{Y Z}$. Also, $U_{a a}$ in CNDO/2 is obtained in a slightly different way from CNDO/1 (Segal, 2012). While the CNDO/1 and CNDO/2 are implemented for ground state calculations, the CNDO/s developed by Bene and Jeffe, was uniquely developed for excitations and spectroscopy calculations (Bene and Jaffe, 1968).

### 2.3.3.7 Intermediate Neglect of Differential Overlap(INDO)

INDO was developed by Pople and co-workers in 1967 as an improvement over CNDO . Unlike the CNDO method, electron spin effect was included in the INDO method (Pople et al., 1967). In contrast to the ZDO approximation and CNDO, the electron-electron repulsion integrals on one-center are non-zero in the INDO model. These integrals are calculated using the Slater-Condon parameters obtained from spectroscopic data. The INDO Fock matrix is define by both diagonal and off-diagonal matrix elements (Pople et al., 1967, Andrew, 2001).

For the diagonal matrix elements ( $a=b$ ),

$$
\begin{equation*}
F_{a a}=U_{a a}+\sum_{b \text { on } Y}\left[P_{b b}\langle a a \mid b b\rangle-\frac{1}{2} P_{b b}\langle a b \mid a b\rangle\right]+\sum_{Y \neq Z}\left(P_{Z Z}-Z A_{Z}\right) \gamma_{a b} . \tag{2.106}
\end{equation*}
$$

For off-diagonal matrix elements ( $a \neq b$, both $a$ and $b$ are on the same atom, say Y),

$$
\begin{equation*}
f_{a b}=\frac{3}{2} P_{a b}\langle a b \mid a b\rangle-\frac{1}{2} P_{a b}\langle a a \mid b b\rangle . \tag{2.107}
\end{equation*}
$$

For off-diagonal matrix elements ( $a \neq b, a$ and $b$ different atoms, say $Y$ and $Z$ ),

$$
\begin{equation*}
F_{a b}=\frac{1}{2}\left(\beta_{Y}+\beta_{Z}\right) S_{a b}-\frac{1}{2} P_{a b} \gamma_{Y Z} . \tag{2.108}
\end{equation*}
$$

An important variant of the INDO is INDO/CI, which was developed in 1971 to calculate lower excitation states for hydrocarbons (Van Catledge, 1971). Zerner and co-workers in 1976, developed the Zerner intermediate neglect of differential overlap for spectroscopy (ZINDO/s), sometimes known as intermediate neglect of differential overlap for spectroscopy (INDO/s). It is an INDO method followed by CIS and it is widely employed for excitations and spectroscopy in organic molecules and transition metal complexes (Ridley and Zerner, 1976, Zerner et al., 1980b). Within the ZINDO/s approximation, the one-centre two-electron repulsion integrals were obtained from spectroscopic data through the Slater-Condon parameters and the resonance integrals $\beta$ by fits to experiments. The electron- nuclear attraction $\gamma_{Y Z}$ is define using the modified Mataga Nishimoto parametric expression (Mataga and Nishimoto, 1957):

$$
\begin{equation*}
\gamma_{Y Z}=\frac{f\left(\gamma_{Y Y}+\gamma_{Z Z}\right)}{2 f+R_{Y Z}\left(\gamma_{Y Y}+\gamma_{Z Z}\right)}, \tag{2.109}
\end{equation*}
$$

where $f$ is and $R_{Y Z}$ is the separation between the centers of atoms $Y$ and $Z$.

### 2.3.3.8 Neglect of Diatomics of Differential Overlap (NDDO)

The most sophisticated neglect of differential overlap method is the NDDO. It includes all one center electron-electron repulsion integrals as INDO but retains the two center integrals of the form $\langle a b \mid c d\rangle$, where $a$ and $b$ are centered on the same atom and $c$ and $d$ also on the same atom. Hence, more terms are evaluated in the NDDO formalism than in the INDO and CNDO (Pople and Segal, 1965). The Fock matrix in NDDO is defined by both diagonal and off-diagonal matrix elements(Andrew, 2001).

For diagonal matrix elements $\left(a=b, F_{a b}=F_{a a}\right)$,

$$
\begin{equation*}
F_{a a}=H_{a a}+\sum_{c o n Y d} \sum_{o n Y} P_{c d}\left[\langle a a \mid c d\rangle-\frac{1}{2}\langle a c \mid a d\rangle\right]+\sum_{Z \neq Y} \sum_{\text {on } Z d} \sum_{\text {on } Z} P_{c d}\langle a a \mid c d\rangle . \tag{2.110}
\end{equation*}
$$

For off-diagonal matrix elements ( $a \neq b$, both $a$ and $b$ are on the same atom, say $Y$ ),

$$
\begin{equation*}
F_{a b}=H_{a b}+\sum_{c \text { on } Y} \sum_{d \text { on } Y} P_{c d}\left[\langle a b \mid c d\rangle-\frac{1}{2}\langle a c \mid b d\rangle\right]+\sum_{Z \neq Y c} \sum_{\text {on } Z d} \sum_{\text {on } Z} P_{c d}\langle a b \mid c d\rangle . \tag{2.111}
\end{equation*}
$$

For off-diagonal matrix elements ( $a \neq b, a$ and $b$ different atoms, say $Y$ and $Z$ ),

$$
\begin{equation*}
f_{a b}=H_{a b}-\frac{1}{2} \sum_{c}{ }_{o n Y} \sum_{d o n ~} P_{c d}\langle a c \mid b d\rangle . \tag{2.112}
\end{equation*}
$$

Various methods based on NDDO have been developed. The first method based on NDDO is the modified neglect of differential overlap (MNDO) introduced in 1976 (Dewar and Thiel, 1977). In the MNDO, the diagonal matrix elements,

$$
\begin{equation*}
H_{a a}=U_{a a}-\sum_{Z \neq Y}-Z_{\alpha}\left\langle a_{Y} a_{Y} \mid s_{Z} s_{Z}\right\rangle \tag{2.113}
\end{equation*}
$$

For off-diagonal matrix elements ( $a \neq b$, both $a$ and $b$ are on the same atom, say $Y$ ),

$$
\begin{equation*}
H_{a b}=-\sum_{Z \neq Y}-Z_{\alpha}\left\langle a_{Y} b_{Y} \mid s_{Z} s_{Z}\right\rangle . \tag{2.114}
\end{equation*}
$$

For off-diagonal matrix elements ( $a \neq b, a$ and $b$ different atoms, say $Y$ and $Z$ ),

$$
\begin{equation*}
H_{a b}=\frac{1}{2} S_{a b}\left(\beta_{a}^{Y}+\beta_{b}^{Z}\right) \tag{2.115}
\end{equation*}
$$

where $Z_{\alpha}$ is the atomic number of atom $Z, s_{Z}$ is s-orbital on atom $\mathrm{Z}, S_{a b}$ is the overlap integral and $\beta_{a}^{Y}$ is a parameter for atomic orbital $a$ centered on atom $Y$.

Other NDDO based methods like the AM1, PM3, MNDO/d, PM6, PM7 are improvements over MNDO and have been applied successfully for ground-state calculations (Silva-Junior and Thiel, 2010).

### 2.4 INDO/s formalism and parametrization

INDO/s is a variant of INDO approximation (Ridley and Zerner, 1973). It is an INDO formalism followed by configuration interaction singles (CIS) calculations. It was uniquely
developed and parameterised for excited states and spectroscopic studies. Its parameters were determined for organic molecules (Ridley and Zerner, 1973, 1976), transition metals (Bacon and Zerner, 1979, Zerner et al., 1980a) and lanthanide complexes (Kotzian et al., 1992).

### 2.4.1 INDO/s formalism and parameterisation for organic molecules

The INDO/S Fock matrix elements follow from those of the INDO approximations(Ridley and Zerner, 1973). See detail of the description of INDO matrix elements in section 2.3.3.7. Definition and determination of parameters within the INDO/S formalism are given as follows:

Considering s,p basis set, the one center integrals $U_{s s}$ and $U_{p p}$ are written as follows:

$$
\begin{gather*}
U_{s s}=-\left(Z_{I}-1\right) F_{s s}^{0}+\frac{1}{6} m G^{1}(s p)+I_{s}  \tag{2.1}\\
U_{p p}=-\left(Z_{I}-1\right) F_{s s}^{0}+\frac{2}{25}(m-1) F_{p p}^{2}+\frac{1}{6} l G^{1}(s p)+I_{p} \tag{2.2}
\end{gather*}
$$

where $Z_{I}$ is the atomic number of atom $I, F^{2}$ and $G^{1}$ are Slater-Condon factors and $I_{s}$ and $I_{p}$ are ionization potentials for electron in $s$ and $p$ orbital, respectively. The values of $F^{2}$ and $G^{1}, I_{\mu}$ are obtained from atomic spectra table (Moore, 1949).

The nuclear attraction integral is written as

$$
\begin{equation*}
V_{I J}=Z_{J} \gamma_{I J}, \tag{2.3}
\end{equation*}
$$

where $\gamma_{I J}$ is a two-center Coulomb integral and defined using the modified Mataga Nishimoto parametric expression (Mataga and Nishimoto, 1957),

$$
\begin{equation*}
\gamma_{I J}=\frac{f\left(\gamma_{I I}+\gamma_{J J}\right)}{2 f+R_{I J}\left(\gamma_{I I}+\gamma_{J J}\right)} . \tag{2.4}
\end{equation*}
$$

$R_{I J}$ is the separation between atomic centers $I$ and $J$ and

$$
\begin{equation*}
\gamma_{J J}=F^{0}(J J)=I_{J}-A_{J}, \tag{2.5}
\end{equation*}
$$

where $I_{J}$ and $A_{J}$ are ionization potential and electron affinity, respectively, obtained from atomic spectra data given by Moore. The one-center two-electron repulsion integrals are defined from Slater-Condon factors (Moore, 1949) as follows:

$$
\begin{gather*}
\langle s s \mid s s\rangle=F^{0}(s s)  \tag{2.6}\\
\langle s p \mid s p\rangle=0.3333 G^{1}(s p)  \tag{2.7}\\
\left\langle p_{x} p_{x} \mid p_{x} p_{x}\right\rangle=F^{0}(p p)+0.16 F^{2}(p p)  \tag{2.8}\\
\left\langle p_{x} p_{x} \mid p_{y} p_{y}\right\rangle=F^{0}(p p)-0.08 F^{2}(p p)  \tag{2.9}\\
\left\langle p_{x} p_{y} \mid p_{x} p_{y}\right\rangle=0.12 F^{2}(p p) \tag{2.10}
\end{gather*}
$$

The $f$ factors and $\beta$ parameters for $\mathrm{C}, \mathrm{H}, \mathrm{O}$, and N of the two-center two-electron repulsion were optimised to reproduce the spectra from benzene and pyridine geometries. $f_{\sigma \sigma}$ and $f_{\pi \pi}$ were set as 1.267 and 0.585 , respectively. The bonding parameters were set as $\beta_{s}=\beta_{p}=\beta_{I}$ and $\beta_{I}$ was obtained from a fit to spectral data (Voityuk, 2013). Without any re-parametrization, calculations of spectra of different organic molecules including naphthalene, pyrazine, pyrimidine, etc were carried out using the model.

### 2.4.2 INDO/s formalism and parametrization for transition metals

Unlike the parametrization of INDO/s for organic molecular that mostly involves the valence sp basis, the transition metal includes the d orbitals also. As such, a bit of modification is introduced into the INDO Hamiltonian as follows (Zerner et al., 1980a, Bacon and Zerner, 1979): The diagonal matrix element (one centre one-electron Fock matrix) is

$$
\begin{equation*}
F_{a a}=U_{a a}-\sum_{J \neq I} \gamma_{\overline{\bar{B}} B}\left\{\left(n_{s}+n_{p}\right)+\left(n_{d}\right)_{J}\right\}+\sum_{c d} p_{c d}\left[\langle\mu \mu \mid c d\rangle-\frac{1}{2}\langle\mu c \mid \mu d\rangle\right]+\sum_{c \notin I} P_{c c} \gamma_{c a} . \tag{2.11}
\end{equation*}
$$

The off-diagonal matrix element (one-centre two-electrons Fock matrix) is given by

$$
\begin{equation*}
F_{a b}=\sum_{c d} p_{c d}\left[\langle\mu \mu \mid c d\rangle-\frac{1}{4}\langle\mu c \mid \mu d\rangle\right] . \tag{2.12}
\end{equation*}
$$

The off-diagonal matrix element (two-centre two electron Fock matrix) is

$$
\begin{equation*}
F_{a b}=\beta_{a b}-\frac{1}{2} P_{a b} \gamma_{a b} . \tag{2.13}
\end{equation*}
$$

The one-electron core, $U_{a a}$ are calculated from ionization processes.
The bonding integral, $\beta_{a b}=\frac{1}{2}\left(\beta_{a}^{A}+\beta_{b}^{B}\right) \bar{S}_{a b}$,
where $\beta_{a}^{A}$ and $\beta_{b}^{B}$ are bonding parameters for orbital $a$ centered on atom $A$ and $b$ centered on atom $B$, respectively. The weighted overlap $\bar{S}_{a b}$ for spd basis are expressed as follows:

$$
\begin{gathered}
\bar{S}_{s s}=S_{s s} \\
\bar{S}_{s p}=S_{s p} \\
\bar{S}_{p p}=g_{\pi} f_{p \pi} S_{p \pi p \pi}+g_{\sigma} f_{p \sigma} S_{p \sigma p \sigma} \\
\bar{S}_{s d}=S_{s d} \\
\bar{S}_{d d}=g f_{d \delta} S_{d \delta d \delta}+g_{\pi} f_{d \pi} S_{p \pi d \pi}+g_{0} f_{d \sigma} S_{p \sigma d \sigma}
\end{gathered}
$$

$g_{a}$ and $f_{a}$ are geometric factors and interaction weighing factors respectively. $f_{p \pi}$ and $f_{p \sigma}$ are set to 0.64 and 1.267, respectively (Ridley and Zerner, 1973, 1976). $f_{d \pi}$ and $f_{d \sigma}$ are set to one. The parameters $\beta_{s}=\beta_{p}$ and $\beta_{d}$ were obtained empirically from spectra. The two-electron Coulomb integral, $\gamma_{I J}$ were in some cases evaluated from the Mataga Nishimoto expression (Zerner et al., 1980a) and in other cases evaluated over Slater-type orbitals (Bacon and Zerner, 1979, Zerner et al., 1980a). The one-centre two electron integrals were obtained from the Slater-Condon parameters.

### 2.4.3 Determination of electronic and optical properties with INDO/s

INDO/s was developed for calculations of ionization potentials, electronic excited states and spectra. It fails for geometry optimisation and some ground state calculations (Voityuk, 2013). INDO/s is employed for predicting low-lying vertical excitation energies and oscillator strength. These are carried out by first, performing a self-consistent field ground state calculation and thereafter, a CIS calculation is performed in order to obtain excita-
tion energies. The oscillator strength, $f_{o s}$ is then calculated with the expression,

$$
\begin{equation*}
f_{o s}=\frac{2}{3} \Delta E\left|\overrightarrow{D_{t r}}\right|^{2}, \tag{2.14}
\end{equation*}
$$

where $\Delta E$ is the excitation energy in eV , and $\overrightarrow{D_{t r}}$ is the transition dipole moment (in Debye)) defined by

$$
\begin{equation*}
\overrightarrow{D_{t r}}=\left\langle\psi_{f}\right| \vec{\mu}\left|\psi_{i}\right\rangle . \tag{2.15}
\end{equation*}
$$

INDO/s, when benchmarked with the well known Thiel data set for electronically excited energies of 28 small organic molecules, gave a mean absolute error $\approx 0.5 \mathrm{eV}$ but was found to overestimate oscillator strength (Silva-Junior and Thiel, 2010, Voityuk, 2013). The accuracy of INDO/s calculation for transition metals has not been reliably verified due to a lack of reliable data and a need for further parameterisation (Voityuk, 2013). Higher excitation energies are poorly predicted in INDO/s since calibrations were performed with low-lying excited state reference data (Silva-Junior and Thiel, 2010). INDO/s is also employed for the calculation of ionization potential ( $I_{P}$ ) and electron affinity $\left(E_{A}\right)$ in the following way:

$$
\begin{equation*}
I_{P}=-\varepsilon_{\text {НОмо }} \tag{2.16}
\end{equation*}
$$

and

$$
\begin{equation*}
E_{A}=-\varepsilon_{L U M O}, \tag{2.17}
\end{equation*}
$$

where $\varepsilon_{\text {Номо }}$ is the energy of the highest occupied molecular orbital (HOMO) while $\varepsilon_{\text {LUMO }}$ is the energy of the lowest unoccupied molecular orbital (LUMO). INDO/s predicts $I_{p}$ for molecules with a typical error $\approx 0.40 \mathrm{eV}$ but fails for transition metal complexes (Voityuk et al., 1999). Furthermore, INDO/s can be applied for calculating polarizability, charge distribution and excitation energy transfer (Voityuk, 2013).

### 2.4.4 Applications of INDO/s Methods

A lot of studies have been performed with INDO/s but here we discuss a few examples. Electronically excited states and UV-vis spectra of carbon nanostructures and their complexes have been largely studied with INDO/s. The results obtained compare well
with experiments (Feng et al., 1990, Tian et al., 2006). In particular, Zerner and coworker reported a low-lying band of $27,300 \mathrm{~cm}^{-1}$ from INDO/s which compare well with $25,900 \mathrm{~cm}^{-1}$ from experiment for $C_{60}$ (Feng et al., 1990). A mean error of 0.38 eV was reported from the calculation of the electronic gap with INDO/s for 60 different organic molecules (Hutchison et al., 2002). INDO/s has also been employed to study complex systems like carbon nanotubes (Kilina et al., 2012). It has also been applied in the study of excitation and charge transfer in DNA (Voityuk, 2006). Prediction of the structure of the material with desired electronic properties with the INDO/s method has been reported (Di Bella et al., 1993, Quarti et al., 2011).

### 2.4.5 Limitations and improvement in INDO/s

INDO/s, no doubt plays a significant role in the study of electronic and optical properties of large systems and nanoparticles. However, it is limited in some ways. First, while its accuracy for predicting the low-lying excited energies still needs improvement, it gives a poor prediction for higher excited states. Moreover, the parameterisation for transition metals are not reliable (Voityuk, 2013). With available powerful computing tools, significant improvement in the performance of INDO/s can be achieved by re-parameterising it with reliable data from experiment and a high-level ab-initio calculations. However, Only little improvements have been achieved in INDO/s since its development in the 70s. Voityuk modified and parameterised INDO/s for $\mathrm{C}, \mathrm{H}, \mathrm{N}$, and O with the theoretical bestestimated data (electronic excitation energies and oscillator strength) (Silva-Junior and Thiel, 2010). The newly obtained model which is called INDO/x performed better than INDO/s. The mean absolute deviate was obtained from INDO/x as 0.26 eV for singlet vertical excitation energies and 0.33 eV for triplets vertical excitation energies. The corresponding deviations for INDO/s are 0.56 eV and 0.64 eV , respectively. INDO/s was parameterised to reproduce spectrum from TDDFT for silver nanoclusters (Gieseking et al., 2016). To the best of our knowledge, re-parametrisation of INDO/s for a lot of elements including transition metals, sulphur and silicon have not been performed.

### 2.5 Quantum Mechanical Methods for Determination of Excitations Energies and

 Absorption SpectraArguably, experiments are generally the best and most accurate methods for the determination of electronic excitations energies and absorption spectra. The spectrophotometer has been extensively employed to determine the UV-Vis absorption spectra even for large systems such as biological molecules (Nilapwar et al., 2011). It has been used to estimate protein concentration(Aitken and Learmonth, 2009), DNA melting point(Nilapwar et al., 2011). Results from experiments have been applied reliably for benchmarking empirical and semi-empirical methods (Winget et al., 2003, Winget and Clark, 2005, Kayi and Clark, 2007, Stewart, 2007, Voityuk, 2014).However, experiments pose some challenges, which include high infrastructure costs, complexity in their procedures, and difficulties in describing vertical excitations (Schreiber et al., 2008, Silva-Junior et al., 2008, Voityuk, 2013). With the advent of powerful computing resources, high-level quantum mechanical methods have demonstrated the ability to yield results that are comparable to experiments (Barone, 2011). Some of the high-level methods for excitation energies and absorption spectra calculations include EOM-CCSD (Christiansen et al., 1995, Caricato et al., 2010), GW (van Setten et al., 2012, Leng et al., 2016), solution of the Bethe Salpeter Equation (BSE) (Nakanishi, 1969, Leng et al., 2016, Blase et al., 2018), Quantum Monte Carlo (QMC) method (Schautz et al., 2004), the CASPT2 (Andersson et al., 1990) and others.

The EOM-CCSD approach is known as the gold-standard for excitation energies and absorption spectra calculations. It is highly accurate with typical errors within 0.1 eV of experimental energies (Caricato et al., 2010, 2011, Bennie et al., 2017). It has been applied to a wide range of problems related to electronic excitations including excitation state vibrionic coupling. Also, the EOM-CCSD method contains no adjustable parameter and one does not have to deal with the problem of choice of active space as is the case in multi-reference reference methods such as CASPT2. It is a benchmark method for other molecular orbital theory methods like TDDFT (Caricato et al., 2011). However, EOM-CCSD is computationally prohibitive for large systems like nano-scale materials (e.g quantum dots) . It is restricted to systems with less than 20 atoms or so, within a moderate basis set (Caricato et al., 2010, Bennie et al., 2017).

Complete Active Space Perturbative Theory (CASPT2) method is a multi-reference approach that successfully describes electron correlations (Andersson et al., 1990). It was employed alongside the coupled-cluster ( CC2, CC3, CCSD) approaches in the well known Thiel's benchmark set for excitation states of 28 molecules (Schreiber et al., 2008, Silva-Junior et al., 2008). Reports have shown that CASPT2 compares well with EOM-CCSD with typical error within $0.2-0.3 \mathrm{eV}$ (Schreiber et al., 2008). CASPT2 has been applied for validation of ab-initio methods (Silva-Junior et al., 2008, Sauri et al., 2010) and calibration of semi-empirical methods (Silva-Junior and Thiel, 2010). However, like the EOM-CCSD, in practice CASPT2 is restricted to small atomic systems. The accuracy of BSE and QMC for excitation calculations have also been reported for small-sized systems.

Good alternative to the accurate methods discussed so far are the TDDFT and CIS(D) (Runge and Gross, 1984, Barone, 2011). They can handle moderate-sized molecules. Report have shown that excitation energies from TDDFT compare with Thiel's benchmark (data from CASPT2, CC methods etc) for 28 medium-sized organic molecules with a typical error of 0.27 eV (Silva-Junior et al., 2008). Also, valence states excitation obtained with TDDFT compare well with experiment with a mean absolute error of 0.23 eV (Leang et al., 2012). Gieseking and co-workers applied TDDFT to benchmark INDO/s absorption spectrum for silver nanoclusters (Gieseking et al., 2016). Hartree Fock followed by configuration interaction singles doubles (CISD) gives roughly comparable results with TDDFT. Nonetheless, TDDFT and CIS(D) are compute-intensive for large atomic clusters ( $\mathrm{N}>60$ atoms) and nano-sized materials.

Generally, semi-empirical quantum mechanical methods have been widely adopted for the studies of excitations and spectroscopy of large atomic clusters and molecules. These methods are computationally cheap but generally less accurate compared with accurate $a b$-initio methods. The most common, reliable and straight forward semi-empirical quantum mechanical methods are those based on the Hartree Fock formalism or molecular orbital theory. In particular, INDO/S was uniquely developed and parameterised for excitations and spectroscopy. It is still widely applied to study organic molecules and some transition complexes (Voityuk, 2013).

### 2.6 Development and Improvement in Semi-empirical Molecular Orbital Theory

The semi-empirical molecular orbital theory dates as far back as the early 1930s when Huckel proposed the Huckel theory applied, which is uniquely applied to $\pi$ molecular systems only. The molecular orbital is expressed in the basis sets of only the $\pi$ electrons. The method gives only a qualitative description of the electronic structure of planar conjugated systems and is useful for teaching in class (Hückel, 1931, Andrew, 2001). In the 1950s, the Pariser-Parr-Pople (PPP) model was developed as an improvement over the Huckel theory though, still applicable only to $\pi$ molecules. It includes the effect of electron-electron repulsion which is not well represented in the Huckel formalism. The PPP method is useful in spectroscopic calculations and for periodic systems (Pariser and Parr, 1953, Zerner, 1991). In 1963, Hoffman extended the Huckel theory to other molecules other than the conjugated $(\pi)$ molecules. The method, which is known as the Extended Huckel Theory (EHT) includes all valence electrons in its calculations. EHT has found application in band structure calculations and is suitable for the study of metallic systems. However, it fails for geometry optimisations (Hoffmann, 1963, Zerner, 1991, Andrew, 2001).

Pople and Co-workers in the 1960s proposed some methods based on zero differential overlap (ZDO) approximations, namely Complete Neglect of Differential Overlap (CNDO), Intermediate Neglect of Differential Overlap (INDO) and Neglect Diatomic of Differential Overlap(NDDO). The CNDO developed in 1965, was the first model to implement ZDO approximation (Pople et al., 1967, Andrew, 2001). The CNDO (also known as CNDO/1) has been fairly successful in predicting some physical properties of a molecular system and sometimes used to generate initial guess in ab-initio methods. However, it predicts equilibrium bonding and heat of formation poorly. Pople and co-workers (1966) proposed CNDO/2(Pople, 1965, Pople et al., 1967), an improved formalism over CNDO/1. In contrast to CNDO/1, penetrating integral effect in CNDO/2 is well defined and, thus, it predicts better equilibrium bond lengths, bond angles and dipole moments. For instance, $\mathrm{CNDO} / 2$ predicts the bond length of carbon monoxide(CO) to be 1.190 a.u which compares well with the experiment value of 1.128 a.u. Unfortunately, CNDO/2 predicts the heat of formation poorly (Andrew, 2001, Segal, 2012). Another variant of CNDO is the CNDO/S, which is developed to calculate electronic spectra.

Here, the resonance integral was redefined and spectroscopic data were included for the optimisation of its parameters (Bene and Jaffe, 1968). However, its shortcomings have been reported (Hata et al., 2006). INDO approximation is a more sophisticated method than CNDO. It was proposed by Pople and co-workers to overcome some problems in the CNDO approximation. In the INDO approximation, in contrast to the CNDO approximation, the one-center two-electron repulsion integrals are not neglected and by this, spin effects are taken into account. While the CNDO formalism does not distinguish between the energy of a singlet and triplet states, INDO does. INDO was found to reproduce similar results as in $\mathrm{CNDO} / 2$. For instance, the dipole moment of $\mathrm{CH}_{2}$ calculated with INDO and CNDO/2 are 2.17 Debye and 2.26 Debye, respectively. The bond angle of $\mathrm{CH}_{2}$ from INDO and $\mathrm{CNDO} / 2$ are $107.2^{\circ}$ and $108.6^{\circ}$, respectively (Segal, 2012).

Important variants of INDO are the INDO/1, INDO/s, SINDO, and SINDO/1. Zener and co-workers (1973) developed INDO/1 employed for geometry optimisation calculations and INDO/s for electronic spectroscopy calculations (Ridley and Zerner, 1973). INDO/s was parameterised at the CIS level with spectroscopic data. It has been widely employed for calculation electronic transition in organic molecules and transition metal complexes (Voityuk, 2013). It does predict well $d \rightarrow d^{*}$ orbital transition and oscillator strengths for weak transitions. However, INDO/s is inaccurate for charge transfer and Rydberg state calculations (Zerner, 1991, Voityuk, 2013).

The NDDO is the most sophisticated and widely employed semi-empirical method. It has received significant improvement since its development. The modified neglect of differential overlap (MNDO), the first method to implement the NDDO approximation, was developed by Dewar and Thiel in 1977 (Dewar and Thiel, 1977). It has been parameterised for a lot of elements (Zerner, 1991). MNDO is successful in predicting, polarizabilities, hyperpolarizabilities, and other properties. Although it has advantages over the MINDO methods, it has the challenge of poor prediction of the hydrogen bond, underestimation of rotational barriers, hypervalent molecules are unstable, underestimation of electronic excitations among other issues (Zerner, 1991, Andrew, 2001). The problems associated with MNDO are partly due to overestimation atom-atom repulsion for the sum of vander Waals equivalent distance (Andrew, 2001). To overcome this problem, the

Dewar group developed the Austin Model 1 (AM1) in 1985. AM1 employs Gaussian functions to describe the atom-atom repulsion (Dewar et al., 1985) thereby increasing the number of parameters in AM1 as compared to MNDO. However, the computation time in both MNDO and AM1 is about the same (Andrew, 2001, Kayi, 2009). Also, in AM1, the reparameterisation of MNDO parameters was performed. This was necessary because MNDO parameterisation was considered unreliable since it was performed in the 1970s when computational resources were limited.(Kayi, 2009). AM1 has been parameterised for main group elements. It performs well for hydrogen bonds and activation energy predictions. However, some hypervalent systems e.g alkyl and peroxide compounds are poorly described within the AM1 model (Kayi, 2009). The parametric model 3 (PM3) (Stewart, 1989) was introduced in 1989. It was parameterised for more elements than AM1 (Zerner, 1991) and was also obtained by modifying the core-core term of MNDO and reparameterising. While AM1 parameters were derived by intuition and chemical knowledge, PM3 parameters, on the other hand were obtained from parameter optimisation. Although AM1 and PM3 have some of their parameters different, they both predict well some molecular structure properties with the same level of accuracy (Andrew, 2001). However, PM3 predicts hydrogen bond to be too short and poorly predicts the amide bond rotations.

Generally, MNDO, AM1, and PM3 are faced with the problem of poor prediction of weakly bonded molecules and parameters of metals in these models are unreliable. MNDO/d was introduced to describe metals and transition metals. It involves the inclusion of the d orbitals into MNDO. Thus, MNDO/d is applied for hypervalent molecules, transition elements and it is helpful in describing the polarization of the second-row elements. MNDO/d performs better than PM3, AM1, MNDO, especially in predicting some properties like the heat of formation for hypervalent compounds and transition metals (Thiel and Voityuk, 1996, Jensen, 2017, Chatfield and Christopher, 2002). MNDO/d has been parameterised for $\mathrm{Cd}, \mathrm{Zr}, \mathrm{Zn}, \mathrm{Mg}$, and Na . Nonetheless, just like the original MNDO, it performs poorly in predictions of hydrogen bonding (Thiel and Voityuk, 1996, Chatfield and Christopher, 2002). Henre and co-worker also added d functions to PM3 to obtain a model called PM3(tm). Its parametrization was based on geometries of systems and as such, it is applied for molecular geometry generations. However, this method seems unreliable since the method is very scarce in literature (Børve et al., 1997,

Chatfield and Christopher, 2002). There is also the AM1(d) model in which d functions were added to AM1 and its core-core interaction term modified. It was first parametrized for molybdenum in the year 2000 by Voityuk and Rosch (Voityuk and Rösch, 2000) and latter optimised for potassium by Lopez and York (Lopez and York, 2003). PM5 was developed by reparametrization of PM3 and the core-core repulsion term replaced with "pure" parameters. It shows a slight improvement over PM3 and has been parameterised for many elements (Kayi, 2009). In 2003, Clark and co-workers introduced the AM1* (Winget et al., 2003) which is based on AM1. With d functions added and the core interaction redefined for some elements other than O, F, N, C, H. Hence, AM1* reproduces the same result as AM1 for O, F, N, C, H (Winget and Clark, 2005, Winget et al., 2003, Kayi and Clark, 2007). Without a need for a change to AM1 formalism, AM1 was parametrized in 2006 to obtain RM1 the model. The reparametrization was performed for ten elements only, namely, I, Br, Cl, F, P, S, O, N, C, H (Rocha et al., 2006).

PM6 was developed in 2007 by Stewart by modifying the NDDO core-core repulsion, including d function and reparametering. Parametrization was performed for seventy elements. Report has shown the improvement of PM6 over the NDDO methods discussed so far. The heats of formation and bond length are better predicted within PM6 than PM3 and AM1 (Stewart, 2007). Some variants of PM6 have also been introduced which include PM6-DH (Rezac et al., 2009), PM6-DH2 (Fanfrlik et al., 2010) and PM6D3H4X. They are improvements over PM6 for non-covalent interactions (Hostavs et al., 2013). The most recent PM series is the PM7 developed in 2013 by Stewart. It is an improvement over PM6 and includes terms to describe non-covalent interactions. It performs better than PM6 in predicting bond length and heat of formation (Stewart, 2013). Recently, Voityuk improved upon INDO/S for C, H, N and O to obtain a model called INDO/x which reproduces the excitation energies of the TBE-2 organic molecules with typical error of 0.26 eV against 0.56 eV from INDO/S (Voityuk, 2014).

### 2.7 Parameterization scheme in NDO semi-empirical methods

The goal of any semi-empirical molecular orbital methods is to reproduce experimental or high-level ab-initio results with less computational effort and resources. In ab-initio
molecular orbital methods, the different center-integrals are responsible for the computation expense. In the semi-empirical scheme, however, the less important integrals like three and four-centered integrals are omitted while the one and two-centered integrals are replaced with parameters or parametric expressions. These parameters are then calibrated to reproduce experimental or high-level ab-initio data. The accuracy of a semi-empirical method depends strongly on the accuracy of their optimised parameters. Provided the essential physics are retained, the optimised parameters can correct for the error introduced by the drastic approximations and make the semi-empirical methods even more comparable to experiments than the ab-initio counterparts (Zerner, 1991).

In parameterisation, the collection of reliable data, application of weighting factors and the parameterisation procedure are important steps in obtaining reliable parameters. (Thiel et al., 2000, Kayi, 2009)

### 2.7.1 Reference Data

Molecular reference data is a set of accurate data comprising the chemical and structural properties of molecular or atomic systems obtained from experiments or high-level $a b$ initio methods. The data are sometimes from experiments or high-level ab-initio (CC, CASPT2, GW/BSE, QMC, B3LYP) calculations or both. These data are expected to represent the main features of their potential applications (Thiel et al., 2000, Kayi, 2009). Some of the important properties captured in the reference data are dipole moments, geometrical structures, the heat of formation, ionization potential and reaction energies. For instance, parametrization for geometrical properties is obtained from fits to reference bond lengths, bond angles, and dihedrals. For spectra and optical properties, calibrations are done using transition dipole moments, oscillator strength, excitation energies. Experimental data and high-level data databases are available from different sources including the Cambridge database (Groom et al., 2016), the NIST Webbook (Linstrom and Mallard, 2001), and Theoretical best estimate(Silva-Junior and Thiel, 2010).

### 2.7.2 Parametrization procedure

Parametrization has to do with optimisation or determination of the parameters of semiempirical methods in order to reproduce experimental or high-level ab-initio data. Optimisations are carried out by varying the model parameters in order to minimize an error function such as (Kayi, 2009, Govender et al., 2014)

$$
\begin{equation*}
f=\sum_{a} \omega_{a}\left[\eta_{a}^{c}-\eta_{a}^{r e f}\right]^{2}, \tag{2.1}
\end{equation*}
$$

where, $\eta_{a}^{r e f}$ and $\eta_{a}^{c}$ are the values of the experiment/ab-initio and computed $a_{t h}$ property of interest,respectively and $\omega_{a}$ is the $a_{t h}$ weighing factor. The optimised parameters are obtained when the following conditions are fulfilled (Govender et al., 2014, Kayi, 2009):

$$
\begin{align*}
& \frac{d f}{d P_{a}}=0  \tag{2.2}\\
& \frac{d^{2} f}{d P_{a}^{2}}>0 \tag{2.3}
\end{align*}
$$

Where $\mathrm{P}_{a}$ is the $a_{t h}$ parameter. In the parameterisation procedure, the guess of the starting parameter can be very challenging. Starting parameters sometimes are obtained from the existing parameters of other methods.

A trial and error approach is not a good option to carry out a successful parametrization. Rather, various algorithms like the steepest descent and non-linear square-least which have been developed, can be employed. Another important aspect of parameterisation is to verify the transferability of the parameters/model obtained. That is, can the optimised parameters produce accurate results for geometries not included in the training set? Usually, not all different set of optimised parameters of a model are transferable. Hence, parameterisations must be performed carefully, rigorously and a constant check of the transferability of each set optimised parameters must be carried out.

### 2.8 Theoretical studies of quantum dots

There are different methods in literature for the studies of large atomic systems and quantum dots. The one-band effective mass approximation method (EMA) is a simple
method that has been extensively employed for the study of nanostructure systems, including quantum dots. The approach assumes holes and electrons in the quantum dot are confined by an effective potential (potential barrier). Although qualitative results are obtained with this method, it has helped to gain some insights into some characteristics features of the nanostructure systems. EMA has been extensively deployed to study the quantum confinement effect in nanostructures (Borah et al., 2018). Reports have shown that EMA describes the electronic gap well for large nanoparticles but deviates largely from the experiment for small nanoparticles (Vatankhah and Ebadi, 2013). EMA has also been employed to study the dependence of the electronic gap and the binding energy of the dot on some physical properties (Baskoutas et al., 2004). EMA fails for medium-sized and smaller quantum dots because it does not account for atomistic effects.(Fu et al., 1998). Also, within this method, the study of the structural, electronic, and optical properties of a system is not straight forward. (Borah et al., 2018).

The k.p method is an improvement over EMA. It is comprised of multiple bands but still limited by the non-inclusion of atomistic effect. As in the EMA case, the k.p methods fail for intermediate and small nanostructures. The empirical pseudopotential method (EPM) (Wang and Zunger, 1994) is a significant advancement over EMA and k.p methods. It was developed in 1960, originally for bulk material calculations but has been modified for nanostructure calculations. Within the EPM frame, atomistic effects are accounted for using empirically determined pseudopotentials (Wang and Zunger, 1994, Galli et al., 2002). EPM has been successfully applied for the determination of the electronic and optical properties of nanostructures. Zunger and co-workers in their work-study employed EPM to determine the electronic and optical properties of silicon quantum dots (Wang and Zunger, 1994, 1997) and CdSe quantum dots (Wang and Zunger, 1996). Bester employed the EPM followed by CIS to study the electronic excitations of nanostructures.(Bester, 2008). However, EPM involves a huge number of plane-wave basis sets for large systems like quantum dots. Solving the corresponding huge matrix requires a lot of computational time and resources. However, Zunger and co-workers developed a method to address this problem in the EPM method (Wang and Zunger, 1996).

Tight binding (TB) method is another semi-empirical method which includes the atomistic effect in its descriptions of electronic structure (see section 2.3.3.1 for a description
of TB). TB methods have been successfully applied for ground state properties of bulk materials. Also, significant improvements of the method has been achieved and it is being employed in the study of finite systems like quantum dots. Suman and co-workers reported a qualitative result of the electronic state of the quantum dots using the tightbinding framework (Dhayal et al., 2014). The method has also been applied to study the optical properties and effect of strain on the electronic state of the quantum dots (Ramaniah and Nair, 1993, Santoprete et al., 2003, Schulz, 2007). A variant of the tightbinding method is the density functional tight binding (DFTB), a method based on DFT (see section 2.3.3.2 for description DFTB). DFTB has been employed for nanostructure optimisations (Zonias et al., 2009, Fedorov et al., 2016),to and study electronic and optical properties of atomic system (Wilson et al., 2014, Darghouth et al., 2015). Despite the vast application of the TB and DFTB, they are limited due to the challenges of transferability. Though, the calculation of excitation energies and spectra have been reported within these methods (Nishimoto, 2015), the calculations are not straight forward because their wavefunctions are not explicitly defined. In addition, DFTB are inaccurate in predictions of properties like the electronic gap, ionization potential and electron affinity. (Darghouth et al., 2015).

Quantum dots can also be studied using methods based on the Hartree Fock formalism. Paramount among these methods are the NDDO, INDO, and CNDO based methods described earlier. These methods are a drastic approximation to Hartree Fock and carry out their calculations with minimal basis sets. They differ from the TB and EPM discussed above in that, their procedure includes self-consistent fields which aids transferability. The advent of powerful computational resources and high-level data have caused a significant improvement in these methods and thus, have been adopted for large atomic clusters and nanocluster calculations. PM3 and AM1, methods based on NDDO, have been applied for the study nanoparticle structure, electronic and optical properties (Robles et al., 1999, Wang et al., 2008). The most recent improvement in NDDO based method, PM6 and PM7, have been employed to compute polarizabilities for molecules and nanoparticles. The accuracy was found to reduce moving from molecules of small size to nanoparticles (Praveen et al., 2015). MSINDO, a method based on INDO, has also been applied for nanoparticle calculations (Jug and Wichmann, 2000, Wahab, 2012). INDO/s, a method widely applied for excitation energies and spectroscopy calculation,
has been employed to study electronic properties (Reimers and Hush, 2001), optical properties of silver nanoclusters (Gieseking et al., 2016), carbon nanotube (Furmanchuk et al., 2012) and absorption spectra of $\mathrm{TiO}_{2}$ nanoparticles (Persson et al., 2000).

### 2.9 Software Packages for quantum molecular structure calculations

Several computer software packages for ab-initio and semi-empirical quantum electronic structures calculations have been developed and implemented. These packages include GAMESS (Guest* et al., 2005), Gaussian, NW-CHEM (Valiev et al., 2010), Q-CHEM (Shao et al., 2015), ORCA (Neese, 2012, 2018), MOPAC (Stewart, 1990), MOLPRO, and so on. While some of these packages are open sources others are not. In particular, the ORCA (Neese, 2012, 2018), a close-source but free code package, has been widely employed for the implementation of DFT, Hartree Fock and post-Hartree Fock methods, and semi-empirical methods. Many semi-empirical methods including AM1, MNDO, PM3, and ZINDO codes have been implemented in ORCA. ORCA implements the ZINDO/1 and ZINDO/2 methods for geometry optimisation and ZINDO/s (also known as INDO/s) for electronic excitations and spectroscopy calculations.

The ZINDO/s executed in the ORCA software has a semi-empirical Hamiltonian which as presented in equations 2.11, 2.12, and 2.13 have the following adjustable Hamiltonian parameters for each atom, where applicable:

1. Interaction factors: $f_{s s \sigma}, f_{s p \sigma}, f_{s d \sigma}, f_{p p \sigma}, f_{p d \sigma}, f_{d d \sigma}, f_{p p \pi}, f_{p d \pi}, f_{d d \pi}$, and $f_{d d \delta}$
2. Core integral: $\mathrm{U}_{s s}, \mathrm{U}_{p p}, \mathrm{U}_{d d}$ and $\mathrm{U}_{f f}$
3. Basis set parameters:
(a) Number of Slater type orbitals: $\mathrm{N}_{s}, \mathrm{~N}_{p}, \mathrm{~N}_{d}$, and $\mathrm{N}_{f}$ for $s, p, d$, and $f$ orbitals, respectively.
(b) Exponents: $\zeta_{s 1}, \zeta_{s 2}, \zeta_{p 1}, \zeta_{p 2}, \zeta_{d 1}, \zeta_{d 2}, \zeta_{f 1}$ and $\zeta_{f 2}$ for $s, p, d$, and $f$ orbitals.
4. Resonance integrals: $\beta_{s}, \beta_{p}, \beta_{d}$ and $\beta_{f}$ for $s, p, d$, and $f$ orbitals, respectively.
5. Number of electrons: $\mathrm{N}_{e l}$
6. Gamma parameters(one-center, two electron integrals): $\gamma_{s s}, \gamma_{s p}, \gamma_{s d}, \gamma_{s f}, \gamma_{p p}, \gamma_{p d}$, $\gamma_{p f}, \gamma_{d d}, \gamma_{d f}$, and $\gamma_{f f}$.
7. Slater-Condon parameters: $\mathrm{F} 2_{p p}, \mathrm{~F} 2_{p d}, \mathrm{~F} 2_{d d}, \mathrm{~F} 4_{d d}, \mathrm{G} 1_{s p}, \mathrm{G} 1_{p d}, \mathrm{G} 2_{s d}, \mathrm{G} 3_{p d}$, $\mathrm{R} 1_{\text {sppd }}, \mathrm{R} 2_{\text {sdpp }}$ and $\mathrm{R} 2_{\text {sddd }}$
8. Nuclear interaction parameters: $\mathrm{NR}_{n}(\mathrm{n}=1,2,3, \ldots, 13)$
9. Parameters for spin orbit coupling: $\mathrm{SOC}_{p}, \mathrm{SOC}_{d}$ and $\mathrm{SOC}_{f}$

## Chapter 3

## MATERIALS AND METHOD

This chapter gives a detailed report on how the research work was carried out. The materials used for the work include high performance computing (HPC) devices, ORCA 4.0 (Neese, 2012), Amoeba Optimizer (Press et al., 2007), MOPAC 7 (Stewart, 1990), Avogadro, QuantumATK (Stradi et al., 2017), Gnuplot 4.6 (Racine, 2006) and Gabedit 2.5.0 (Allouche, 2008, 2011). For more information on these materials, the reader is referred to appendix A .

In this work, a new method capable of producing accurate excitation energies and absorption spectra for large clusters and quantum dots was developed. It involves parameterising the Intermediate Neglect of Differential Overlap for spectroscopy (INDO/s) Hamiltonian model to reproduce excitation energies for homogeneous diatomics. This new method was tested on silicon, zinc, cadmium, sulphur, zinc sulphide and cadmium sulphide clusters. In the new method which is presented in this thesis:
(i) for a given diatomics e.g $\mathrm{Si}_{2}$, determine $\mathrm{V}_{e}$ EOM-CCSD excitation energies for different atom-atom separations of the system. We need $\mathrm{V}_{e}=8$ lowest vertical excitations. We note that, in principle, the EOM-CCSD excitation energies can be replaced by energies from any high-level ab-initio method such as GW, BSE, MRCI. While these methods are prohibitive for large systems, the computational resources required for small systems such as diatomics is affordable.
(ii) Parameterise the model Hamiltonian (in this case, INDO/S) to reproduce the $a b$ initio excitation energies calculated in (i) above.
(iii) The one-electron terms $\mathrm{U}_{s s}, \mathrm{U}_{p p}, \mathrm{U}_{d d}$ are shifted in order to reproduce accurate first ionization potential for each atom. This does not change the excitation energies since these are differences in energies and, thus, are not affected by a constant shift of all the energies. However, this shift is important in ensuring transferability when inhomogeneous diatomics (e.g CdS) are being investigated.

Note that:
(i) Since only diatomics are involved, only one and two-center integrals feature in this approach. This is perfectly consistent with the NDDO Hamitonian (sect. 2.3.3.8) which contain terms involving one and two-center integrals ( with no three- and four-center integrals in the approximation). Thus, the approach presented captures all the integrals present in NDDO. This is in contrast to common methods which parameterise using data from systems containing more than two atoms. The $a b-$ initio data for systems with more than two atoms contain effects from three-center and more (if more than three atoms are in the systems). Thus, parameterising NDDO Hamiltonian to reproduce properties of systems with more than two atoms would force the one and two-center integrals become 'effective parameter' capturing the effect of three- and four-center integrals in an effective or average way. It is clear that this approach would lead to parameters which change with the surrounding atomic configurations i.e. these parameters/models will not typically be transferable. New parameters will be required for different atomic environments.
(ii) As far as we know, this is the first work that parameterises semi-empirical Hamiltonians to the energies of only one and two-atom systems.
(iii) Since the new method presented in this work completely neglects the three- and four-center integrals, when used for clusters with more than two atoms, new terms beyond the NDDO formalism need to be included. This will be the topic for future work (see chapter 5).
(iv) Having obtained the parameters for diatomics, without any changes, these parameters are used for the calculations of properties of clusters with more than two atoms. The performance of these parameters for these kind of clusters shows the extent to which they are transferable parameters.

### 3.1 Calculation of benchmark excited state energies and UV-Vis absorption spectra from ab-initio methods

The semi-empirical Hamiltonian model training (parameterisations) and validation data sets are composed of benchmark excitation energies and absorption spectra from highlevel ab-initio methods. The high-level ab-initio methods employed were EOM-CCSD, for the training on diatomics and TDDFT, and CIS(D) for validation. All calculations were done with a large basis set namely, DEF2-TZVPP basis set (see Ref. (Weigend and Ahlrichs, 2005)). The calculations were performed using the ORCA 4.0 package.

### 3.1.1 Calculation of excited state energies for dimer geometries

The training data sets used for parameterisations in this work are vertical excitation energies from EOM-CCSD calculations for homogeneous diatomics. The vertical excitation energies calculations were carried out for $\mathrm{Si}_{2}, \mathrm{~S}_{2}, \mathrm{Cd}_{2}$ and $\mathrm{Zn}_{2}$ of different separations at the EOM-CCSD/DEF2-TZVPP level. (i) For $\mathrm{Si}_{2}$, the calculations were performed at different separations between 1.8 and $3.0 \AA$ in steps of $0.2 \AA$ with the triplet state of the ground state configurations. Eight lowest vertical excitation energies were obtained for each separation, giving a total of 64 excitations for all eight separations considered. (ii) For $S_{2}$, a total of 20 excitation energies were calculated at five different separations from 1.8 and $2.6 \AA$ in steps $0.2 \AA$. The $S_{2}$ excitations were also performed from the triplet ground state configurations. (iii) $\mathrm{Cd}_{2}$ vertical excitation energies were obtained from singlet ground-state configurations at different separations from 2.0 and $4.0 \AA$ in steps of $0.25 \AA$. A total of 88 vertical excited state energies were obtained for $\mathrm{Cd}_{2}$. (iv) For $\mathrm{Zn}_{2}$, the configurations used are the same as for $\mathrm{Cd}_{2}$ described above.

### 3.2 Parameterisation of the ZINDO/s (INDO/s) Hamiltonian model

Using the amoeba optimisation algorithm, the Hamiltonian matrix, H of the Zerner Intermediate Neglect of Differential Overlap for spectroscopy (ZINDO/s) also known as INDO/s (Intermediate Neglect of Differential Overlap for spectroscopy) was parameterised. The ORCA 4.0 software was then used to solve the Fock secular equation fol-
lowed by diagonalisation of the CIS matrix.

The secular equation is given as

$$
\begin{equation*}
H C=C \varepsilon \tag{3.1}
\end{equation*}
$$

where $H$ is the Hamiltonian matrix, $C$ is the orbital coefficient and $\varepsilon$ is the orbital energy. The elements of the Hamiltonian matrix $H$ as detailed in sect. 2.3.3.7 are expressed as:

- $H_{a a}$ : diagonal elements

$$
\begin{equation*}
H_{a a}=U_{a a}+\sum_{c} P_{c c}\left[\langle a a \mid c c\rangle-\frac{1}{2}\langle a c \mid a c\rangle\right]+\sum_{B \neq A}\left(P_{B B} \gamma_{A B}-Z_{B} \gamma_{A B}\right), \tag{3.2}
\end{equation*}
$$

where $U_{a a}$ is the one center integral, $\mathrm{P}_{c c}$ is the density matrix, $\langle a a \mid c c\rangle$ and $\langle a c \mid a c\rangle$ are the Coulomb and exchange integrals, respectively. $\gamma_{A B}$ is the two-center twoelectron repulsion integral and $Z_{B}$ is the atom $B$ effective atomic number.
$P_{B B}$ is expressed as

$$
\begin{gather*}
P_{B B}=\sum_{b \in B} P_{b b}  \tag{3.3}\\
P_{a b}=2 \sum_{i} C_{a i} C_{b i}, \tag{3.4}
\end{gather*}
$$

where $a, b$, and $c$ are atomic orbitals. $\mathrm{C}_{a i}$ and $\mathrm{C}_{b i}$ are orbital coefficients.

- $H_{a b}$ : where both $a$ and $b$ are on the same atom

$$
\begin{equation*}
H_{a b}=\frac{1}{2} P_{a b}[3\langle a b \mid a b\rangle-\langle a a \mid b b\rangle] \tag{3.5}
\end{equation*}
$$

- $H_{a b}$ : where $a$ and $b$ are on different atoms $A$ and $B$, respectively

$$
\begin{equation*}
H_{a b}=\frac{1}{2}\left(\beta_{a}+\beta_{b}\right) S_{a b}-\frac{1}{2} P_{a b} \gamma_{A B}, \tag{3.6}
\end{equation*}
$$

where $\beta_{a}$ and $\beta_{b}$ are resonance integrals for atoms $A$ and $B$, respectively. $S_{a b}$ is the overlap integral calculated using atomic orbitals with exponents, $\zeta_{a}$ and $\zeta_{b}$ for orbitals $\psi_{a}$ and $\psi_{b}$, respectively.

The Hamiltonian matrix adjustable parameters are namely:

1. One-center integral, $U_{a a}(a=s, p, d \ldots)$
2. One-center Coulomb integrals ( $\langle a a \mid b b\rangle$ and $\langle a a \mid c c\rangle$ ), $\gamma_{a b}$
3. Slater Condon parameters $\left(F^{0}, G^{1}, F^{1}, F^{2}\right.$ etc) used in defining one-center exchange integrals ( $\langle a b \mid a b\rangle$ and $\langle a c \mid a c\rangle$ ).
4. Bonding parameters, $\beta_{a}$ and $\beta_{b}$
5. The gammas ( $\gamma_{A A}$ and $\gamma_{B B}$ ) parameters upon which two-center coulomb integral $\left(\gamma_{A B}\right)$ depend (Mataga and Nishimoto, 1957):

$$
\begin{equation*}
\gamma_{A B}=\frac{f\left(\gamma_{A A}+\gamma_{B B}\right)}{2 f+R_{A B}\left(\gamma_{A A}+\gamma_{B B}\right)} \tag{3.7}
\end{equation*}
$$

$R_{A B}$ is the separation between atomic centers A and B .
6. Exponents ( $\zeta$ ) of the atomic orbital $\left(\phi(r) \propto e^{-\zeta r}\right)$

After solving Eq. 3.1, a CIS calculation is carried out. The CIS is a post-Hartree Fock method whose wavefunction, $\Psi$ is given as

$$
\begin{equation*}
\Psi_{C I S}=l_{0} \Phi_{0}+\sum_{i, a} l_{i}^{a} \Phi_{i}^{a} \tag{3.8}
\end{equation*}
$$

where $\Phi_{0}$ and $\Phi_{i}^{a}$ are the Hartree Fock groundstate wavefunction and singly-excited configuration Slater determinants, respectively Szabo and Ostlund (2012). Excited states are obtained by diagonalizing the CIS matrix, $\mathbf{A}=\left\langle\Psi_{C I S}\right| \hat{H}\left|\Psi_{C I S}\right\rangle$, whose matrix elements for a closed-shell system are given by

$$
\begin{equation*}
\mathbf{A}_{i j}^{a b}=\left\langle\Phi_{i}^{a}\right| \hat{H}\left|\Phi_{j}^{b}\right\rangle=\delta_{i j} \delta_{a b}\left(\varepsilon_{a}-\varepsilon_{i}\right)-J_{i a}+2 K_{i a}, \tag{3.9}
\end{equation*}
$$

where $\varepsilon_{j}$ and $\varepsilon_{i}$ are the orbital energies and $J_{i j}$ and $K_{i j}$ are the Coulomb and exchange integrals.

The ZINDO/s Hamiltonian matrix was parameterised with benchmark excitation energies obtained from EOM-CCSD for $\mathrm{Si}, \mathrm{S}, \mathrm{Zn}$, and Cd diatomics of different separations
(section 3.1.1). The parameters were adjusted to minimize the error function $\chi^{2}$ given as

$$
\begin{equation*}
\chi^{2}=\frac{1}{N} \sum_{j=1}^{N}| | E_{j}^{\mathrm{f}}-E_{j}^{\mathrm{a}}|-\varepsilon| \tag{3.10}
\end{equation*}
$$

The set of parameters that minimizes $\chi^{2}$ were chosen as the optimised parameters and the mean absolute errors were calculated using

$$
\begin{equation*}
M A E=\frac{1}{N} \sum_{j=1}^{N}\left|E_{j}^{\mathrm{f}}-E_{j}^{\mathrm{a}}\right| \tag{3.11}
\end{equation*}
$$

where $E_{j}^{f}$ and $E_{j}^{a}$ are the jth excitation energies from the fit and ab-initio calculations, respectively, N is the total number of $j$ th excitation energies and $\varepsilon$ gives the extent of the accuracy to be achieved.

For $\mathrm{Si}_{2}$ and $\mathrm{S}_{2}$, eleven parameters each were optimised, namely $U_{s s}, U p p, \zeta_{s}, \zeta_{p}, \beta_{s}$, $\beta_{p}, \gamma_{s s}, \gamma_{s p}, \gamma_{p p}, G_{s p}$ and $F_{s}$. $\mathrm{N}=64$ for $\mathrm{Si}_{2}$ and 20 for $\mathrm{S}_{2}$. The value of $\varepsilon$ was set equal to 0.1 eV for both $\mathrm{Si}_{2}$ and $\mathrm{S}_{2}$.

Twenty-four and twenty-five parameters were optimised for $\mathrm{Zn}_{2}$ and $\mathrm{Cd}_{2}$, respectively. The parameters include $U_{s s}, U_{p p}, U_{d d}, \zeta_{s}, \zeta_{p}, \zeta_{d}, \beta_{s}, \beta_{p}, \beta_{d}, \gamma_{s s}, \gamma_{s p}, \gamma_{p p}, \gamma_{s d}, \gamma_{p d}, \gamma_{d d}$, $G 1_{s p}, G 1_{p d}, G 2_{s d}, G 3_{p d}, F 2_{p p}, F 2_{p d}, F 2_{d d}$ and $F 4_{d d} . \mathrm{N}=88$ excitations and $\varepsilon=0.0 \mathrm{eV}$ for each of $\mathrm{Zn}_{2}$ and $\mathrm{Cd}_{2}$.

As mentioned in sect. 2.9, the third and final step of this new method is to shift $U_{i i}$ ( $i$ $=s, p, d)$ in order to reproduce first ionization potentials. During the parameterisation, the $U_{i i}$ ( $\mathrm{i}=\mathrm{s}, \mathrm{p}, \mathrm{d}$ ) were not directly optimised but their differences, $U_{p p}-U_{s s}$ and $U_{d d}-U_{s s}$ were. Keeping the differences constant $\left(U_{p p}-U_{s s}\right.$ and $\left.U_{d d}-U_{s s}\right), U_{i i}(\mathrm{i}=\mathrm{s}, \mathrm{p}, \mathrm{d})$ were optimised to reproduce the first ionization potential (IP) obtained with Coupled-Cluster Singles and Doubles (CCSD)/DEF2-TZVPP for $\mathrm{Si}, \mathrm{S}, \mathrm{Zn}$, and Cd . Ionization potential (IP) was obtained as follows:

$$
\begin{equation*}
I P=E(M)-E(M-1) \tag{3.12}
\end{equation*}
$$

Where $E(Y)$ is the energy of the atom/ion with Y electrons.

Following the parameterisations and shifting of the $U_{i i}(\mathrm{i}=\mathrm{s}, \mathrm{p}, \mathrm{d})$ parameters of the ZINDO/s (INDO/s) Hamiltonian, the newly obtained model was called optimised for excitation Intermediate Neglect of Differential Overlap (oeINDO). The parameters for all the systems studied in this work are listed in chapter four. However, more importantly, the method used is described in this thesis and can be engaged for developing parameters for other atoms.

### 3.3 Validation of the oeINDO

An important question to ask is, is the newly obtained Hamiltonian model (i.e oeINDO model) transferable? That is, can the new model give accurate results for geometries that were not used during the parameterisation? To verify the transferability of the oeINDO, calculation of the excitation energies and absorption spectra were carried out with the oeINDO model for complex homo-nuclear and hetero-nuclear geometries not included in the training geometry sets. The geometries considered were $\mathrm{Si}_{n}(\mathrm{n}=3,4,5,19,40), \mathrm{S}_{n}$ $(\mathrm{n}=3,5,6,10,20), \mathrm{Zn}_{n}(\mathrm{n}=3,4,6,8,16,24), \mathrm{Cd}_{n}(\mathrm{n}=3,4,6,8,16),(\mathrm{ZnS})_{n}(\mathrm{n}=2,3,4$, 10), $(C d S)_{n}(\mathrm{n}=2,3,4,10)$ and $\mathrm{Cd}_{x} \mathrm{Zn}_{y} \mathrm{~S}_{19}(\mathrm{x}=5,10,15 ; \mathrm{y}=14,9,4)$. (See next section 3.3.1 below for how the structures were obtained) The same calculations were performed with the original ZINDO/s model. The oeINDO and ZINDO/s results obtained were compared with those obtained from EOM-CCSD. For larger atomic clusters for which EOM-CCSD is expensive, oeINDO and ZINDO/s results were compared with those from TDDFT and CIS(D). MAEs (equation 3.11) were computed to compare results. Gnuplot and Gabedit were employed to visualize and plot excitation energies and UVVIS absorption spectra.

### 3.3.1 Calculation of excited state energies and Ultraviolet-Visible (UV-Vis) absorption spectra for complex structures

To obtain the semi-empirical Hamiltonian validation data sets, excitations and absorption spectra were computed with high-level methods for complex structures outside the training structures (diatomics). The calculations were performed both for homo-nuclear and hetero-nuclear complex structures using EOM-CCSD, B3LYP/TDDFT and CIS(D)
with large basis set DEF2-TZVPP. The homo-nuclear structures include $\operatorname{Si}_{n}(n=3,4,5$, $19,40), \mathrm{S}_{n}(n=3,5,6,10,20), \mathrm{Zn}_{n}(n=3,4,6,8,16,24)$ and $\mathrm{Cd}_{n}(n=3,4,6,8,16)$ while the hetero-nuclear structure considered were $(\mathrm{ZnS})_{n}(n=2,3,4,10)$ and $(\mathrm{CdS})_{n}(n=2,3$, 4,10). For large structures, EOM-CCSD is prohibitively expensive and so, only TDDFT and CIS(D) were employed for their calculations. EOM-CCSD calculations were performed only for $\mathrm{Si}_{n}(n=3,4,5), \mathrm{S}_{n}(n=3,5), \mathrm{Zn}_{n}(n=3,4)$ and $\mathrm{Cd}_{4}$. The structures chosen were the equilibrium structures for these complexes. The equilibrium structures of some of the atomic clusters were sourced from Literature while others were obtained by carrying out geometry optimisation using ab-initio or semi-empirical methods. The equilibrium structure of $\mathrm{Si}_{n} n=3,4,5,19,40,148$ and $\mathrm{S}_{n} n=3,4,5,6,10,20$ (where $n$ is the number of atoms) were sourced from literatures (Raghavachari, 1986, Raghavachari and Rohlfing, 1988, Tam et al., 2015, Jin et al., 2015, Jackson and Jellinek, 2016). The equilibrium structures of $\mathrm{Zn}, \mathrm{Cd}, \mathrm{ZnS}$, and CdS clusters and quantum dots were obtained by performing geometry optimisation calculations.

### 3.3.2 Geometry Optimisation

The geometry optimisation for $\mathrm{Zn}_{n}(\mathrm{n}=3,4,6,8,16,24)$ and $\mathrm{Cd}_{n}(\mathrm{n}=3,4,6,8,16$, 20) clusters were carried out by first generating their starting geometry coordinates using the Avogadro software. The geometries obtained were then optimised using the three-parameters functional of Becke, Lee Yang and Parr (B3LYP) (Becke, 1993) and the valence triple-zeta with two sets of polarization functions (DEF2-TZVPP) basis sets (Weigend and Ahlrichs, 2005). The vibrational frequencies of each optimised structure were computed to ascertain that the structure obtained was a global minimum. These calculations were performed using ORCA 4.0. For quantum dots of $\mathrm{Si}, \mathrm{Zn}, \mathrm{Cd}, \mathrm{S}, \mathrm{ZnS}$, and Cds , the starting geometries were built from the builder Wulff constructor in quantumATK software. For large atomic clusters like quantum dots, geometry optimisations with ab-initio (e.g B3LYP/TZVPP or even with smaller basis sets) are computationally expensive. Hence, the computationally cheap and moderately accurate semi-empirical, PM7 (Hostavs et al., 2013), was employed in carrying out geometry optimisation calculations for quantum dots. These calculations were performed using PM7 as executed in the MOPAC7 package.

### 3.4 Study of large atomic clusters and quantum dots using the oeINDO model

After the validation of the oeINDO model, it was employed to calculate and predict excitation energies and UV-VIS absorption spectra for $\mathrm{Si}, \mathrm{S}, \mathrm{Zn}, \mathrm{Cd}, \mathrm{ZnS}, \mathrm{CdS}$ and CdZnS large clusters and quantum dots. Also, it was employed to predict theoretical optimal size and shape of ZnS and CdS quantum dots for solar cell applications. The Gnuplot and Gabedit were used to visualize and plot absorption spectra plots.

## Chapter 4

## RESULTS and DISCUSSION

### 4.1 Results of Parameterization

The results of the parameterisations of the INDO/s Hamiltonian model using excitation energies obtained from EOM-CCSD/DEF2-TZVPP for diatomics are presented in Figures 4.1, 4.2, 4.3, and 4.4. The figures are scatter plots of excitation energies obtained from the optimised INDO/s (oeINDO) and unoptimised INDO/s (ZINDO/s) in comparison with EOM-CCSD (the benchmark) for $\mathrm{Si}_{2}, \mathrm{~S}_{2}, \mathrm{Zn}_{2}$ and $\mathrm{Cd}_{2}$. The blue circles in these figures represent excitation energies from oeINDO while the red plus signs represent ZINDO/s excitations. The straight green line represents excitation energies from EOM-CCSD.

The Mean Absolute Error (MAE) was computed for sets of points (excitation energies). So that, if the MAE of a particular set of points is zero relative to the benchmark, then, the points lie exactly on the straight green line in the figure.


Figure 4.1: A scatter plot of $\mathrm{Si}_{2}$ excitation energies obtained using ZINDO/s and oeINDO against those from EOM-CCSD (the benchmark). The blue circles are the oeINDO excitation energies while the red plus signs are the ZINDO/s excitation energies. The straight green line represent the benchmark.


Figure 4.2: Scatter plot of $S_{2}$ excitation energies obtained using ZINDO/s and oeINDO against those from EOM-CCSD (the benchmark). The blue circles are the oeINDO excitation energies while the red plus signs are the ZINDO/s excitation energies. The straight green line represents the benchmark.


Figure 4.3: Scatter plot of $\mathrm{Cd}_{2}$ excitation energies obtained using ZINDO/s and oeINDO against those from EOM-CCSD (the benchmark). The blue circles are the oeINDO excitation energies while the red plus signs are the ZINDO/s excitation energies. The straight green line represents the benchmark.


Figure 4.4: Scatter plot of $\mathrm{Zn}_{2}$ excitation energies obtained using $\mathrm{ZINDO} / \mathrm{s}$ and oeINDO against those from EOM-CCSD (the benchmark). The blue circles are the oeINDO excitation energies while the red plus signs are the ZINDO/s excitation energies. The straight green line represents the benchmark.

Figure 4.1 is a scatter plot of 64 excitation energies each obtained using oeINDO and ZINDO/s against those from EOM-CCSD for $\mathrm{Si}_{2}$. It can be observed from the figure that the oeINDO energies (blue circles) agree reasonably well with the EOM-CCSD energies (the straight green line). Also, the oeINDO energies are closely matched than the ZINDO/s energies (red plus signs) to the EOM-CCSD energies. The MAE of the oeINDO energies as compared with those from EOM-CCSD was computed to be 0.21 eV while that for ZINDO/s was computed to be 0.87 eV (about four times the error observed in oeINDO). Also, ZINDO/s produced some negative excitation energies which are not acceptable because excitation energy is the difference between the energy of the unoccupied electronic state (higher energy) and the occupied electronic state (lower energy).

Figure 4.2 is a scatter plot of excitation energies of sulphur diatomics produced with oeINDO and ZINDO/s and compared with EOM-CCSD excitation energies. The figure shows that excitation energies calculated with oeINDO matched well and better than ZINDO/s excitation energies to the EOM-CCSD ones. The MAE of the oeINDO excitation energies relative to those from EOM-CCSD is calculated to be 0.19 eV . The ZINDO/s excitation energies, however, deviate from those of EOM-CCSD with a large error of 2.38 eV . It can also be observed that ZINDO/s gave negative excitations at some dimer separations, which are unacceptable.

For $\mathrm{Cd}_{2}$ geometries (Figure 4.3), oeINDO excitation energies (blue circles) were also found to align more closer than ZINDO/s energies (red plus signs) to the EOM-CCSD energies (straight green line). The MAE of oeINDO energies relative to those of EOMCCSD is 0.29 eV . However, ZINDO/s energies deviate more with MAE 0.67 eV (over twice the error noticed for oeINDO).

Figure 4.4 presents a total of 88 excitation energies each obtained with oeINDO and ZINDO/s for $\mathrm{Zn}_{2}$ geometries and compared with those from EOM-CCSD (benchmark). From the figure, the blue circles (oeINDO energies) are found to match closer to the straight green line (EOM-CCSD energies) than the red plus signs (ZINDO/s energies). The oeINDO matches with MAE 0.22 eV while ZINDO/s matches with a larger error of MAE 1.09 eV .

Table 4.1: MAEs of excitation energies (eV) from ZINDO/s and oeINDO relative to EOM-CCSD energies for various diatomics

| Diatomics | oeINDO | ZINDO/s |
| :--- | :---: | :---: |
| $\mathrm{Si}_{2}$ | 0.21 | 0.87 |
| $\mathrm{~S}_{2}$ | 0.19 | 2.38 |
| $\mathrm{Zn}_{2}$ | 0.22 | 1.09 |
| $\mathrm{Cd}_{2}$ | 0.29 | 0.67 |

A summary of the MAEs of oeINDO and original INDO/s excited state energies relative to EOM-CCSD excited state energies for the different diatomic systems are given in Table 4.1. On the average, the oeINDO energies compare with EOM-CCSD energies with a typical error of 0.23 eV while ZINDO/s energies deviate with a larger MAE of 1.25 eV .

After the parameterisations, the one center integrals, $U_{i i}$ were shifted to reproduce first ionization potential (IP) at CCSD/DEF2-TZVPP level. First ionization potentials of silicon, zinc, cadmium and sulphur calculated at CCSD/DEF2-TZVPP level are 8.069, $8.8971,8.665$, and 12.363 eV , respectively. The shift of $U_{s s}$ and $U_{p p}$ produced silicon atom first IP of value 8.069 eV which compares well with the experimental value of 8.1517 eV . However, for Zn and Cd atoms, the shift in $U_{s s}, U_{p p}$ and $U_{d d}$ produced IPs with the error of about 1.1 eV relative to IPs calculated with CCSD/DEF2-TZVPP.

The parameters of ZINDO/s and the one obtained from the parameterisations of INDO/s (oeINDO) are presented in the following Tables 4.2-4.5 and discussed.

Table 4.2: $\mathrm{ZINDO} / \mathrm{s}$ and oeINDO parameters for $\mathrm{Si}_{2}$

| Parameters | ZINDO/s | oeINDO |
| :--- | :--- | :--- |
| $\zeta_{s}$ | 1.52 bohrs | 1.430753 bohrs |
| $\zeta_{p}$ | 1.52 bohrs | 1.411963 bohrs |
| $U_{s s}$ | -36.235929 eV | -25.4244 eV |
| $U_{p p}$ | -28.594917 eV | -13.6400 eV |
| $\beta_{s}$ | 13.0 eV | 29.06189 eV |
| $\beta_{p}$ | 13.0 eV | 9.052134 eV |
| $\gamma_{s s}$ | 7.57 eV | 2.311795 eV |
| $\gamma_{s p}$ | 7.57 eV | 2.311795 eV |
| $\gamma_{p p}$ | 7.57 eV | 2.311795 eV |
| $F_{p p}^{2}$ | 2.2627 eV | 1.750264 eV |
| $G_{s p}$ | 4.8122 eV | 3.132682 eV |

Table 4.3: oeINDO and ZINDO/s parameters for $\mathrm{S}_{2}$

| Parameters | oeINDO | ZINDO/s |
| :--- | :--- | :--- |
| $U_{s s}$ | -39.7751 eV | -36.235929 eV |
| $U_{p p}$ | -33.2246 eV | -28.594917 eV |
| $\zeta_{s}$ | 2.169004 bohrs | 1.52 bohrs |
| $\zeta_{p}$ | 1.586553 bohrs | 1.52 bohrs |
| $\beta_{s}$ | 11.524900 eV | 13.0 eV |
| $\beta_{p}$ | 12.301215 eV | 13.0 eV |
| $\gamma_{s s}$ | 6.013544 eV | 7.57 eV |
| $\gamma_{s p}$ | 6.013544 eV | 7.57 eV |
| $\gamma_{p p}$ | 6.013544 eV | 7.57 eV |
| $F_{p p}^{2}$ | 6.431554 eV | 2.2627 eV |
| $G_{s p}$ | 3.455990 eV | 4.8122 eV |

Table 4.4: oeINDO and ZINDO/s parameters for $\mathrm{Zn}_{2}$

| Parameters | oeINDO | ZINDO/s |
| :--- | :--- | :--- |
| $U_{s s}$ | -111.667 eV | -110.620093 eV |
| $U_{p p}$ | -105.918 eV | -105.224805 eV |
| $U_{d d}$ | -159.416 eV | -161.738551 eV |
| $\zeta_{s}$ | 1.561604 bohrs | 1.5090 bohrs |
| $\zeta_{p}$ | 1.417918 bohrs | 1.5090 bohrs |
| $\zeta_{d}$ | 3.645080 bohrs | 4.6261 bohrs |
| $\beta_{s}$ | 10.184300 eV | 10.0 eV |
| $\beta_{p}$ | 4.392070 eV | 10.0 eV |
| $\beta_{d}$ | 33.844518 eV | 34.0 eV |
| $\gamma_{s s}$ | 5.172389 eV | 7.98 eV |
| $\gamma_{s p}$ | 5.172389 eV | 7.98 eV |
| $\gamma_{p p}$ | 5.172389 eV | 7.98 eV |
| $\gamma_{s d}$ | 9.930301 eV | 9.39 eV |
| $\gamma_{p d}$ | 9.930301 eV | 9.39 eV |
| $\gamma_{d d}$ | 14.589140 eV | 14.55 V |
| $F_{p p}^{2}$ | 1.275683 eV | 1.1778 eV |
| $F_{p d}^{2}$ | 1.386388 eV | 1.4878 eV |
| $F_{d d}^{2}$ | 1.497962 eV | 11.4063 eV |
| $F_{d d}^{4}$ | 10.747871 eV | 7.6249 eV |
| $G 1_{s p}$ | 10.088581 eV | 2.5292 eV |
| $G 1_{p d}$ | 0.159041 eV | 0.8679 eV |
| $G 2_{s d}$ | eV |  |
| $G 3_{p d}$ |  |  |

Table 4.5: oeINDO and ZINDO/s parameters for $\mathrm{Cd}_{2}$

| Parameters | oeINDO | ZINDO/s |
| :--- | :--- | :--- |
| $U_{s s}$ | -88.9433 eV | -110.620093 eV |
| $U_{p p}$ | -83.5324 eV | -105.224805 eV |
| $U_{d d}$ | -117.034 eV | -161.738551 eV |
| $\zeta_{s}$ | 1.784273 bohrs | 1.5090 bohrs |
| $\zeta_{p}$ | 1.586890 bohrs | 1.5090 bohrs |
| $\zeta_{d 1}$ | 5.565263 bohrs | 4.6261 bohrs |
| $\zeta_{d 2}$ | 2.818233 bohrs | 4.6261 bohrs |
| $\beta_{s}$ | 8.743760 eV | 10.0 eV |
| $\beta_{p}$ | 5.270200 eV | 10.0 eV |
| $\beta_{d}$ | 32.520550 eV | 34.0 eV |
| $\gamma_{s s}$ | 5.820365 eV | 7.98 eV |
| $\gamma_{s p}$ | 5.820365 eV | 7.98 eV |
| $\gamma_{p p}$ | 5.820365 eV | 7.98 eV |
| $\gamma_{s d}$ | 7.632756 eV | 9.39 eV |
| $\gamma_{p d}$ | 7.632756 eV | 9.39 eV |
| $\gamma_{d d}$ | 0.463479 eV | 14.55 eV |
| $F_{p p}^{2}$ | 10.301339 eV | 1.1778 eV |
| $F_{p d}^{2}$ | 2.848384 eV | 1.4878 eV |
| $F_{d d}^{2}$ | 1.338146 eV | 11.4063 eV |
| $F_{d d}^{4}$ | 6.415983 eV | 7.6249 eV |
| $G 1_{s p}$ | 4.067983 eV | 2.5292 eV |
| $G 1_{p d}$ | 0.89679 eV |  |
| $G 2_{s d}$ | 0.6199 eV |  |
|  |  |  |

The oeINDO and the original ZINDO/s parameters for silicon, sulphur, zinc and cadmium are presented in Tables 4.2, 4.3, 4.4, and 4.5, respectively. A total of 11 parameters each were optimised for $\mathrm{Si}_{2}$ and $\mathrm{S}_{2}, 23$ parameters for $\mathrm{Zn}_{2}$ and 24 parameters for $\mathrm{Cd}_{2}$.

The new set of parameters (oeINDO) for silicon atoms are different from those of the original ZINDO/s (see Table 4.2). For the ZINDO/s, the exponents, $\zeta_{s}$ and $\zeta_{p}$ are equal. Also, the bonding parameters, $\beta_{s}$ and $\beta_{p}$ have equal values. However, for oeINDO, $\zeta_{s}$ and $\zeta_{p}$ are slightly different while the $\beta_{s}$ and $\beta_{p}$ largely differ in values. The values of $\gamma$ parameters (parameters upon which the two center one electron integrals depend) for oeINDO are less than those of original ZINDO/s. The difference between $U_{s s}$ and $U_{p p}$ is $\approx 12 \mathrm{eV}$ for oeINDO and $\approx 8 \mathrm{eV}$ for ZINDO/s. The difference between the $F^{2}$ parameters of oeINDO and ZINDO/s is $\approx 0.5 \mathrm{eV}$ and between their $G^{1}$ parameters is $\approx 1.7 \mathrm{eV}$, respectively.

Significant differences were also observed in oeINDO and ZINDO/s parameters for sulphur atom (see Table 4.3). The difference between the $U_{s s}$ and $U_{p p}$ is $\approx 6 \mathrm{eV}$ for oeINDO and $\approx 8 \mathrm{eV}$ for ZINDO/s. The values exponents $(\zeta)$ are equal for ZINDO/s but differ significantly for oeINDO. The bonding parameters ( $\beta \mathrm{s}$ ) differ by $\approx 0.7 \mathrm{eV}$ for oeINDO but are equal for ZINDO/s. The values of $\gamma$ s for oeINDO are less than those of ZINDO/s by $\approx 1.5 \mathrm{eV}$

For a zinc atom, a significant difference of about 5.6 eV was observed between oeINDO and ZINDO/s p-orbital bonding parameters. Also, their gamma parameters differ by about 2.8 eV .

For cadmium, the oeINDO $\beta, \gamma$, and $\zeta_{d}$ deviate significantly from those of ZINDO/s. The differences between the one center electron integral $U_{s s}$ and $U_{d d}$ are $\approx 28 \mathrm{eV}$ for oeINDO and $\approx 51 \mathrm{eV}$ for ZINDO/s.

Significant deviations were observed in the values of oeINDO parameters from the original ZINDO/s ones and this is responsible for the good performance of oeINDO in reproducing the EOM-CCSD energies for diatomics with a minimal error.

### 4.2 Validation of the oeINDO model

The parameters obtained for the oeINDO model as presented in Tables 4.3, 4.4, 4.5 and 4.2 were validated by verifying their transferability and accuracy for complex geometries not included in training geometry sets. To achieve these, excitation energies and absorption spectra obtained for complex equilibrium geometries/structures using oeINDO and ZINDO/s were compared with excitation energies and spectra from ab-initio methods (EOM-CCSD, TDDFT and CIS(D)).

### 4.2.1 Equilibrium Structures

This subsection gives information about the complex structures employed for the new model validation and predictions. Both homogeneous and heterogeneous structures were considered. The small- and medium-sized homogeneous equilibrium structures include $\mathrm{Si}_{n}(\mathrm{n}=148,40,19,5,4,3),, \mathrm{S}_{n}(\mathrm{n}=20,10,6,5,4,3), \mathrm{Zn}_{n}(\mathrm{n}=3,4,6,8,16,24)$, and $\mathrm{Cd}_{n}$ ( $\mathrm{n}=3,4,6,8,16$ ). The small- and medium-sized heterogeneous equilibrium structures are $(\mathrm{ZnS})_{n},(\mathrm{CdS})_{n}(\mathrm{n}=2,3,4,10)$ and $\mathrm{Cd}_{x} \mathrm{Zn}_{y} \mathrm{~S}_{19}$. For nano-sized systems (quantum dots), the homogeneous and heterogeneous structures used for predictions with the new model have sizes ranging from $0.8-3.0 \mathrm{~nm}$ (38-779 atoms).

The $\mathrm{Si}_{n}(\mathrm{n}=148,40,19,5,4,3$,$) and \mathrm{S}_{n}(\mathrm{n}=20,10,6,5,4,3)$ equilibrium geometries and ground-state configurations were sourced from Literature (Tam et al., 2015, Raghavachari, 1986, Raghavachari and Rohlfing, 1988, Jin et al., 2015, Jackson and Jellinek, 2016). The equilibrium silicon nano-sized structures were obtained from geometry optimisation calculations using PM 7 and presented in Figure B. 5 in the appendix. The structure sizes are 1.6 nm (a structure with 128 silicon atoms), 1.8 nm (190 silicon atoms), 2.0 nm ( 244 silicon atoms) and 3.0 nm ( 779 silicon atoms).

Equilibrium structures obtained for $\mathrm{Zn}_{n}(\mathrm{n}=3,4,6,8,1624)$ clusters at the b3lyp/def2tzvpp level are presented in the appendix in Figure B.1. Equilibrium $\mathrm{Zn}_{3}$ and $\mathrm{Zn}_{4}$ clusters were observed to be equilateral and tetragonal in shape, respectively. The optimised zinc nano-clusters (quantum dots) obtained at the PM7 level are namely 1.0 nm zinc cluster (containing 38 zinc atoms), 1.2 nm ( 62 zinc atoms), 1.4 nm ( 104 zinc atoms), 1.6 nm (128 zinc atoms), 1.8 nm (190 zinc atoms) and 2.0 nm ( 244 zinc atoms), and are
presented in Figure B.3.

The $\mathrm{Cd}_{n}(\mathrm{n}=3,4,6,8,16)$ equilibrium structures obtained, are shown in Figure B. 2 in the appendix. The equilibrium $\mathrm{Cd}_{3}$ and $\mathrm{Cd}_{4}$ clusters assumed equilateral and tetragonal shapes, respectively. The sizes of the optimised Cd nano-clusters at the level of PM7 are in the range of $1.0-2.0 \mathrm{~nm}$.

Different nano-sized structures obtained at the PM7 level for sulphur, cadmium sulphide and zinc sulphide are shown in Figures B.4, B. 7 and B.6, respectively. The sizes of sulphur quantum dots range from 1.0 to 2.0 nm . For cadmium sulphide, the range of the dot sizes is 1.0 to 2.2 nm while for zinc sulphide quantum dots, the sizes are $1.0,1.2$, 1.4, 1.8, 2.0 and 2.2 nm .

### 4.2.2 Transferability of oeINDO silicon parameters

In order to verify how accurate the oeINDO silicon parameters were, its eight lowest excited state energies were compared with benchmark for $\operatorname{Si}_{n}(n=3,4,5,19,40)$. The first eight lowest excited state energies of $\operatorname{Si}_{n}(n=3,4,5,19,40)$ were computed with EOM-CCSD, TDDFT, CIS(D), oeINDO and ZINDO/s and presented in Tables 4.6, 4.8, 4.9, 4.10 and 4.11. The values from these Tables were presented as scatter plots in Figure 4.11 and the mean absolute errors (MAEs) were computed and summarized in Tables 4.7 and 4.12 . Also, UV-vis absorption spectra, which are displayed in Figures 4.5, 4.6, 4.7, 4.8 , and 4.9 were used to further validate the oeINDO model.

Table 4.6: First Eight Lowest Excited State Energies (eV) for $\mathrm{Si}_{3}$

| EOM-CCSD | oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: | :---: |
| 1.406 | 1.310 | -0.047 | 1.267 | 1.447 |
| 1.478 | 1.342 | 2.258 | 1.343 | 1.653 |
| 1.981 | 1.497 | 2.797 | 1.896 | 2.118 |
| 2.197 | 2.316 | 3.292 | 2.142 | 2.307 |
| 2.583 | 2.458 | 3.630 | 2.475 | 2.787 |
| 2.998 | 2.538 | 3.875 | 3.026 | 3.230 |
| 3.494 | 3.211 | 4.274 | 3.364 | 3.965 |
| 3.619 | 3.342 | 4.341 | 3.535 | 3.995 |
|  |  |  |  |  |



Figure 4.5: Absorption spectra for $\mathrm{Si}_{3}$ equilibrium geometry obtained from different methods. The intensities have been scaled so that the highest intensity is equal to unity.

Table 4.7: MAEs in eV of semi-empirical methods, $\mathrm{CIS}(\mathrm{D})$ and TDDFT from ab-initio methods for Si equilibrium geometries

| Methods | EOM-CCSD | TDDFT | CIS(D) |
| :--- | :--- | :--- | :--- |


| $\mathbf{S i}_{3}$ |  |  |  |
| :---: | :---: | :---: | :---: |
| oeINDO | 0.22 | 0.17 | 0.39 |
| ZINDO/s | 0.92 | 0.98 | 0.74 |
| TDDFT | 0.07 | - | - |
| CIS(D) | 0.20 | - | - |
| $\mathbf{S i}_{4}$ |  |  |  |
| oeINDO | 0.18 | 0.24 | 0.31 |
| ZINDO/S | 1.38 | 1.71 | 1.25 |
| TDDFT | 0.12 | - | - |
| CIS(D) | 0.29 | - | - |
| $\mathbf{S i}_{5}$ |  |  |  |
| oeINDO | 0.09 | 0.04 | 0.16 |
| ZINDO/S | 1.47 | 1.52 | 1.39 |
| TDDFT | 0.09 | - | - |
| CIS(D) | 0.05 | - | - |
| $\mathbf{S i}_{19}$ |  |  |  |
| oeINDO | - | 0.32 | 0.36 |
| ZINDO/S | - | 1.42 | 1.46 |
| $\mathbf{S i}_{40}$ |  |  |  |
| oeINDO | - | 0.12 | 0.03 |
| ZINDO/S | - | 0.68 | 0.80 |

Table 4.8: First Eight Lowest Excited State Energies (eV) for $\mathrm{Si}_{4}$

| EOM-CCSD | oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: | :---: |
| 1.580 | 1.451 | 2.722 | 1.499 | 1.622 |
| 1.836 | 1.514 | 3.423 | 1.579 | 1.882 |
| 2.300 | 2.257 | 3.937 | 2.184 | 2.450 |
| 2.958 | 2.520 | 3.980 | 2.716 | 3.187 |
| 3.047 | 2.894 | 4.204 | 2.905 | 3.307 |
| 3.179 | 3.057 | 4.526 | 2.976 | 3.311 |
| 3.197 | 3.113 | 4.581 | 3.152 | 3.873 |
| 3.539 | 3.185 | 5.516 | 3.448 | 4.518 |
|  |  |  |  |  |



Figure 4.6: Absorption spectra of $\mathrm{Si}_{4}$ equilibrium geometry obtained from different methods. The intensities have been scaled so that the highest intensity is equal to unity.

Considering the first eight lowest vertical excited energies for $\mathrm{Si}_{3}$ equilibrium structure presented in Table 4.6, the first lowest excited state energy of oeINDO ( 1.31 eV ) as well as those from TDDFT ( 1.27 eV ) and CIS(D) $(1.45 \mathrm{eV})$ compared well with the 1.40 eV , first excited state from EOM-CCSD (the benchmark). The ZINDO/s energy is however much deviated and gives negative excitation energy value of -0.047 eV , which is unacceptable. The MAE (see Table 4.7) calculated for the eight excitation energies from oeINDO compared to the EOM-CCSD ones was 0.22 eV . The ZINDO/s excitations, however, compared with EOM-CCSD excitations with a large MAE of 0.98 eV . Table 4.7 shows that ab-initio methods (TDDFT and CIS(D)) agree with EOM-CCSD with MAE 0.07 eV and 0.20 eV , respectively. In comparison with other ab-initio methods, oeINDO excitations compared with TDDFT and CIS(D) excitations with MAEs 0.17 eV and 0.39 eV , respectively. Figure 4.5 shows the absorption spectra from EOM-CCSD, oeINDO, CIS(D), TDDFT and ZINDO/s for $\mathrm{Si}_{3}$ structure. The oeINDO spectrum agrees reasonably with that of EOM-CCSD. The highest peak of oeINDO is red-shifted by about 0.50 eV from that of EOM-CCSD.

The oeINDO reproduced the EOM-CCSD first eight lowest excitations energies (table 4.8 ) with MAE 0.18 eV (Table 4.7 ) for the $\mathrm{Si}_{4}$ equilibrium structure. Relative to EOMCCSD, although oeINDO error is slightly higher than that of TDDFT (MAE $=0.12 \mathrm{eV}$ ), it performs better than the $a b$-initio, CIS(D) (MAE $=0.29 \mathrm{eV}$ ). The excitation energies from the original ZINDO/S deviate much from those from EOM-CCSD with MAE of 1.38 eV . The first excited state energy (Table 4.8) from oeINDO ( 1.45 eV ) shows good agreement with that of the benchmark, EOM-CCSD (1.58 eV). The TDDFT and CIS(D) energies 1.50 and 1.62 eV , respectively are also in good agreement with the EOM-CCSD one. Absorption spectra plots for $\mathrm{Si}_{4}$ equilibrium structure in Figure 4.6, show that oeINDO agrees better than CIS(D) (ab-initio) to EOM-CCSD. The highest peak of oeINDO is blue-shifted from that of EOM-CCSD by 0.11 eV while ZINDO/s and CIS(D) peaks are blue-shifted with a larger error of 0.46 eV and 0.22 eV , respectively. TDDFT spectrum shows a good match with the EOM-CCSD one with error $<0.1 \mathrm{eV}$.

Table 4.9: First Eight Lowest Excited State Energies (eV) for $\mathrm{Si}_{5}$

| EOM-CCSD | oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: | :---: |
| 2.535 | 2.465 | 3.445 | 2.470 | 2.557 |
| 2.575 | 2.466 | 3.445 | 2.622 | 2.619 |
| 2.794 | 2.657 | 4.097 | 2.655 | 2.844 |
| 2.844 | 2.687 | 4.380 | 2.749 | 2.905 |
| 2.844 | 2.770 | 4.380 | 2.749 | 2.905 |
| 2.859 | 2.770 | 4.666 | 2.759 | 2.988 |
| 2.859 | 2.835 | 4.983 | 2.759 | 2.988 |
| 3.097 | 2.835 | 4.983 | 2.824 | 3.209 |
|  |  |  |  |  |



Figure 4.7: Absorption spectra for $\mathrm{Si}_{5}$ equilibrium geometry obtained from different methods.. The intensities have been scaled so that the highest intensity is equal to unity.

Table 4.10: First Eight Lowest Excited State Energies (eV) for $\mathrm{Si}_{19}$

| oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: |
| 1.649 | 2.447 | 1.773 | 1.761 |
| 1.731 | 2.769 | 1.927 | 1.830 |
| 2.390 | 3.452 | 1.993 | 1.931 |
| 2.482 | 3.639 | 2.100 | 2.021 |
| 2.613 | 3.861 | 2.108 | 2.077 |
| 2.638 | 3.948 | 2.175 | 2.081 |
| 2.670 | 3.979 | 2.234 | 2.272 |
| 2.703 | 4.026 | 2.265 | 2.322 |
|  |  |  |  |

Table 4.11: First Eight Lowest Excited State Energies (eV) for $\mathrm{Si}_{40}$

| oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: |
| 0.394 | 1.221 | 0.722 | 0.318 |
| 0.420 | 1.348 | 0.848 | 0.540 |
| 0.581 | 1.384 | 0.932 | 0.587 |
| 0.614 | 1.434 | 1.072 | 0.694 |
| 0.689 | 1.491 | 1.089 | 0.707 |
| 0.754 | 1.641 | 1.126 | 0.767 |
| 0.802 | 1.649 | 1.133 | 0.839 |
| 0.818 | 1.740 | 1.189 | 0.877 |



Figure 4.8: Absorption spectra for $\mathrm{Si}_{19}$ equilibrium geometry obtained from different methods.. The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.9: Absorption spectra for $\mathrm{Si}_{40}$ equilibrium geometry obtained from different methods.. The intensities have been scaled so that the highest intensity is equal to unity.

For the $\mathrm{Si}_{5}$ equilibrium cluster (Figure 4.7), oeINDO also showed a good performance. Its absorption spectra (Figure 4.7) and excitations (Table 4.9) are in good agreement with those from EOM-CCSD. The oeINDO spectrum and those from other ab-initio methods including the benchmark method are found in the same energy range (2.5-3.5 eV) but ZINDO/s one is much shifted from this range. The oeINDO spectrum maximum is shifted from the EOM-CCSD maximum with a slight error of 0.07 eV while ZINDO/s peak is shifted with a larger error $>1.7 \mathrm{eV}$. From Table 4.9 , it can be seen that the first and second excited state energies from oeINDO ( 2.47 and 2.47 eV ) as well as those from TDDFT ( 2.47 and 2.62 eV ) and CIS(D) ( 2.56 and 2.62 eV ) are in good agreement with the EOM-CCSD ones ( 2.54 and 2.58 eV ). However, the corresponding ZINDO/s energies ( 3.45 and 3.45 eV ) deviate more. The first eight excited state energies from oeINDO match with those from EOM-CCSD with a small error of MAE 0.09 eV while the ZINDO/s ones deviate much more with 1.47 eV .

For larger structures, $\mathrm{Si}_{19}$ and $\mathrm{Si}_{40}$, $\mathrm{EOM}-\mathrm{CCSD}$ computations are prohibitively expensive. Hence, oeINDO results for these structures were compared to TDDFT and CIS(D) results. For the $\mathrm{Si}_{19}$ equilibrium structure, the Table 4.10 shows that the first excited energy from oeINDO ( $\approx 1.7 \mathrm{eV}$ ) match better than ZINDO/s ( $\approx 2.5 \mathrm{eV}$ ) one to ab-initios ( $\approx 1.8 \mathrm{eV}$ ). The MAEs (Table 4.7) of the first eight lowest excitations from oeINDO relative to $\mathrm{CIS}(\mathrm{D})$ and TDDFT are 0.36 eV and 0.32 eV , respectively. For $\mathrm{Si}_{40}$, oeINDO excitation energies agree with those obtained with TDDFT and CIS(D) with small MAEs of 0.12 eV and 0.03 eV , respectively. As seen earlier Table 4.7, TDDFT agrees with EOM-CCSD with a typical error of about 0.09 eV . Hence, oeINDO excitations for $\mathrm{Si}_{40}$ compared with EOM-CCSD with a typical error of about 0.21 eV . The absorption spectra of $\mathrm{Si}_{19}$ and $\mathrm{Si}_{40}$ are presented in Figure 4.8 and Figure 4.9, respectively. The figures show that the oeINDO spectra are comparable with TDDFT spectra. For $\mathrm{Si}_{19}$, the spectrum from oeINDO is comparable with the TDDFT spectrum pattern and its spectrum highest peak agrees well with those from CIS(D) and TDDFT with an error of 0.05 eV . For $\mathrm{Si}_{40}$ (Figure 4.9), oeINDO gives similar spectrum pattern as that of CIS(D) in the same energy range.


Figure 4.10: Highest peaks against number of units $n$ in equilibrium $\operatorname{Si}_{n}$ structures.


Figure 4.11: Semi-empirical energies against TDDFT energies for $\operatorname{Si}_{n} \quad(n=$ $3,4,5,19,40$ ) equilibrium. structures.


Figure 4.12: Scatter plot of different (non-equilibrium and equilibrium) $\operatorname{Si}_{n}(n=5,4,3)$ clusters.

Table 4.12: MAEs (eV) of excitations from various methods as compared to EOMCCSD excitations for different (non-equilibrium and equilibrium) Si clusters

| Methods | $\mathrm{Si}_{5}$ | $\mathrm{Si}_{4}$ | $\mathrm{Si}_{3}$ |
| :--- | :--- | :--- | :--- |
| oeINDO | 0.11 | 0.27 | 0.25 |
| ZINDO/S | 1.31 | 1.32 | 0.87 |
| TDDFT | 0.08 | 0.11 | 0.06 |
| CIS(D) | 0.06 | 0.15 | 0.13 |

The plot of absorption spectra of the highest peaks as a function of the number of Si atoms is presented in Figure 4.10. It shows that the oeINDO peak positions compare reasonably well with TDDFT peaks with a typical error of 0.22 eV . The positions of the ZINDO/S peaks deviate much with a typical error of 1.03 eV .

Table 4.7 summarizes the MAEs of excitations energies from oeINDO and ZINDO/s as compared with EOM-CCSD, CIS(D), and TDDFT for the $\operatorname{Si}_{n}(n=3,4,5,19,40)$ equilibrium geometries. Evidently, oeINDO is more accurate than ZINDO/s. Also, the scatter plot of a total of 40 excitation energies from all the Si equilibrium geometries presented in Figure 4.11, is a further proof that oeINDO agrees better than ZINDO/s with ab-initio methods. The MAE of oeINDO excitations as compared to TDDFT is 0.15 eV (that is, about 0.24 eV relative to EOM-CCSD) while for ZINDO/s, the MAE is 1.23 eV relative to TDDFT

Calculations were also performed with non-equilibrium silicon cluster structures. A total of 12 silicon structures were obtained by perturbing the $\operatorname{Si}_{n}(n=3,4,5)$ equilibrium structures with scale factors $0.9,1.0,1.1$ and 1.2. The scatter plots of 48 excitation energies each from different methods for these structures are presented in Figure 4.12. The summary of the MAEs of excitation energies from the 12 equilibrium and nonequilibrium structures are displayed in Table 4.12. Obviously, the results from the table and the figure show that oeINDO is also gives good result for non-equilibrium structures. Table 4.7 shows, on the average, that oeINDO excitations agree with those from EOMCCSD with MAE of 0.27 eV . However, the ZINDO/s deviates from EOM-CCSD with an MAE of 1.17 eV , much larger than for oeINDO. On the other hand, TDDFT and CIS(D) compare with EOM-CCSD with 0.08 eV and 0.12 eV , respectively.

### 4.2.3 Transferability of oeINDO Zinc Parameters

The validity of oeINDO model for zinc was verified by comparing its first eight excited state energies with those from EOM-CCSD, TDDFT and CIS(D). The energies for $\mathrm{Zn}_{3}$ are presented in Table 4.13 while for other Zn cluster structures are given in Tables C.2C. 6 in the appendix. The scatter plots comparing excitations from other methods (including oeINDO) with benchmark (EOM-CCSD), semi-empirical methods with TDDFT and semi-empirical methods with CIS(D) are presented in Figures 4.13, 4.14 and 4.15, respectively. Absorption spectra for Zn cluster structures obtained with different methods and presented in Figures 4.16-4.21, were also employed for model validation.

Table 4.13: First Eight Lowest Excited State Energies (eV) for $\mathrm{Zn}_{3}$

| EOM-CCSD | oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: | :---: |
| 3.942 | 3.803 | 2.837 | 3.831 | 3.979 |
| 3.946 | 3.808 | 2.843 | 3.833 | 3.983 |
| 4.225 | 4.012 | 3.430 | 4.034 | 4.251 |
| 4.259 | 4.260 | 4.071 | 4.036 | 4.340 |
| 4.262 | 4.373 | 4.147 | 4.213 | 4.344 |
| 4.608 | 4.373 | 4.148 | 4.279 | 4.662 |
| 5.105 | 4.654 | 4.894 | 5.126 | 5.117 |
| 5.105 | 4.660 | 4.901 | 5.134 | 5.118 |
|  |  |  |  |  |



Figure 4.13: Comparing excitation energies obtained using different methods with those calculated at EOM-CCSD/TZVPP level for different $\mathrm{Zn}_{n}(n=3,4)$ equilibrium and nonequilibrium structures.


Figure 4.14: Excitation energies obtained using semi-empirical methods (oeINDO (blue circles) and ZINDO/s (red plus signs) against TDDFT energies for $\mathrm{Zn}_{n}(n=$ $3,4,6,8,16,24$ ) equilibrium structures


Figure 4.15: Excitation energies obtained using semi-empirical methods against CIS(D) energies for $\mathrm{Zn}_{n}(n=3,4,6,8,16,24)$ equilibrium structures.

From Table 4.13, it can be seen that excited state energies from oeINDO as well as those from TDDFT and CIS(D) are close to the EOM-CCSD ones. For instance, oeINDO and EOM-CCSD first excited state energies, $\approx 3.8$ and $\approx 3.9 \mathrm{eV}$, respectively differ by only 0.1 eV . However ZINDO/s first excited state energy, $\approx 2.8$, differ from that of EOM-CCSD by a larger error of 1.1 eV . Figure 4.13 is a scatter plot of 32 excitation energies each obtained using different methods for eight different $\mathrm{Zn}_{n}(\mathrm{n}=3,4)$ equilibrium and non-equilibrium structures. The plot compares the excitation energies from semiempirical methods and other ab-initio methods with those from EOM-CCSD. It can be observed that oeINDO excitations have a better match than TDDFT and ZINDO/s excitations to EOM-CCSD excitations. The oeINDO excitations match the EOM-CCSD excitations with MAE 0.25 eV while the TDDFT and ZINDO/s excitations match with 0.28 and 0.70 eV , respectively. The CIS(D)results compare with the EOM-CCSD ones with a smaller MAE of 0.03 eV . It can also be observed that the ZINDO/s model gave some negative excitation energy values, which indicate bad predictions by the model; excitation energy is defined as the difference between the lowest unoccupied molecular orbital (higher energy level) and the highest occupied molecular orbital (lower energy level).

For larger $\mathrm{Zn}_{n}(n>4)$ clusters, EOM-CCSD is compute intensive. Thus, results from other ab-initio methods (TDDFT and CIS(D)) were used for the validation of the oeINDO model. Calculation of excitations and absorption spectra were carried for $\mathrm{Zn}_{n}(\mathrm{n}=3,4,6$, $8,16,24)$ equilibrium structures using oeINDO, ZINDO/s, TDDFT and CIS(D). A scatter plot comparing the excitation energies from the semi-empirical methods (oeINDO and ZINDO/s) with those from TDDFT for all the equilibrium structures are shown in Figure 4.14. The figure shows that oeINDO excitation energies (blue circles) are much closer than ZINDO/s ones (red plus signs) to TDDFT excitation energies (the straight line). The excitations from oeINDO agree well with TDDFT with MAE 0.24 eV while ZINDO/s excitations deviate with a larger MAE of 0.63 eV .

Despite the fact that UV-Vis absorption spectra of Zn diatomics were not included in the training data sets, oeINDO produced spectra that qualitatively agree with those from the benchmark. The absorption spectra for $\mathrm{Zn}_{n}(\mathrm{n}=3,4,6,8,16,24)$ equilibrium clusters are well predicted by oeINDO as shown in Figures 4.16, 4.17, 4.18, 4.19, 4.20 and 4.21.


Figure 4.16: Absorption spectra for $\mathrm{Zn}_{3}$ equilibrium geometry obtained using different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.17: Absorption spectra for $\mathrm{Zn}_{4}$ equilibrium geometry obtained using different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.18: Absorption spectra for $\mathrm{Zn}_{6}$ equilibrium geometry obtained using different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.19: Absorption spectra for $\mathrm{Zn}_{8}$ equilibrium geometry obtained using different methods.The intensities have been scaled so that the highest intensity is equal to unity.

For $\mathrm{Zn}_{3}$ absorption spectra (Figure 4.16), the absorption bands of the spectra from the different methods are prominent within the ultraviolet region ( $3.0-30.0 \mathrm{eV}$ ). The highest peak of the oeINDO spectrum is red-shifted from the EOM-CCSD (the benchmark) by about 0.5 eV while ZINDO/s is red-shifted by a larger error of about 1.0 eV . The peaks of the spectra from the ab-initio methods (CIS(D) and TDDFT), however, have good match with that of EOM-CCSD.

The absorption spectra of $\mathrm{Zn}_{4}$ equilibrium structure presented in Figure 4.17 show that it absorbs in the ultraviolet region (3.0-30.0 eV) for all methods except ZINDO/s, which shows absorption partly in the visible region ( $1.5-3.0 \mathrm{eV}$ ) and partly in the ultraviolet region. The highest peak of the oeINDO spectrum compared better with the EOMCCSD one than does ZINDO/s and CIS(D), although, in general, intensities from CIS based methods are expected to compare only qualitatively with accurate ab-initio or experimental results.

For $\mathrm{Zn}_{n}(n>4)$ clusters, the semi-empirical model results were compared with only the TDDFT and CIS(D) results since EOM-CCSD is expensive for these clusters. In the spectra for $\mathrm{Zn}_{6}$ equilibrium structure (Figure 4.18), the absorption band of the $a b$ initio methods as well as that of oeINDO occur at about the same range. The peak relative intensity of oeINDO is red-shifted from TDDFT by about 0.70 eV while that from ZINDO/s is shifted by 1.60 eV which is more than twice that of oeINDO.

For $\mathrm{Zn}_{8}$ (Figure 4.19), oeINDO matches ab-initio results better than do ZINDO/s. The highest peak of the oeINDO is red-shifted from TDDFT and CIS(D) spectra by about 0.5 eV and 0.3 eV , respectively while ZINDO/s, on the other hand, is red-shifted by larger errors of about 2.5 eV and 2.2 eV from TDDFT and CIS(D), respectively.


Figure 4.20: Absorption spectra for $\mathrm{Zn}_{16}$ equilibrium geometry obtained using different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.21: Absorption spectra for $\mathrm{Zn}_{24}$ equilibrium geometry obtained using different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.22: Highest peaks against number of units $n$ in equilibrium $\mathrm{Zn}_{n}$ structures

The absorption spectra for $\mathrm{Zn}_{16}$ and $\mathrm{Zn}_{24}$ from oeINDO displayed in Figures 4.20 and 4.21 , respectively agree reasonably with those from TDDFT and CIS(D). For $\mathrm{Zn}_{16}$, the highest peak of the oeINDO absorption spectrum is blue-shifted with a small error of about 0.1 eV from TDDFT but the ZINDO/s peak is blue-shifted by about 0.80 eV , which is eight times larger. Absorption spectra of $\mathrm{Zn}_{24}$ (Figure 4.21) shows that the position of the oeINDO highest peak matches that from TDDFT with a typical error of about 0.25 eV .

Plots of the energy of highest peaks of the absorption spectra as a function of the number of atoms is shown in Figure 4.22 for different methods. The oeINDO method reproduced the same trend as observed for TDDFT; the peaks are red-shifted (decreased in peak energies) with increase in the number of atoms. However, for ZINDO/s the trend broke for the sixteen atoms Zn cluster. The MAEs of the peak energies from oeINDO and ZINDO/s as compared to TDDFT are 0.40 eV and 0.83 eV respectively, with the latter being roughly two times less accurate than the former in this case.

From the results discussed so far, oeINDO is transferable and compares favourably well with accurate ab-initio methods.

### 4.2.4 Transferability of oeINDO Cadmium parameters

The accuracy of the oeINDO parameters for cadmium was also checked. The oeINDO cadmium parameters were tested with cadmium complex structures, $\mathrm{Cd}_{n}(n=3,4,6$, 8, 16, 20, 25, 30). Eight lowest excited state energies for each of these structures were calculated using various methods including oeINDO. The results are presented in Tables (Tables 4.14, C.7, C.8, C. 9 and C.10) and given in form of scatter plots for comparison. Absorption spectra computed using different methods for the same set of structures were also used for the test, even though spectra were not part of the parameterisation data.

Table 4.14: First Eight Lowest Excited State Energies (eV) for $\mathrm{Cd}_{3}$

| EOM-CCSD | oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: | :---: |
| 4.077 | 3.970 | 5.615 | 3.691 | 4.225 |
| 4.091 | 3.981 | 5.618 | 3.702 | 4.241 |
| 4.339 | 4.028 | 5.739 | 3.895 | 4.468 |
| 4.367 | 4.045 | 5.833 | 3.905 | 4.524 |
| 4.379 | 4.488 | 5.833 | 4.077 | 4.538 |
| 4.720 | 4.512 | 5.834 | 4.125 | 4.834 |
| 5.189 | 4.907 | 5.835 | 4.965 | 5.309 |
| 5.191 | 4.913 | 5.860 | 4.970 | 5.312 |
|  |  |  |  |  |



Figure 4.23: Semi-empirical methods against TDDFT energies for $\mathrm{Cd}_{n}(n=3,4,6,8,16$, $20,25,30$ ) equilibrium structures. The straight green line represents energies obtained using TDDFT.

Table 4.14 shows the excited state energies obtained for $\mathrm{Cd}_{3}$ equilibrium structure using oeINDO, EOM-CCSD, ZINDO/s, TDDFT, and CIS(D). The table shows that oeINDO produced first and second excited state energies of 3.970 and 3.981 eV , respectively, which in close agreement with corresponding energies of 4.077 and 4.091 eV from EOM-CCSD. The MAE of the eight lowest excited state energies from oeINDO relative to the EOM-CCSD ones is 0.21 eV . It can be observed that, relative to EOM-CCSD, oeINDO performed better than TDDFT (MAE=0.37 eV) and ZINDO/s (MAE=1.22 eV). However, CIS(D) appreciably out performs oeINDO with the MAE of its energies relative to the EOM-CCSD energies being 0.13 eV . The tables displaying the excitation energies of other $\mathrm{Cd}_{n}$ structures can be found in the appendices C.7-C.10.

Seventy-two (72) excitation energies each were obtained from calculations with oeINDO and ZINDO/s for $\mathrm{Cd}_{n}(\mathrm{n}=3,4,6,8,16,20,25,30)$ equilibrium structures. These energies were compared with those from TDDFT as shown in Figure 4.23. The figure shows that the excitation energies (blue circles) from oeINDO match closely with TDDFT (the straight green line). However, the ZINDO/s excitations (red plus signs) are much deviated from the straight line. The MAE of the oeINDO energies relative to those of the TDDFT is 0.22 eV . The ZINDO/s energies however, deviated much more from TDDFT with MAE of 2.83 eV , more than ten times the size of the MAE from oeINDO.

Although the UV-Vis absorption spectra were not used as reference data during the parameterisation, oeINDO produced absorption spectra that are qualitatively comparable to spectra from accurate ab-initio approaches for $\mathrm{Cd}_{n}(n=3,4,6,8,16,20,25,30)$ equilibrium structures.


Figure 4.24: Absorption spectra for $\mathrm{Cd}_{3}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.25: Absorption spectra for $\mathrm{Cd}_{4}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.26: Absorption spectra for $\mathrm{Cd}_{6}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.27: Absorption spectra for $\mathrm{Cd}_{8}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.

The absorption spectra of $\mathrm{Cd}_{3}$ obtained using the EOM-CCSD, TDDFT, oeINDO and ZINDO/s methods are presented in Figure 4.24. The absorption bands of the spectra from all the different methods lie in the ultraviolet region. The oeINDO band matches well with that of the TDDFT; the highest absorption peaks of TDDFT and oeINDO are both red-shifted (decreased in energy) by about 0.10 eV from EOM-CCSD. However, the ZINDO/s peak is blue-shifted (increased in energy) by about 0.50 eV from EOM-CCSD or five times more than the oeINDO error.

Figure 4.25 shows absorption spectra from TDDFT, oeINDO and ZINDO/s for $\mathrm{Cd}_{4}$. Because of the limited computational resources, the spectrum from EOM-CCSD was not computed. All the methods predict $\mathrm{Cd}_{4}$ equilibrium structure to absorb electromagnetic radiation within the ultraviolet energy range. The oeINDO absorption energy range and the spectrum highest peak agrees well with those from TDDFT. Its peak is shifted from that of the TDDFT by a small error, less than 0.10 eV . The peak of the ZINDO/s, however, is blue-shifted by a larger error of about 1.50 eV .

The absorption spectra for $\mathrm{Cd}_{6}$ in Figure 4.26, further reveal that oeINDO spectra compare well with the ab-initio ones. The TDDFT and oeINDO both predict absorption spectra in the same energy range ( $2.60 \mathrm{eV}-4.0 \mathrm{eV}$ ). The oeINDO spectrum highest peak is blue-shifted from TDDFT by a small error of 0.30 eV while the ZINDO/s one is shifted by error of about 2.0 eV , which is roughly seven times more.

For $\mathrm{Cd}_{8}$, the oeINDO spectrum matches almost exactly with that from TDDFT as shown in Figure 4.27. The ZINDO/s spectrum, on the other hand, deviates from TDDFT by about 2.40 eV .


Figure 4.28: Absorption spectra of $\mathrm{Cd}_{16}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.29: Absorption spectra for $\mathrm{Cd}_{20}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.30: Absorption spectra for $\mathrm{Cd}_{25}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.31: Absorption spectra for $\mathrm{Cd}_{30}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.32: Highest peaks against number of units $n$ in equilibrium $\mathrm{Cd}_{n}$ structures

The spectra of the $\mathrm{Cd}_{16}$ cluster as shown in Figure 4.28 corroborates the good performance of oeINDO. Both oeINDO and TDDFT showed that the cluster absorbs is in the visible region of the electromagnetic spectrum. In contrast, ZINDO/s showed absorption the ultraviolet region for the same cluster. The positions of the peaks of the oeINDO and ZINDO/s spectra intensity relative to that of TDDFT are about 0.30 eV and 3.30 eV , respectively. The absorption spectra for $\mathrm{Cd}_{20}, \mathrm{Cd}_{25}$ and $\mathrm{Cd}_{30}$ in Figures 4.29, 4.30, and 4.29 , respectively, also show that the oeINDO spectra agree well with those from the ab-initio TDDFT and CIS(D) method. The ZINDO/s spectra, however, deviate much more up to $\approx 3.3 \mathrm{eV}$

Figure 4.32 summarizes how the peak maxima from the oeINDO and ZINDO/s absorption spectra compare with those from TDDFT. It can be observed for all methods that the energies of the peaks decrease with the number of atoms. The peaks of oeINDO agree well with those from TDDFT with MAE of 0.17 eV while ZINDO/s peaks deviate from TDDFT with a larger MAE of 1.98 eV .

### 4.2.5 Transferability of oeINDO Sulphur Parameters

The oeINDO method was also validated by verifying its accuracy for complex sulphur structures that were not included in the training geometries. Excitation energies and absorption spectra were computed with different methods for $S_{n}(n=3,5,6,10,20)$ equilibrium structures and their results are presented in Figures 4.33, 4.34, 4.35, 4.36, 4.37, and 4.37.


Figure 4.33: Semi-empirical methods against TDDFT energies for $S_{n}(n=5,6,10,20)$ equilibrium structures


Figure 4.34: Absorption spectra for $S_{3}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.35: Absorption spectra for $\mathrm{S}_{5}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.36: Absorption spectra for $S_{6}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.

Figure 4.33 is a scatter plot that shows how the excitation energies from oeINDO and ZINDO/s compare with those from TDDFT for $\mathrm{S}_{n}(\mathrm{n}=5,6,10,20)$ structures. A total of thirty-two excitation energies were obtained with of this each method. The plot shows that oeINDO excitation energies (blue circles) are reasonably close to the $a b$ initio TDDFT excitations while ZINDO/s energies (red plus signs) deviate more. The oeINDO excitations compared favorably with the TDDFT ones with MAE of 0.36 eV while ZINDO/s excitations (red plus signs) compared with a larger MAE of 2.51 eV .

The absorption spectra of $S_{3}$ equilibrium structure obtained with different methods are presented in Figure 4.34. Similar spectra patterns are observed for all methods. The EOM-CCSD (benchmark), oeINDO and ZINDO/s absorption band appear in the visible and the ultra-violet energy regions but that of TDDFT is found only in the ultraviolet region. The highest peak of the spectra from TDDFT, oeINDO and ZINDO/s are shifted from EOM-CCSD by 0.26 eV (blue-shifted), 0.61 eV (red-shifted), and 0.70 eV (redshifted), respectively. Relative to EOM-CCSD, the oeINDO performed better than the original ZINDO/s model.

The spectra for the $\mathrm{S}_{5}$ equilibrium structure in Figure 4.35 show that oeINDO agrees well with EOM-CCSD. It reproduced the EOM-CCSD spectra pattern. The oeINDO has a comparable performance with TDDFT. Both oeINDO and TDDFT spectra highest peaks are shifted from EOM-CCSD by approximately 0.44 eV . While the oeINDO peak is blue-shifted, the TDDFT one is red-shifted. The spectrum peak from ZINDO/s however, is much deviated from EOM-CCSD spectrum highest peak with a larger error of 2.94 eV .

For the sulphur structures with six or more atoms, EOM-CCSD is computationally intensive and the available computational resources could not handle them. Thus, comparisons were made with TDDFT, a reasonable alternative to EOM-CCSD. In Figure 4.36, a very good agreement was observed between the oeINDO and TDDFT absorption spectra for the $\mathrm{S}_{6}$ equilibrium structures. Both have similar spectra pattern and show absorption in the energy range $3.50-4.80 \mathrm{eV}$. However, ZINDO/s spectrum is far shifted (blue-shifted) from that of the TDDFT by 2.39 eV .


Figure 4.37: Absorption spectra for $S_{10}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.38: Absorption spectra for $S_{20}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.

The performance of the oeINDO in favorably reproducing spectra from ab-initio calculations was also confirmed with the absorption spectra obtained for $S_{10}$ and $S_{20}$ structures as shown in Figures 4.37 and 4.38, respectively.

For $\mathrm{S}_{10}$ equilibrium structure, oeINDO reproduced the spectrum pattern observed in the TDDFT one. The first prominent peak from oeINDO is shifted from the first and second peak of the TDDFT spectrum by about 0.2 and 0.72 eV , respectively.

A good match of the spectra from oeINDO and TDDFT was also observed for $\mathrm{S}_{20}$ equilibrium structure (Figure 4.38). Both show that the $\mathrm{S}_{20}$ structure absorbs in the ultraviolet region with their highest peaks matching exactly. In contrast, the spectra from ZINDO/s is blue-shifted from TDDFT by a large amount: 1.56 eV .

The average absolute shift of oeINDO absorption spectra highest peaks from those of the TDDFT for all $\mathrm{S}_{n}(\mathrm{n}=5,6,10,20)$ equilibrium structures is 0.5 eV . The ZINDO/s is however shifted by a larger amount of 2.02 eV .

### 4.2.6 Transferability of oeINDO to ZnS clusters

Despite the fact that the $\mathrm{Zn}-\mathrm{S}$ diatomics were not directly parameterised, the transferability and accuracy of the oeINDO model to complex ZnS clusters were also found to be good. Calculations for these clusters were carried out using the oeINDO $\mathrm{Zn}_{2}$ and $\mathrm{S}_{2}$ parameters.


Figure 4.39: Scatter plot of $\mathrm{Zn}-\mathrm{S}$ diatomics excitation energies from ZINDO/s and oeINDO against those from EOM-CCSD (the benchmark). The blue circles are the oeINDO excitation energies while the red plus signs are the ZINDO/s excitation energies. The straight green line represents the benchmark.


Figure 4.40: Semi-empirical methods against TDDFT energies for $(\mathrm{ZnS})_{n}(n=$ $2,3,4,10,19)$ equilibrium structures.

Figure 4.39 is a scatter plot of 48 excitation energies each obtained using oeINDO and ZINDO/s against those obtained with EOM-CCSD for $\mathrm{Zn}-\mathrm{S}$ diatomics. The oeINDO energies compared with the EOM-CCSD energies with MAE of 0.52 eV while ZINDO/s compared with a slightly larger error of 0.57 eV .

The excitation energies computed for $(\mathrm{ZnS})_{n}(\mathrm{n}=2,3,4,10,19)$ equilibrium structure using the oeINDO, ZINDO/s and TDDFT method are presented in Tables C. 17 - C.21. A scatter plot showing a total of 40 excitation energies obtained using oeINDO and ZINDO/s against those from TDDFT for all $(\mathrm{ZnS})_{n}(\mathrm{n}=2,3,4,10,19)$ equilibrium structures are displayed in Figure 4.41. The plot shows that oeINDO agrees well with the TDDFT and performs better than ZINDO/s. The MAE of the excitations from oeINDO relative to those from TDDFT is 0.23 eV while MAE of the excitations from ZINDO/s relative to TDDFT is much larger: 1.33 eV . Also, it can be observed from Table C. 17 and Figure 4.41 that ZINDO/s has its first and second-lowest excitation energies as negative values for the $(\mathrm{ZnS})_{2}$ structure.

The oeINDO transferability was further verified with the absorption spectra obtained for the different ZnS complex clusters. The oeINDO spectra as shown in Figures 4.41, 4.42, $4.43,4.44$, and 4.45 agree reasonably with those from TDDFT.


Figure 4.41: Absorption spectra for $(\mathrm{ZnS})_{2}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.42: Absorption spectra for $(\mathrm{ZnS})_{3}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.43: Absorption spectra for $(\mathrm{ZnS})_{4}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.44: Absorption spectra for $\mathrm{Zn}_{10} \mathrm{~S}_{10}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.45: Absorption spectra for $\mathrm{Zn}_{19} \mathrm{~S}_{19}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.46: Highest peaks against number of units $n$ in equilibrium $(\mathrm{ZnS})_{n}$ structures

The absorption spectra of $(\mathrm{ZnS})_{2}$ (Figure 4.41), from oeINDO, CIS(D), TDDFT and ZINDO/s peaks at about 2.23 eV (in visible region of the spectrum), 3.63 eV (ultraviolet region), 3.93 eV (ultraviolet region) and 5.20 eV (ultraviolet region), respectively. The oeINDO spectrum highest peak is red-shifted by 1.70 eV from that TDDFT while that of ZINDO/s is blue-shifted by 1.20 eV .

The spectra from different methods for the $(\mathrm{ZnS})_{3}$ cluster in Figure 4.16 show that the cluster absorbs in the ultraviolet region for all methods employed. oeINDO reproduced the TDDFT absorption spectrum with its highest peak shifted from TDDFT by 0.13 eV . However, the CIS(D) and ZINDO/s spectra peaks are shifted from TDDFT by 0.45 eV and 1.45 eV , respectively.

A very good match of the oeINDO and TDDFT was observed for $(\mathrm{ZnS})_{4}$ (Figure 4.43). The oeINDO spectrum highest peak matched that of TDDFT with a small error of 0.08 eV .

The absorption spectra from oeINDO in Figure 4.37 and Figure 4.45 for $(Z n S)_{n}(\mathrm{n}=10$, 19) support the transferability ability of the oeINDO parameters. For $(\mathrm{ZnS})_{10}$ cluster, the oeINDO, TDDFT and CIS(D) spectra have similar pattern and they all absorb in the visible region (1.50-3.0 eV). ZINDO/s however, absorb in the ultraviolet region (3.0$30.0 \mathrm{eV})$. oeINDO reproduces the spectrum from the TDDFT for $(\mathrm{ZnS})_{19}$ cluster with a good match in absorption spectra maxima.

Figure 4.46 shows how the spectra maxima from oeINDO and ZINDO/s compare with those from TDDFT. Obviously, the oeINDO compares better than ZINDO/s with TDDFT as the benchmark. The MAE of the spectra peaks from oeINDO relative to TDDFT is 0.50 eV while that for ZINDO/s relative to TDDFT is 1.90 eV .

### 4.2.7 Transferability of oeINDO to CdS clusters

Just as in the case of ZnS clusters, the Cd-S diatomics were not directly parameterised rather, the parameters obtained for $\mathrm{Cd}_{2}$ and $\mathrm{S}_{2}$ were employed to calculate the excitation energies and absorption spectra for the $(\mathrm{CdS})_{n}(\mathrm{n}=2,3,4,10)$ equilibrium structures.


Figure 4.47: Scatter plot of Cd-S diatomics excitation energies from ZINDO/s and oeINDO against those from EOM-CCSD (the benchmark). The blue circles are the oeINDO excitation energies while the red plus signs are the ZINDO/s excitation energies. The straight green line represents the benchmark.


Figure 4.48: Semi-empirical methods against TDDFT energies for $(\mathrm{CdS})_{n}(n=$ $2,3,4,10$ ) equilibrium structures

Figure 4.47 shows the scatter plot of oeINDO and ZINDO/s excited state energies against the EOM-CCSD energies for Cd-S diatomic of different separations. A total of 48 excitation energies obtained with each method. The oeINDO compared better than ZINDO/s to EOM-CCSD. The MAE of oeINDO excited energies relative to EOM-CCSD energies is 0.51 eV . The MAE of ZINDO/s however, is much larger: 0.81 eV .

A total of thirty-two vertical excitation energies were computed each using oeINDO and ZINDO/s for $(\mathrm{CdS})_{n}(n=2,3,4,10)$ equilibrium structures and compared with corresponding energies from TDDFT. Comparisons were not made with EOM-CCSD because the available computational resource is limited and cannot handle the huge calculations involved for these structures.

The scatter plot in Figure 4.48 shows the comparison of the excitation energies from oeINDO and ZINDO/s to those from TDDFT for $(\mathrm{CdS})_{n}(\mathrm{n}=2,3,4,10)$ equilibrium structures. The oeINDO excitation energies matched with the TDDFT ones with MAE of 0.36 eV . ZINDO/s performed poorly. Its excitation energies deviate from TDDFT energies with larger MAE of 0.96 eV . More so, it gave some negative excitations (Figure 4.48 and Table C.22), which are unacceptable because the excitation energy difference (HOMO -LUMO) cannot be a negative value.


Figure 4.49: Absorption spectra for $(\mathrm{CdS})_{2}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.50: Absorption spectra for $(\mathrm{CdS})_{3}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.51: Absorption spectra for $(\mathrm{CdS})_{4}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.52: Absorption spectra for $(\mathrm{CdS})_{10}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.53: Absorption spectra for $(\mathrm{CdS})_{19}$ equilibrium geometry obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.54: Highest peaks against number of units $n$ in equilibrium $(\mathrm{CdS})_{n}$ structures. The oeINDO curve and not the ZINDO/s curve qualitatively reproduces the TDDFT one.

Figures $4.49,4.50,4.51,4.52$, and 4.53 further confirm the capability of oeINDO to give reasonably accurate results for structures not included in the parameterisation training sets.

For the $(\mathrm{CdS})_{2}$ equilibrium structure, the absorption spectra pattern is similar for all methods as shown in Figure 4.49. Although the highest peaks of the spectra from both oeINDO and ZINDO/s are red-shifted from that of TDDFT, the oeINDO peak is in better agreement with TDDFT results.

Both oeINDO and ZINDO/s methods reasonably reproduced the absorption spectrum from TDDFT for the $(\mathrm{CdS})_{3}$ equilibrium structure. The absorption bands for all three methods are within the ultraviolet energy range. The highest peaks of the absorption spectra from both oeINDO and ZINDO/s are shifted from TDDFT by about 0.1 eV .

The situation with spectra obtained for the $(\mathrm{CdS})_{4}$ equilibrium structure (Figure 4.51) are not different from those of the $(\mathrm{CdS})_{3}$ equilibrium structure. In Figure 4.51, it can be observed that the absorption patterns and bands from all methods are similar. The ZINDO/s and oeINDO spectra agree well with TDDFT. However, the spectrum highest peak from ZINDO/s is better compared to TDDFT than for oeINDO to TDDFT. The peaks of spectra from ZINDO/s and oeINDO are shifted from that of TDDFT by 0.07 eV and 0.13 eV , respectively.

The absorption spectra of the larger CdS clusters, $\left((\mathrm{CdS})_{n}(\mathrm{n}=10,19)\right)$ in Figures 4.52 and 4.53 , show that the oeINDO spectra compare qualitatively with TDDFT. The absorption bands and pattern of the oeINDO and TDDFT in Figure 4.52 are comparable. For $(\mathrm{CdS})_{19}$ spectra (Figure 4.53), both the oeINDO and TDDFT spectra peak were found in the ultraviolet region of the electromagnetic spectrum. The oeINDO spectrum peak is shifted from that of TDDFT by about 0.87 eV . The highest peak of the ZINDO/s spectrum is in the visible region and is shifted from TDDFT highest peak by 2.21 eV .

Figure 4.54 displays the plot of absorption highest peaks for $(\mathrm{CdS})_{n}$ against number of units $n$. It can be seen that oeINDO highest absorption peaks produced a similar trend observed in that for TDDFT. The average shift of the oeINDO peaks from TDDFT peaks is 0.67 eV while ZINDO/s spectra peaks shifted by 1.15 eV (almost double the error observed in oeINDO).


Figure 4.55: Plot comparing excitation energies from semi-empirical methods (ZINDO/s and oeINDO) with those from TDDFT for $\mathrm{Cd}_{5} \mathrm{Zn}_{14} \mathrm{~S}_{19}, \mathrm{Cd}_{10} \mathrm{Zn}_{9} \mathrm{~S}_{19}$ and $\mathrm{Cd}_{15} \mathrm{Zn}_{4} \mathrm{~S}_{19}$.


Figure 4.56: Absorption spectra for $\mathrm{Cd}_{5} \mathrm{Zn}_{14} \mathrm{~S}_{19}$ equilibrium structure obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.57: Absorption spectra for $\mathrm{Cd}_{10} \mathrm{Zn}_{9} \mathrm{~S}_{19}$ equilibrium structure obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.58: Absorption spectra for $\mathrm{Cd}_{15} \mathrm{Zn}_{4} \mathrm{~S}_{19}$ equilibrium structure obtained from different methods.The intensities have been scaled so that the highest intensity is equal to unity.

### 4.2.8 Transferability oeINDO parameters to $\mathrm{Cd}_{x} \mathrm{Zn}_{y} \mathbf{S}_{19}$ clusters

The transferability of oeINDO model was also verified with cadmium-zinc sulphide, $\mathrm{Cd}_{x} \mathrm{Zn}_{y} \mathrm{~S}_{19}$ structures (where $\mathrm{x}, \mathrm{y}$ are the number atoms), that is, how well did the model reproduce the ab-initio model (TDDFT) results. The accuracy of the model was discussed using scatter plot of excitation energies from oeINDO and ZINDO/s as compared to TDDFT for $\mathrm{Cd}_{5} \mathrm{Zn}_{14} \mathrm{~S}_{19}, \mathrm{Cd}_{10} \mathrm{Zn}_{9} \mathrm{~S}_{19}$ and $\mathrm{Cd}_{15} \mathrm{Zn}_{4} \mathrm{~S}_{19}$ equilibrium structures. UV-vis absorption spectra for the same set of structures were also employed for the discussion.

Figure 4.55 displays a plot comparing excitation energies from semi-empirical methods (ZINDO/s and oeINDO) with those from TDDFT for $\mathrm{Cd}_{5} \mathrm{Zn}_{14} \mathrm{~S}_{19}, \mathrm{Cd}_{10} \mathrm{Zn}_{9} \mathrm{~S}_{19}$ and $\mathrm{Cd}_{15} \mathrm{Zn}_{4} \mathrm{~S}_{19}$ structures. It can be observed that blue circles (oeINDO energies) are much more closely packed than the red plus signs (ZINDO/s energies) to the green line (representing the ab-initio, TDDFT energies). The oeINDO excitations match well with excitations from TDDFT with MAE of 0.08 eV while ZINDO/s excitations deviate with a larger MAE: 0.60 eV .

The absorption spectra of $\mathrm{Cd}_{5} \mathrm{Zn}_{14} \mathrm{~S}_{19}$ structure obtained from TDDFT, oeINDO and ZINDO/s are presented in Figure 4.56. A good match of the absorption spectra from TDDFT and oeINDO methods can be observed with their prominent peaks matching with a small error of 0.04 eV . Also, the spectra are found in about the same energy range 2.3-3.5 eV. However, ZINDO/s spectrum energy is 3.2-5.0 eV (almost outside the benchmark energy range). Its highest peak located at 4.51 eV is blue-shifted from TDDFT one by a much larger error of 1.39 eV .

The absorption spectra of $\mathrm{Cd}_{10} \mathrm{Zn}_{9} \mathrm{~S}_{19}$ equilibrium structure from TDDFT and oeINDO match reasonably well (see fig. 4.57). The absolute difference in their highest peak is 0.14 eV . ZINDO/s spectrum maximum is, however, shifted from that of the TDDFT by 0.62 eV .

For the $\mathrm{Cd}_{15} \mathrm{Zn}_{4} \mathrm{~S}_{19}$ equilibrium structure, the absorption spectra obtained from TDDFT and oeINDO are also in good agreement. Their highest peaks, which are observed at 3.05 eV (TDDFT) and 3.07 eV (oeINDO), match with a small error of 0.02 eV . The ZINDO/s spectrum peak on the other hand is observed at 2.67 eV being shifted from that of TDDFT by 0.38 eV .

Table 4.15: First eight (8) lowest vertical excitation energies (eV) for large silicon clusters

| $\mathrm{S} / \mathrm{N}$ | $\mathrm{Si}_{124}$ | $\mathrm{Si}_{147}$ | $\mathrm{Si}_{172}$ | $\mathrm{Si}_{244}$ | $\mathrm{Si}_{779}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.008 | 0.235 | 0.039 | 0.017 | 0.016 |
| 2 | 0.038 | 0.305 | 0.048 | 0.062 | 0.017 |
| 3 | 0.050 | 0.391 | 0.058 | 0.071 | 0.019 |
| 4 | 0.053 | 0.408 | 0.064 | 0.075 | 0.025 |
| 5 | 0.070 | 0.533 | 0.079 | 0.101 | 0.036 |
| 6 | 0.094 | 0.560 | 0.081 | 0.110 | 0.038 |
| 7 | 0.101 | 0.587 | 0.090 | 0.134 | 0.042 |
| 8 | 0.110 | 0.677 | 0.100 | 0.148 | 0.054 |



Figure 4.59: Absorption spectra of $\operatorname{Si}_{n}(n=124,147,172,779)$ equilibrium geometries obtained from oeINDO.The intensities have been scaled so that the highest intensity is equal to unity.

Table 4.16: First eight (8) vertical excitation energies (eV) for Zn quantum dots of different sizes

| $\mathrm{S} / \mathrm{N}$ | $\mathrm{Zn}_{62}$ <br> $(1.2 \mathrm{~nm})$ | $\mathrm{Zn}_{104}$ <br> $(1.4 \mathrm{~nm})$ | $\left.\mathrm{Zn}_{128}\right)$ <br> $(1.6 \mathrm{~nm})$ | $\mathrm{Zn}_{190}$ <br> $(1.8 \mathrm{~nm})$ | $\left(\mathrm{Zn}_{244}\right)$ <br> 2.0 nm |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.080 | 0.080 | 0.048 | 0.055 | 0.042 |
| 2 | 0.203 | 0.094 | 0.077 | 0.096 | 0.068 |
| 3 | 0.211 | 0.098 | 0.124 | 0.112 | 0.084 |
| 4 | 0.296 | 0.145 | 0.133 | 0.123 | 0.099 |
| 5 | 0.304 | 0.161 | 0.139 | 0.135 | 0.115 |
| 6 | 0.325 | 0.171 | 0.159 | 0.148 | 0.126 |
| 7 | 0.349 | 0.180 | 0.188 | 0.157 | 0.147 |
| 8 | 0.395 | 0.191 | 0.210 | 0.169 | 0.149 |



Figure 4.60: Absorption spectra of Zn quantum dots of different sizes (diameters) obtained from oeINDO.The intensities have been scaled so that the highest intensity is equal to unity.

### 4.3 Study of large clusters and quantum dots using the oeINDO model

Having validated the oeINDO model in the previous section, it was employed for the study of large clusters and quantum dots, for which accurate ab-initio methods (e.g EOM-CCSD, TDDFT) are computationally expensive or sometimes prohibitive. The oeINDO was used to compute excitation energies and absorption spectra of silicon, zinc, cadmium, sulphur, cadmium sulphide and zinc sulphide large clusters and quantum dots of various sizes. Results obtained are presented in Tables 4.15-4.21 and Figures 4.59 -4.66 and discussed.

Table 4.15 and Figure 4.59 show the excitation energies and absorption spectra, respectively obtained for $124,147,172,244,779 \mathrm{Si}$ atom clusters (ranging from 1.0 to 3.0 nm Si quantum dots) using the oeINDO method. It can be observed that the first excitation energy (4.15) and the absorption edge (4.59) of each silicon cluster are less than 0.04 eV except for $\mathrm{Si}_{147}$ which has a value of 0.23 eV . This indicates that large silicon clusters and quantum dots are metallic or 'near metallic' systems. This result is in agreement with the result obtained by Jacksons et al. (Jackson and Jellinek, 2016). Furthermore, the first lowest excitation energy (electronic gap) 4.59 of 0.24 eV determined for $\mathrm{Si}_{147}$ using oeINDO compares well with the electronic gap (of roughly 0.30 eV ) obtained in the same report (Jackson and Jellinek, 2016).

The results of excitation energies and absorption spectra for zinc quantum dots of diameters 1.2, 1.4, 1.6, 1.8 and 2.0 nm obtained with oeINDO are presented in Table 4.16 and Figure 4.60. The first excitation energies and the absorption spectra edge of all the Zn quantum dots are $<0.1 \mathrm{eV}$ which suggest that the systems are all metallic. This agrees with previous reports (Kostko et al., 2005, Aguado et al., 2018). An important feature of nano-sized systems, confinement effect, was also observed. It involves a red-shift (decrease in energy) in the absorption spectra maxima with an increase in the size of the dot and a blue-shift (increase in energy) as the size decreases. From the Zn dot spectra plot, one can readily observe that confinement effect set in at dot size 1.6 nm with maximum peak at 0.21 eV . The peak energy decreased to 0.17 eV for dot size 1.8 nm and 0.14 eV for size 2.0 nm .

Table 4.17: Vertical excitation energies for Cadmium quantum dots of different sizes

| $\mathrm{S} / \mathrm{N}$ | $\mathrm{Cd}_{38}$ <br> $(1.0 \mathrm{~nm})$ | $\mathrm{Cd}_{62}$ <br> $(1.2 \mathrm{~nm})$ | $\mathrm{Cd}_{104}$ <br> $(1.4 \mathrm{~nm})$ | $\mathrm{Cd}_{128}$ <br> $(1.6 \mathrm{~nm})$ | $\mathrm{Cd}_{190}$ <br> $(1.8 \mathrm{~nm})$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.741 | 0.401 | 0.070 | 0.078 | 0.088 |
| 2 | 0.759 | 0.436 | 0.090 | 0.101 | 0.092 |
| 3 | 0.800 | 0.464 | 0.131 | 0.150 | 0.110 |
| 4 | 0.831 | 0.487 | 0.145 | 0.197 | 0.146 |
| 5 | 0.858 | 0.519 | 0.177 | 0.222 | 0.153 |
| 6 | 0.863 | 0.584 | 0.186 | 0.241 | 0.157 |
| 7 | 0.884 | 0.601 | 0.214 | 0.250 | 0.161 |
| 8 | 0.899 | 0.618 | 0.231 | 0.273 | 0.171 |



Figure 4.61: Absorption spectra of Cd quantum dots of different sizes (diameters) obtained from oeINDO.The intensities have been scaled so that the highest intensity is equal to unity.

Table 4.18: Lowest eight (8) vertical excitation energies(eV) for sulphur quantum dots of different sizes

| $\mathrm{S} / \mathrm{N}$ | $\mathrm{S}_{38}$ <br> $(1.0 \mathrm{~nm})$ | $\mathrm{S}_{62}$ <br> $(1.2 \mathrm{~nm})$ | $\mathrm{S}_{104}$ <br> $(1.4 \mathrm{~nm})$ | $\mathrm{S}_{128}$ <br> $(1.6 \mathrm{~nm})$ | $\mathrm{S}_{244}$ <br> $(2.0 \mathrm{~nm})$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1 | 2.357 | 0.230 | 0.411 | 0.075 | 0.117 |
| 2 | 2.384 | 0.284 | 0.472 | 0.155 | 0.141 |
| 3 | 2.551 | 0.407 | 0.692 | 0.165 | 0.216 |
| 4 | 2.578 | 0.593 | 0.831 | 0.271 | 0.231 |
| 5 | 2.681 | 0.871 | 0.861 | 0.317 | 0.272 |
| 6 | 2.698 | 0.998 | 0.913 | 0.346 | 0.308 |
| 7 | 2.738 | 1.022 | 0.916 | 0.384 | 0.340 |
| 8 | 2.748 | 1.118 | 0.965 | 0.430 | 0.343 |
|  |  |  |  |  |  |



Figure 4.62: Absorption spectra of $S$ quantum dots of different sizes (diameters) obtained from oeINDO.The intensities have been scaled so that the highest intensity is equal to unity.

Table 4.17 and Figure 4.61 show the results of the excitation energies and absorption spectra of nano-sized cadmium structures ( $1.0-1.8 \mathrm{~nm}$ ) obtained using oeINDO. It can be observed that the spectra maxima are red-shifted and become more metallic with an increase in the size of the Cd quantum dot. This agrees with earlier reports (Kohaut and Springborg, 2016, Kohaut, 2017). In the report by Kohaut and Springborg (Kohaut and Springborg, 2016), using DFT, the electronic gap of $\mathrm{Cd}_{38}\left(1.0 \mathrm{~nm} \mathrm{Cd}\right.$ dot) and $\mathrm{Cd}_{60}$ were computed to be 0.563 eV and 0.29 eV , respectively. Computation of gaps (first excitation energies) for the same structures (4.17) with oeINDO yielded results that are about $\approx 0.2 \mathrm{eV}$ greater. But, DFT is known for typical underestimation of electronic gaps (Tran and Blaha, 2017). Hence, oeINDO prediction of the electronic gaps, which are higher than those from DFT, should be more accurate.

The oeINDO results of the calculation of excitation energies and absorption spectra for different sulphur quantum dots are presented in Table 4.18 and Figure 4.62. From spectra plot, absorption maximum peaks are located at $2.73 \mathrm{eV}, 1.0 \mathrm{eV}, 0.80 \mathrm{eV}, 0.45 \mathrm{eV}$ and 0.25 eV for the $1.0 \mathrm{~nm}, 1.2 \mathrm{~nm}, 1.4 \mathrm{~nm}, 1.8 \mathrm{~nm}$ and 2.0 nm sulphur dots, respectively. It can be observed that the spectra prominent peaks are red-shifted (decrease in energy) as the size of the cluster increases -confinement effect. In addition, the spectra and the first excitation energies from the Table 4.18 show a transition of the sulphur dots from non-metallic to metallic somewhere between 1.0 nm and 1.6 nm dot size.

Table 4.19: First eight lowest vertical excitation energies (eV) for ZnS quantum dots of different sizes

| $\mathrm{S} / \mathrm{N}$ | $\mathrm{Zn}_{19} \mathrm{~S}_{19}$ <br> $(1.0 \mathrm{~nm})$ | $\mathrm{Zn}_{25} \mathrm{~S}_{25}$ <br> $(1.2 \mathrm{~nm})$ | $\mathrm{Zn}_{52} \mathrm{~S}_{52}$ <br> $(1.4 \mathrm{~nm})$ | $\mathrm{Zn}_{95} \mathrm{~S}_{95}$ <br> $(1.8 \mathrm{~nm})$ | $\mathrm{Zn}_{122} \mathrm{~S}_{122}$ <br> $(2.0 \mathrm{~nm})$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.179 | 1.909 | 1.595 | 1.156 | 0.057 |
| 2 | 1.352 | 1.994 | 1.769 | 1.270 | 0.085 |
| 3 | 1.631 | 2.039 | 1.885 | 1.311 | 0.231 |
| 4 | 1.811 | 2.188 | 1.904 | 1.343 | 0.307 |
| 5 | 1.859 | 2.225 | 1.929 | 1.399 | 0.333 |
| 6 | 1.974 | 2.363 | 2.005 | 1.475 | 0.376 |
| 7 | 1.983 | 2.390 | 2.034 | 1.541 | 0.407 |
| 8 | 2.078 | 2.424 | 2.070 | 1.614 | 0.456 |
|  |  |  |  |  |  |



Figure 4.63: Absorption spectra of ZnS quantum dots of different sizes (diameters) obtained from oeINDO.The intensities have been scaled so that the highest intensity is equal to unity.


Figure 4.64: Plot of electronic gap for $(\mathrm{CdS})_{n}$ and $(\mathrm{ZnS})_{n}$ quantum dots for different number of units, n

Table 4.20: First eight lowest vertical excitation energies (eV) for CdS quantum dots of different sizes

| $\mathrm{S} / \mathrm{N}$ | $(\mathrm{CdS})_{19}$ <br> $(1.0 \mathrm{~nm})$ | $\mathrm{Cd}_{52} \mathrm{~S}_{52}$ <br> $(1.4 \mathrm{~nm})$ | $\mathrm{Cd}_{95} \mathrm{~S}_{95}$ <br> $(1.8 \mathrm{~nm})$ | $\mathrm{Cd}_{159} \mathrm{~S}_{159}$ <br> $(2.2 \mathrm{~nm})$ |
| :--- | :---: | :---: | :---: | :---: |
| 1 | 1.674 | 1.773 | 1.073 | 0.322 |
| 2 | 1.797 | 1.829 | 1.353 | 0.412 |
| 3 | 1.894 | 1.980 | 1.418 | 0.521 |
| 4 | 1.949 | 2.006 | 1.443 | 0.682 |
| 5 | 1.994 | 2.044 | 1.491 | 0.695 |
| 6 | 2.094 | 2.051 | 1.534 | 0.704 |
| 7 | 2.096 | 2.090 | 1.595 | 0.711 |
| 8 | 2.184 | 2.132 | 1.638 | 0.791 |



Figure 4.65: Absorption spectra of CdS quantum dots of different sizes (diameters) obtained from oeINDO.The intensities have been scaled so that the highest intensity is equal to unity.

The absorption spectra and the excitation energies obtained from oeINDO for 1.0 2.2 nm (corresponding to 59-319 atoms) ZnS quantum dots are presented in Figure 4.63 and Table 4.19, respectively. A confinement effect can be observed and it sets in at the 1.2 nm ZnS quantum dot; a redshift is observed in the spectra maxima and first excitation energy with an increase in the size of the dots. The plot of electronic gaps (first excitation energies) against number of atoms (Figure 4.64) shows a decrease in ZnS cluster gap with increase in number of atoms, starting from $(\mathrm{ZnS})_{59}$. This confirm confinement effect in ZnS nano-systems, which set in at $(\mathrm{ZnS})_{59}$. The $1.0-1.8 \mathrm{~nm} \mathrm{ZnS}$ quantum dots absorb radiation within the visible region of the electromagnetic spectrum with highest peaks at $1.99 \mathrm{eV}, 2.21 \mathrm{eV}, 1.92 \mathrm{eV}$ and 1.70 eV for $1.0,1.2,1.4$ and 1.8 nm dot size, respectively. However, the spectra highest peaks for $1.0 \mathrm{~nm}, 1.4 \mathrm{~nm}, 1.8 \mathrm{~nm} \mathrm{ZnS}$ quantum dots, which correspond to the reddish visible region of the electromagnetic spectrum, are already approaching infrared radiation. The 1.2 nm ZnS dot has its absorption band span 1.60 eV to 2.50 eV with two prominent peaks, one at 2.00 eV (orange colour) and the other at 2.21 eV (green colour). The 1.2 nm spherical-like quantum dot (optimised at PM7 level) is theoretically optimal for solar cell application whereas bulk ZnS , which has a wide band gap of about 3.68 eV in the ultraviolet region of the electromagnetic spectrum is not suitable for solar cell application. The optimal band gap for solar cell material is about 1.38 eV for $\approx 33 \%$ efficiency (Rühle, 2016).

The eight lowest excitation energies and absorption spectra obtained with the oeINDO model for $1.0,1.4,1.8$ and 2.2 nm CdS quantum dots are shown in Table 4.20 and Figure 4.65. Confinement effects set in at the size 1.4 nm CdS dot since from this dot size, a continuous decrease in electronic gap energy (red-shift) with increase in the dot size is observed (Figure 4.63). Also, a red-shift is observed in the absorption spectra as the dot size increases. The 1.0 and 1.4 nm CdS quantum dots were predicted to absorb light in the visible region of the electromagnetic spectrum, with highest peaks at 1.80 eV (red portion of the visible spectrum) and 2.06 eV (the orange portion of the visible spectrum), respectively. The highest peak of the spectrum for 1.4 nm CdS quantum dot is deeper in the visible region than that from 1.0 nm CdS dot. Thus, the 1.4 nm CdS dot, optimised at the PM7 level is potential for solar cell applications.

Table 4.21: Eight lowest vertical excitation energies (eV) for different CdZnS quantum dots

| $\mathrm{S} / \mathrm{N}$ | $\mathrm{Zn}_{95} \mathrm{~S}_{95}$ | $\mathrm{Cd}_{38} \mathrm{Zn}_{57} \mathrm{~S}_{95}$ | $\mathrm{Cd}_{57} \mathrm{Zn}_{38} \mathrm{~S}_{95}$ | $\mathrm{Cd}_{76} \mathrm{Zn}_{19} \mathrm{~S}_{95}$ | $\mathrm{Cd}_{95} \mathrm{~S}_{95}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.156 | 1.231 | 1.149 | 1.108 | 1.090 |
| 2 | 1.270 | 1.323 | 1.321 | 1.270 | 1.229 |
| 3 | 1.311 | 1.346 | 1.326 | 1.318 | 1.298 |
| 4 | 1.343 | 1.351 | 1.349 | 1.341 | 1.335 |
| 5 | 1.399 | 1.448 | 1.408 | 1.350 | 1.470 |
| 6 | 1.475 | 1.463 | 1.431 | 1.423 | 1.553 |
| 7 | 1.541 | 1.541 | 1.525 | 1.491 | 1.642 |
| 8 | 1.614 | 1.666 | 1.630 | 1.576 | 1.655 |



Figure 4.66: Absorption spectra of CdZnS quantum dots obtained from oeINDO.The intensities have been scaled so that the highest intensity is equal to unity.

Table 4.22: Computational time (in minutes) expended in carrying out excitation energies and absorption spectra for some atomic clusters using different methods

| Atomic clusters | EOM-CCSD | TDDFT | CIS(D) | oeINDO |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Si}_{3}$ | 84.53 | 12.8 | 48.27 | 0.07 |
| $S i_{4}$ | 159.07 | 15.33 | 14.67 | 0.01 |
| $S i_{5}$ | 510.13 | 20.13 | 53.87 | 0.01 |
| $S i_{19}$ | - | 1524.48 | 421.60 | 0.70 |
| $S i_{40}$ | - | 17505 | 13146.4 | 0.03 |
| $S i_{244}$ | - | - | - | 13.38 |
| Si ${ }_{779}$ | - | - | - | 768.73 |
| $S_{3}$ | 194.5 | 26.0 | 39.0 | 0.21 |
| $S_{5}$ | 2102.5 | 11.6 | 13.2 | 0.01 |
| $S_{3} 9$ | - | 2591.5 | 4327.5 | 0.17 |
| $S_{190}$ | - | - | - | 13.98 |
| $C d_{3}$ | 12053.3 | 37.83 | 36.33 | 0.33 |
| $C d_{20}$ | - | 1911.2 | 7071.7 | 0.58 |
| $C d_{60}$ | - | - | - | 2.37 |
| $Z n_{3}$ | 554.8 | 10.17 | 23.33 | 0.01 |
| $Z n_{4}$ | 7913.3 | 68.03 | 39.33 | 0.01 |
| $(Z n S)_{10}$ | - | 1778.67 | 2684.53 | 0.15 |
| $(\mathrm{ZnS})_{159}$ | - | - | - | 76.73 |
| $(C d S)_{19}$ | - | 23138.93 | - | 0.91 |
| $(C d S)_{95}$ | - | - | - | 13.33 |

The results of the excited state energies and UV-Vis absorption spectra of cadmium-zinc sulphide quantum dots obtained from oeINDO model are presented in Table 4.21 and Figure 4.61 respectively. The effect of cadmium-zinc mixing can clearly be seen in Figure 4.61. Starting from the $\mathrm{Zn}_{95} \mathrm{~S}_{95}$ quantum dot, a red-shift (decrease in energy) in spectrum highest peak and the first excitation energy (electronic gap) is observe as cadmium content (replacing the zinc contents) is increased. The trend observed agrees with previous reports (Wageh and Badr, 2008, Gaur and Jeevanandam, 2016, Sethi et al., 2007, Jabeen et al., 2016). The electronic gap decreased from $1.156 \mathrm{eV}\left(\mathrm{Zn}_{95} \mathrm{~S}_{95}\right)$ to 1.090 eV $\left(\mathrm{Cd}_{95} \mathrm{~S}_{95}\right)$ and so is with the absorption spectra highest peaks observed at 1.705 eV , $1.231 \mathrm{eV}, 1.149 \mathrm{eV}, 1.108 \mathrm{eV}$ and 1.090 eV for $\mathrm{Zn}_{95} \mathrm{~S}_{95}, \mathrm{Cd}_{38} \mathrm{Zn}_{57} \mathrm{~S}_{95}, \mathrm{Cd}_{57} \mathrm{Zn}_{38} \mathrm{~S}_{95}$, $\mathrm{Cd}_{76} \mathrm{Zn}_{19} \mathrm{~S}_{95}$ and $\mathrm{Cd}_{95} \mathrm{~S}_{95}$, respectively.

### 4.4 Computation Time Expended for Excitation Energies and Absorption Spectra Calculations Using Different Methods

The computation times expended in the calculation of excited state energies and absorption spectra for some atomic clusters using the oeINDO and ab-initio methods are presented in Table 4.22. EOM-CCSD was found to be compute-intensive and prohibitive for large clusters. It was practically limited to clusters with less than 10 atoms. The B3LYP/TDDFT and CIS(D) methods are less expensive for small-sized clusters but become compute-intensive as clusters grew bigger since the needed computational resources/time grew as $\mathrm{N}^{3}-\mathrm{N}^{4}$, where N is the number of electrons in the system. The oeINDO method, however, is computationally cheap. It uses only a small fraction of the time expended in the computations with EOM-CCSD, TDDFT and CIS(D) (see Table 4.22). For instance, with 1 central processing unit (CPU) of the computer, calculations of excited state energies and absorption spectra for $\mathrm{Cd}_{3}$ were achieved in over 200 hrs (over 8 days) using EOM-CCSD, 39 minutes with TDDFT, 36 minutes with CIS(D) and 0.33 minutes with oeINDO. The oeINDO computation time is less than a thousandth and hundredth of the computation time of EOM-CCSD and TDDFT/CIS(D), respectively. An important milestone achieved with the oeINDO is the study of bigger systems with reduced computational resources and time. For instance, with $1 \mathrm{CPU}, \approx 12 \mathrm{hrs}$ was expended for the computation of excitation energies and absorption spectra of $\mathrm{Si}_{779}$
cluster 4.22 using oeINDO. Even with the available huge computational power, a cluster of this size is prohibitively expensive with EOM-CCSD, TDDFT and CIS(D).

## Chapter 5

## CONCLUSION AND RECOMMENDATIONS

### 5.1 Summary and Conclusion

Despite the availability of powerful computing resources, high-level ab-initio methods are still computational prohibitive for the calculation of excitation energies and absorption spectra of quantum dots (containing over 1000 electrons). In this work, a new semi-empirical Hamiltonian that is computationally cheap and capable of giving accurate vertical excited state energies and ultra-violet absorption spectra for large atomic clusters and quantum dots was used. The semi-empirical Hamiltonian was parameterised using high-level first principle data from homogeneous dimer geometries. The data were obtained from Equation-Of-Motion Coupled-Cluster with Single and Double (EOM-CCSD) excitations, a method generally regarded as the 'gold standard' in computational chemistry that gives accurate results, which are comparable to experiments. The Zerner Intermediate Neglect of Differential Overlap for spectroscopy(ZINDO/s) or INDO/s Hamiltonian was parameterised using benchmark excitation energies from EOM-CCSD for homogeneous diatomics at different separations.

In this method, benchmark excitation energies for diatomics were calculated using highlevel ab-initio. The INDO/s Hamiltonian was parameterised to fit the benchmark excitation energies. In addition, the one-center one electron integral parameters, which were not initially directly optimised, were shifted to reproduce first ionization potential energies. The optimised INDO/s parameters were then validated with homogeneous and inhomogeneous atomic clusters not included in the training geometries. Thereafter, the parameters were used to predict excitation energies and absorption spectra for large
atomic clusters and quantum dots.

The parameterisations were performed for silicon, sulphur, cadmium and zinc diatomics and a typical error of 0.23 eV was obtained for the fits. The model obtained from the parameterisations was called optimised for excitations Intermediate Neglect of Differential Overlap (oeINDO).

Without a need for re-parametrization, it was demonstrated that oeINDO is transferable; that is, the optimised parameters obtained give accurate results for complex geometries that are not included in the training geometry sets. Although the Hamiltonian was not directly optimised for the $\mathrm{Zn}-\mathrm{S}$ and $\mathrm{Cd}-\mathrm{S}$ heterogeneous diatomics, good results were also obtained for ZnS and CdS clusters The oeINDO excitation energies were found to be accurate with typical error of about 0.30 eV relative to the benchmark (EOMCCSD) for complex cluster structures. However, the original ZINDO/s has a typical error of about 1.11 eV for the same structures. Even though the absorption spectra were not included in the training data sets, the relative intensities obtained with oeINDO compared well qualitatively with those from accurate ab-initio methods ( EOM-CCSD, CIS(D) and TDDFT). The oeINDO UV-VIS absorption spectra highest peaks are shifted from those of TDDFT with on MAE of 0.40 eV while the original ZINDO/s spectra highest peaks are shifted with an MAE of 1.49 eV which is $3-4$ times larger.

After the validation of oeINDO, it was employed for calculations and predictions of excitation energies and UV-VIS absorption spectra of silicon, sulphur, zinc, cadmium, zinc sulphide and cadmium sulphide quantum dots of different sizes ranging from 1.0 to 3.0 nm (40-779 atom systems). It predicted quantum confinement effects in the quantum dots; a red-shift (decrease in energy) in the electronic gap and UV-VIS absorption spectra of the quantum dots as their sizes increase and a blue-shift (increase in energy) as the sizes decrease. The oeINDO Hamiltonian also predicted silicon, zinc and cadmium quantum dots as metallic in agreement with earlier literature reports. In addition, the electronic energy gap of 0.24 and 0.401 eV were obtained for $\mathrm{Si}_{147}$ and a 1.2 nm Cd cluster, respectively are consistent with reports in literature. The transition of the zinc and cadmium clusters to bulk-like (metallic) behaviour was observed at a cluster size of about $N=20$ atoms.

The oeINDO was employed in the prediction of optimal size and shape of the quantum dot for solar cell application. It predicted that the 1.2 nm spherical-like ZnS quantum dot, which has its highest absorption peak at 2.21 eV (corresponding to the green portion of the visible region of electromagnetic spectrum), is theoretically optimal for solar cell application. In contrast to bulk ZnS , which has a wide band gap of about 3.68 eV (corresponding to the ultra-violet portion of electromagnetic spectrum), the 1.2 nm sphericallike ZnS quantum dot has its gap to be 1.909 eV (visible region). The 1.4 nm sphericallike CdS quantum dot was predicted theoretically optimal for solar cell application. It has a maximum absorption at 2.06 eV (the orange portion of the visible region) and an electronic gap of 1.773 eV .

Although, both bulk CdS (with band gap 2.42 eV ) and quantum dots are suitable for solar cells application, they are not environmentally friendly. The environmentally friendly ZnS quantum dot can serve as a good replacement for CdS for solar cell applications.

A red-shift was found in the electronic gap and absorption peak of ZnS quantum dot as cadmium is increasingly substituted for its zinc contents. This agrees with the trend observed in earlier reports.

It is important to mention the computation times for the different methods in order to appreciate the significance of the newly developed approach. The oeINDO achieved accuracy close to EOM-CCSD using less than a hundredth of its computational time and resources. Furthermore, oeINDO obtained results reasonably close to TDDFT and sometimes better, with time and resources less than a hundredth of those of TDDFT.

### 5.2 Contribution to Knowledge

A new method of obtaining accurate semi-empirical Hamiltonian model parameters from ab-initio diatomic data was developed. In particular, the method was employed to obtain a model called optimised for excitation Intermediate Neglect of Differential Overlap (oeINDO), which yielded vertical excitation energies and UV-VIS absorption spectra that are comparable to those from high-level ab-initio data at a fraction of the computation cost. The new model, oeINDO, is a significant improvement over the well known

INDO/s (or ZINDO/s) model.

### 5.3 Recommendations

The oeINDO model, like current semi-empirical models based on the Hartree Fock formalism, neglects the three- and four-center integrals. Future work can be to incorporate the three-center integral implicitly into the oeINDO model in order to improve its accuracy. The four-center effect is less significant and so it can be neglected. In addition, to improve the accuracy of oeINDO model for heterogeneous systems, the combination rule for $\beta$ may need to be multiplied by 'interaction factors' and further work can determine the values of these factors for heterogeneous diatomics. Also, a new strategy need to be developed for shifting the one center integrals in order to accurately reproduce ionization potentials.

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## Appendix A

## Materials Used in the Research Work

The materials employd for the research are broadly divided into two, namely hardware and software.

## A. 1 Hardware

Calculations for the determination of equilibrium structure, ground-state properties, excitation energies, and absorption spectra of large atomic clusters and nano-sized materials were carried out using both high-level and semi-empirical methods. The former calculations are compute-intensive and require large computation resources and time. In order to achieve these, High-Performance Computing (HPC) facilities at the Center for High-Performance Computing (CHPC) in South Africa and HPC at the International Centre for Theoretical Physics Trieste, Italy were employed.

## A. 2 Software algorithms and Packages

The software packages used for this research were ORCA 4.0, Amoeba Optimizer, MOPAC 7, Avogadro, QuantumATK, Gnuplot 4.6 and Gabedit 2.5.0.

## A.2.1 ORCA 4.0

ORCA is a self-consistency field molecular orbital package developed by Frank Neese for the execution of first principle and semi-empirical molecular orbital theory approaches (Neese, 2012). In particular, ORCA 4.0 was employed in this research to carry out geometry optimisations and calculate ground state and excitation energies, and UV-VIS absorption spectra both using ab-initio and semi-empirical methods. In addition, the calculations with new accurate semi-empirical method developed in this work were carried out using the ORCA 4.0 platform.

## A.2.2 Amoeba Optimiser Algorithm

The Amoeba optimiser is a method based on the simplex algorithm used for finding the minimum point of a multidimensional function. It was proposed by Mead and Nelder in 1965 (Nelder and Mead, 1965). The method involves moving and continuous transformation (shrinking, reflecting and stretching) of a simplex (e.g triangle or polygon) around an optimal point until a tolerance error is reached. Each vertex of the simplex corresponds to a set of parameters of the function to be minimized. The detailed algorithm and implementation of this method have been reported in the reference: (Press et al., 2007). Although the simplex method is slow in speed, it is reliable and has been widely employed for optimisation of models parameters (Singer and Nelder, 2009).

In this work, the Amoeba optimizer algorithm was used for the optimisation of semiempirical Hamiltonian parameters. The required libraries for the code were taken from Numerical Recipes (Press et al., 2007) in the C++ computer language and a C++ code and a bash linux scripts adopted as drivers to drive the libraries.

## A.2.3 MOPAC7

MOPAC is an acronym for Molecular Orbital Package. It was developed by James Stewart (Stewart, 1990) to execute semi-empirical methods based on NDDO including the PM3, PM6, and PM7.In this research, the PM7 in the MOPAC7 was employed to perform structure optimisation for large atomic clusters and quantum dots for which $a b$ initio methods are prohibitively expensive. The ZINDO semi-empirical Hamiltonian is
not coded in MOPAC. Thus, we could not use MOPAC for calculations of the absorption spectra and excitation energies which were based on the ZINDO/S Hamiltonian.

## A.2.4 QuantumATK

QuantumATK is a powerful tool for modelling different kinds of materials including nanomaterials. It employs a variety of methods like DFT, tight-binding, classical force fields, etc. for its modelling and simulations. It is also composed of a builder used for building varieties of atomic systems like bulk, one-dimensional, two-dimensional and zero-dimensional materials (Stradi et al., 2017). In this work, the Wulff constructor in the quantumATK builder was employed to build large atomic clusters and nano-sized materials (quantum dots).

## A.2.5 Gnuplot 4.6

Gnuplot (Racine, 2006) is a command-line visualization tool used to carry out different plots including scatter plots of the excitation energies and plots of the absorption spectra. In particular, version 4.6 was utilized in this research.

## A.2.6 Gabedit 2.5.0

Gabedit is a computational package, which interfaces with some molecular orbital packages like MOPAC, Gaussian, ORCA etc. It has different features, which include the display of spectra and geometries from the output files of different packages (Allouche, 2008, 2011). In this work, gabedit 2.5.0 was employed to visualize UV-Vis absorption spectra from ORCA excitation and spectra calculations output files.

## Appendix B

## Optimized atomic structures

## B. 1 Zinc clusters



Figure B.1: Equilibrium structures of $\mathrm{Zn}_{n}(n=3,4,6,8,16,24)$. The structures were obtained from geometry optimizations using B3LYP/DEF2-TZVPP for small clusters and B3LYP/DEF2-SVP for moderate-sized clusters.

## B.1.1 Cartesian coordinates of equilibrium zinc clusters in Angstrom

| $\mathrm{Zn}_{3}$ |  |  |  |
| :---: | :---: | :---: | :---: |
|  | x | y | z |
| Zn | 0.977980 | $-0.563480$ | 3.710250 |
| Zn | 2.665000 | -3.488950 | 3.710250 |
| Zn | 4.352020 | -0.563480 | 3.710250 |
| $\mathrm{Zn}_{4}$ |  |  |  |
|  | x | y | z |
| Zn | 1.048700 | -0.769410 | 3.712820 |
| Zn | 2.665010 | 2.399990 | 3.707680 |
| Zn | 2.665010 | -3.938440 | 3.707680 |
| Zn | 4.281000 | -0.769410 | 3.712820 |
| $\mathrm{Zn}_{6}$ |  |  |  |
|  | x | y | z |
| Zn | -1.361706 | 0.353221 | -0.155692 |
| Zn | -0.757604 | 3.442835 | -0.822890 |
| Zn | 0.513636 | 1.602957 | 1.567345 |
| Zn | 1.271224 | 0.878638 | -1.083022 |
| Zn | 0.909574 | -1.502764 | 0.994363 |
| Zn | -4.287594 | -0.693600 | -0.500105 |
| $\mathrm{Zn}_{8}$ |  |  |  |
|  | x | y | z |
| Zn | -3.892005 | 1.053449 | $-0.000563$ |
| Zn | -1.320401 | 2.811341 | 0.001949 |
| Zn | 1.825857 | 3.181664 | -0.001794 |
| Zn | 2.834073 | -1.463988 | 0.001509 |
| Zn | 0.258744 | 0.368232 | 0.002342 |
| Zn | -3.240845 | -2.071732 | -0.001667 |
| Zn | -0.160721 | -2.519134 | -0.000238 |
| Zn | 4.400197 | 1.305980 | -0.001539 |
| $\mathrm{Zn}_{16}$ |  |  |  |
|  | x | y | Z |
| Zn | 1.309724 | -0.272694 | 1.697209 |
| Zn | 0.592158 | 1.145879 | 5.607923 |
| Zn | 2.665000 | 6.283164 | 3.622037 |
| Zn | 2.665000 | 3.583849 | 4.892289 |
| Zn | 2.665002 | -2.572512 | 2.466663 |
| Zn | 2.665002 | -1.138558 | 6.077919 |
| Zn | 4.020277 | -0.272692 | 1.697210 |
| Zn | 4.737841 | 1.145881 | 5.607919 |
| Zn | 0.592227 | -1.145853 | 4.286125 |
| Zn | 1.309718 | 0.272708 | 8.196827 |
| Zn | 2.664998 | 1.138534 | 3.816186 |
| Zn | 2.664998 | 2.572488 | 7.427307 |
| Zn | 2.664999 | -3.583809 | 5.001698 |
| Zn | 2.664994 | -6.283241 | 6.271740 |
| Zn | 4.737776 | -1.145854 | 4.286124 |
| Zn | 4.020286 | 0.272711 | 8.196824 |


|  | $\mathrm{Zn}_{24}$ |  |  |
| :--- | :---: | :---: | :---: |
|  | x | y | z |
| Zn | 1.597912 | 0.940148 | 2.700771 |
| Zn | 0.618412 | 0.177612 | 5.203401 |
| Zn | 1.436436 | 0.677550 | 10.220961 |
| Zn | 1.162049 | 3.565947 | 3.193973 |
| Zn | 1.325607 | 2.927668 | 5.846840 |
| Zn | 2.425704 | 3.193143 | 10.650837 |
| Zn | 3.983035 | -0.384343 | 2.923070 |
| Zn | 1.444607 | -2.129759 | 6.552442 |
| Zn | 1.110823 | -3.164985 | 10.451074 |
| Zn | 5.789436 | 0.143101 | 4.956837 |
| Zn | 5.046609 | 1.050892 | 7.567449 |
| Zn | 4.885845 | 2.604712 | 9.703134 |
| Zn | 1.526950 | -1.663384 | 3.292700 |
| Zn | 0.402669 | 0.395812 | 7.818179 |
| Zn | 3.145742 | -1.408135 | 10.976535 |
| Zn | 3.251933 | 0.969379 | 5.419575 |
| Zn | 2.536852 | 2.181045 | 8.137949 |
| Zn | 3.721663 | 1.108407 | 11.748347 |
| Zn | 2.586800 | -3.833030 | 4.537654 |
| Zn | -0.512227 | -1.989407 | 8.605535 |
| Zn | 4.059498 | -2.727034 | 13.484239 |
| Zn | 4.002320 | -1.801284 | 5.797204 |
| Zn | 3.074436 | -0.670401 | 8.372065 |
| Zn | 5.336890 | -0.163653 | 9.931228 |

## B. 2 Cadmium Clusters


$\mathrm{Cd}_{3}$
$\mathrm{Cd}_{4}$

$\mathbf{C d}_{6}$

$\mathrm{Cd}_{8}$

Figure B.2: Equilibrium structures of $\mathrm{Cd}_{n}(n=3,4,6,8,16)$. The structures were obtained from geometry optimization using B3LYP/DEF2-TZVPP for small clusters and B3LYP/DEF2-SVP for moderate-sized clusters.

## B.2.1 Cartesian coordinates of equilibrium cadmium clusters in Angstrom

|  | $\mathrm{Cd}_{3}$ |  |  |  | $\mathrm{Cd}_{16}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | x | y | z |  | x | y | Z |
| Cd | 0.912857 | -0.522724 | 3.710250 | Cd | 1.091584 | -0.338999 | 1.224444 |
| Cd | 2.665000 | -3.570468 | 3.710250 | Cd | 0.252570 | 1.358260 | 5.708279 |
| Cd | 4.417143 | $-0.522724$ | 3.710250 | Cd | 2.665005 | 7.055048 | 3.598561 |
|  |  |  |  | Cd | 2.665003 | 4.001191 | 4.802245 |
|  | $\mathrm{Cd}_{4}$ |  |  | Cd | 2.665006 | $-3.002380$ | 2.126086 |
|  | x | y | z | Cd | 2.665002 | -1.170568 | 6.224168 |
| Cd | 0.966913 | -0.769323 | 3.710207 | Cd | 4.238412 | -0.338993 | 1.224444 |
| Cd | 2.665005 | 2.468298 | 3.710293 | Cd | 5.077426 | 1.358264 | 5.708275 |
| Cd | 2.665005 | -4.006928 | 3.710292 | Cd | 0.252618 | -1.358352 | 4.185746 |
| Cd | 4.363078 | $-0.769323$ | 3.710208 | Cd | 1.091590 | 0.339060 | 8.669612 |
|  |  |  |  | Cd | 2.664996 | 1.170544 | 3.669924 |
|  | $\mathrm{Cd}_{6}$ |  |  | Cd | 2.664995 | 3.002417 | 7.767868 |
|  |  | $\mathrm{Cd}_{6}$ |  | Cd | 2.664998 | -4.001159 | 5.091708 |
| Cd | -1.479310 | $\underset{0.280929}{\mathrm{y}}$ | Z -0.167017 | Cd | 2.664996 | -7.055049 | 6.295286 |
| Cd | -1.4796582 | 3.649948 | -0.1690809 | Cd | 5.077385 | -1.358350 | 4.185744 |
| Cd | 0.635055 | 1.744607 | 1.772220 | Cd | 4.238414 | 0.339065 | 8.669610 |
| Cd | 1.495986 | 0.917573 | -1.243600 |  |  |  |  |
| Cd | 1.081351 | -1.727707 | 1.095999 |  |  |  |  |
| Cd | -4.658969 | -0.784062 | $-0.566794$ |  |  |  |  |

## $\mathrm{Cd}_{8}$

|  | x | y | z |
| :--- | :---: | :---: | :---: |
| Cd | 0.274865 | 1.740542 | -0.994800 |
| Cd | -2.020659 | 2.966742 | 0.912465 |
| Cd | 1.180767 | 3.162451 | 1.752450 |
| Cd | 1.972044 | -1.271420 | -0.527539 |
| Cd | -0.452511 | 0.368938 | 2.084672 |
| Cd | -1.116844 | -1.466207 | -1.321866 |
| Cd | -0.148486 | -2.806631 | 1.641417 |
| Cd | 1.015725 | -0.028603 | -3.546800 |

## B. 3 Zinc Quantum Dots


1.0 nm Zn quantum dot

1.8 nm Zn quantum dot

1.2 nm Zn quantum dot

1.6 nm Zn quantum dot

2.0 nm Zn quantum dot

Figure B.3: Equilibrium structures of Zn quantum dots of sizes 1.0, 1.2, 1.4, 1.6, 1.8 and 2.0 nm . The structures were obtained from geometry optimization using PM7 as implemented in MOPAC.

## B.3.1 Cartesian coordinates of equilibrium zinc quantum dots in Angstrom

|  | $\mathrm{Zn}_{62}(1.2 \mathrm{~nm})$ |  |  | $\mathrm{Zn}_{104}(1.4 \mathrm{~nm})$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| Zn | 2.629584 | -0.870628 | -9.826427 | Zn | 2.629584 | -0.870628 | -9.82642 |
| Zn | -0.041493 | -4.843919 | -7.080275 | Zn | -0.041493 | -4.843919 | -7.08 |
| Zn | 2.953517 | -3.549539 | -7.676692 | Zn | 2.953517 | -3.549539 | -7.67 |
| Zn | 4.077043 | -2.401456 | -4.702567 | Zn | 4.077043 | -2.401456 | -4.702 |
| Zn | -1.318283 | -7.952943 | -3.2628 | Zn | -1.318283 | -7.9529 | -3.2 |
| Zn | 1.324073 | -6.516969 | -4.569175 | Zn | 1.324073 | -6.516969 | -4. |
| Zn | 4.245731 | -6.477817 | $-2.565587$ | Zn | 4.24573 | -6.477817 |  |
| Zn | 6.495514 | $-4.487786$ | -3.8949 | Zn | 6.49551 | -4.487786 | -3.8 |
| Zn | 6.319579 | -2.851299 | -1.0498 | Zn | 6.319579 | -2.851 | -1.0 |
| Zn | -2.548582 | 0.147201 | -8.67543 | Zn | -2.54858 | 0.1472 | -8.6 |
| Zn | -4.331853 | -2.242587 | -7.23622 | Zn | -4.33185 | -2.24258 | -7.23 |
| Zn | -1.886756 | -2.858980 | -5.169355 | Zn | -1.88675 | -2.85898 | -5.16 |
| Zn | 0.229947 | -0.999325 | -7.413047 | Zn | 0.229947 | -0.99932 | -7.41304 |
| Zn | 0.687804 | 2.277681 | -7.013707 | Zn | 0.687804 | 2.277681 | -7.013 |
| Zn | 3.114602 | 2.370099 | -9.193018 | Zn | 3.114602 | 2.370099 | -9.193018 |
| Zn | 5.405170 | 2.618568 | -6.765772 | Zn | 5.405170 | 2.618568 | -6.765772 |
| Zn | -6.038533 | -4.694297 | -5.747148 | Zn | -6.038533 | -4.694297 | -5.747148 |
| Zn | -3.542814 | -5.344626 | -3.701779 | Zn | -3.542814 | -5.344626 | -3.701779 |
| Zn | 0.151477 | -4.060252 | -2.679671 | Zn | 0.151477 | -4.060252 | -2.679671 |
| Zn | 2.458891 | -0.174461 | -1.024263 | Zn | 2.458891 | -0.174461 | -1.024263 |
| Zn | 1.089103 | $-0.801923$ | -4.123150 | Zn | 1.089103 | -0.801923 | -4. |
| Zn | 2.985121 | 1.777073 | -4.621217 | Zn | 2.985121 | 1.777073 | -4.6 |
| Zn | 7.608959 | 1.994609 | -4.1679 | Zn | 7.60895 | 1.99460 | -4.1 |
| Zn | 5.196799 | 3.868441 | -2.680507 | Zn | 5.19679 | 3.86844 | -2.6 |
| Zn | -0.763572 | -7.931061 | 2.472 | Zn | -0.763572 | -7.9310 | 2.47288 |
| Zn | -0.822723 | -6.281532 | -0.38635 | Zn | -0.82272 | -6.28153 | -0.3 |
| Zn | 2.364818 | -5.675362 | 0.835820 | Zn | 2.364818 | -5.67536 | 0.83 |
| Zn | 2.976062 | -3.507547 | -1.683677 | Zn | 2.976062 | -3.507547 | -1.683677 |
| Zn | 4.235945 | -1.551206 | 1.584581 | Zn | 4.235945 | -1.551206 | 1.5845 |
| Zn | 5.323464 | -0.026117 | -2.623528 | Zn | 5.323464 | -0.026117 | -2.623528 |
| Zn | 6.286153 | 1.038355 | 0.484140 | Zn | 6.286153 | 1.038355 | 0.484140 |
| Zn | 8.392288 | 3.100358 | -0.931117 | Zn | 8.392288 | 3.100358 | -0.931117 |
| Zn | 2.445283 | -7.447333 | 3.587461 | Zn | 2.445283 | -7.447333 | 3.587461 |
| Zn | 5.445488 | -5.995948 | 4.157065 | Zn | 5.445488 | -5.995948 | 4.157065 |
| Zn | 5.719707 | -5.696375 | 0.662145 | Zn | 5.719707 | -5.696375 | 0.662145 |
| Zn | 6.827986 | -3.021863 | 3.060954 | Zn | 6.827986 | -3.021863 |  |
| Zn | 8.581821 | -1.257401 | 0.866518 | Zn | 8.581821 | -1.257401 | 0.86 |
| Zn | -5.525517 | $-0.041738$ | -5.113705 | Zn | -5.525517 | -0.041738 | -5.11 |
| Zn | -3.502602 | 2.311383 | -6.369773 | Zn | -3.502602 | 2.311383 | -6.36 |
| Zn | -1.207687 | 5.116472 | -6.920310 | Zn | -1.207687 | 5.11647 | -6.92 |
| Zn | -7.336220 | -2.422662 | -3.714347 | Zn | -7.336220 | -2.422662 | -3.71434 |
| Zn | -5.024147 | -3.164109 | -1.530306 | Zn | -5.024147 | -3.16410 | -1. |
| Zn | -3.172272 | $-0.764296$ | -2.822159 | Zn | -3.172272 | -0.7642 | -2.8 |
| Zn | -0.936169 | 1.963139 | $-0.479650$ | Zn | -0.936169 | 1.96313 | -0.4 |
| Zn | -1.254518 | 1.572192 | -4.098591 | Zn | -1.254518 | 1.57219 | -4.09859 |
| Zn | 1.174191 | 5.032685 | $-4.360632$ | Zn | 1.174191 | 5.032685 | $-4.360632$ |
| Zn | 4.380797 | 5.557889 | -5.435772 | Zn | 4.380797 | 5.55788 | -5.435772 |
| Zn | 4.66147 | 7.520684 | -2.522154 | Zn | 4.661474 | 7.520684 | $-2.522154$ |
| Zn | -4.22586 | -6.965849 | -0.525015 | Zn | -4.225863 | -6.965849 | -0.525015 |
| Zn | -2.546457 | -3.650948 | 0.848222 | Zn | -2.546457 | -3.650948 | 0.848222 |
| Zn | -0.673964 | -0.454392 | 2.370050 | Zn | -0.673964 | -0.454392 | 2.370050 |
| Zn | -0.824723 | -1.487355 | -0.903927 | Zn | -0.824723 | -1.487355 | -0.903927 |
| Zn | 1.876834 | 1.447007 | 2.059672 | Zn | 1.876834 | 1.447007 | 2.059672 |
| Zn | 2.067254 | 3.332314 | -1.423756 | Zn | 2.067254 | 3.332314 | -1.423756 |
| Zn | 4.112118 | 3.821440 | 1.173577 | Zn | 4.112118 | 3.821440 | 1.173577 |
| Zn | 6.622065 | 5.810179 | $-0.014783$ | Zn | 6.622065 | 5.810179 | $-0.014783$ |
| Zn | -3.954341 | -8.635440 | 2.408646 | Zn | -3.954341 | -8.635440 | 2.408 |
| Zn | -4.399299 | $-6.002198$ | 4.449300 | Zn | -4.399299 | $-6.002198$ | 4.44930 |
| Zn | -1.407660 | -4.887389 | 3.766720 | Zn | -1.407660 | -4.887389 | 3.76672 |
| Zn | 1.485561 | -1.948906 | 5.781042 | Zn | 1.485561 | -1.948906 | 5.78104 |
| Zn | 1.299412 | -3.028644 | 2.608464 | Zn | 1.299412 | -3.028644 | 2.60846 |
| Zn | 3.750501 | 0.273109 | 4.547099 | Zn | 3.750501 | 0.273109 | 4.54709 |
| Zn | 6.134119 | 2.520473 | 3.683212 | Zn | 6.134119 | 2.520473 | 3.683212 |
| Zn | 7.571209 | 5.557010 | 3.142581 | Zn | 7.571209 | 5.557010 | 3.14258 |
| Zn | 1.065237 | $-5.663402$ | 5.929042 | Zn | 1.065237 | -5.663402 | 5.929042 |
| Zn | 4.148344 | -4.050339 | 6.585060 | Zn | 4.148344 | $-4.050339$ | 6.585060 |
| Zn | 6.150307 | -1.554457 | 5.867838 | Zn | 6.150307 | $-1.554457$ | 5.867838 |
| Zn | -8.709485 | -0.043444 | -1.759298 | Zn | -8.709485 | -0.043444 | -1.759298 |
| Zn | -6.660627 | 2.165991 | -2.887231 | Zn | -6.660627 | 2.165991 | -2.887231 |
| Zn | -6.399591 | 5.472157 | -2.537085 | Zn | -6.399591 | 5.472157 | -2.537085 |
| Zn | -4.028738 | 5.466798 | -4.864067 | Zn | -4.028738 | 5.466798 | $-4.8640$ |


| Zn | -2.079011 | 8.062799 | -3.168412 |
| :--- | ---: | ---: | ---: |
| Zn | -8.372367 | -2.796864 | 2.411443 |
| Zn | -6.467873 | -0.988077 | 0.441015 |
| Zn | -4.084526 | 1.837633 | 2.919965 |
| Zn | -4.372768 | 1.598246 | -0.627903 |
| Zn | -4.130340 | 4.943030 | -0.254324 |
| Zn | -1.693245 | 4.623259 | -2.600219 |
| Zn | 0.256614 | 6.018195 | -0.054248 |
| Zn | 1.262738 | 7.773767 | -2.680050 |
| Zn | -5.738232 | -4.798030 | 1.720488 |
| Zn | -6.632513 | -1.560456 | 4.998608 |
| Zn | -4.188169 | -1.356042 | 2.835282 |
| Zn | -1.616453 | 1.389403 | 5.244716 |
| Zn | -1.626662 | 4.012231 | 1.820710 |
| Zn | 0.602239 | 3.649491 | 4.265984 |
| Zn | 2.276181 | 6.003239 | 2.678354 |
| Zn | 4.903623 | 6.867958 | 4.635921 |
| Zn | -4.699707 | -3.515642 | 6.755307 |
| Zn | -1.776841 | -2.085347 | 8.777748 |
| Zn | -2.001364 | -2.146219 | 5.348214 |
| Zn | 0.180751 | 0.506147 | 7.772269 |
| Zn | 2.635787 | 2.441744 | 6.608327 |
| Zn | 5.437175 | 4.181976 | 6.516112 |
| Zn | 1.151041 | -3.634840 | 8.835851 |
| Zn | -7.451233 | 2.314489 | 1.186745 |
| Zn | -8.220336 | 3.323174 | 4.318625 |
| Zn | -5.588795 | 5.043727 | 2.723815 |
| Zn | -2.863179 | 6.782299 | 3.407057 |
| Zn | -2.485426 | 7.853784 | 0.180747 |
| Zn | -5.849572 | 1.655543 | 5.704191 |
| Zn | -3.266672 | 3.584271 | 7.024579 |
| Zn | -1.070903 | 5.824876 | 5.983720 |
| Zn | -1.687459 | 2.514850 | 9.661516 |
|  |  |  |  |


| $\mathrm{Zn}_{128}(1.6 \mathrm{~nm})$ |  |  |  |
| :---: | :---: | :---: | :---: |
|  | X | y | Z |
| Zn | 2.856009 | -1.992959 | -8.349380 |
| Zn | 1.905763 | 1.642131 | -8.138038 |
| Zn | 0.906421 | -5.705138 | -6.217056 |
| Zn | 3.982520 | -4.941109 | -7.322336 |
| Zn | 4.749978 | -3.234884 | -4.570032 |
| Zn | 4.695799 | -0.133343 | -6.213961 |
| Zn | 7.178954 | 2.387223 | -5.518305 |
| Zn | 1.140042 | -9.076122 | -1.156339 |
| Zn | 1.843662 | -6.558966 | -3.146017 |
| Zn | 3.925382 | -4.530832 | -1.606867 |
| Zn | 7.363711 | -5.157768 | -3.596019 |
| Zn | 6.849721 | -0.720666 | -3.220457 |
| Zn | 9.727878 | 1.028397 | -3.716635 |
| Zn | 3.722814 | -8.523210 | 1.107989 |
| Zn | 5.867125 | -7.277791 | -1.344224 |
| Zn | 7.510774 | -5.751012 | 0.982132 |
| Zn | 7.480649 | -3.049371 | -0.958382 |
| Zn | -2.740857 | 1.573132 | -9.222411 |
| Zn | -0.579352 | 3.656866 | -7.389828 |
| Zn | -3.996499 | -1.211777 | -7.060843 |
| Zn | -1.893405 | -2.569683 | -4.583902 |
| Zn | -0.544389 | -0.697570 | -7.924898 |
| Zn | -2.107406 | 1.386683 | -5.287230 |
| Zn | 1.475976 | 0.767468 | -4.758904 |
| Zn | 3.842599 | 3.443554 | -5.707785 |
| Zn | -2.635778 | -5.600122 | -5.839856 |
| Zn | -3.893565 | -6.258308 | -2.763888 |
| Zn | -0.929524 | -4.938827 | -2.133474 |
| Zn | 1.381109 | -2.520428 | -1.872164 |
| Zn | 1.245921 | -2.505587 | -5.304194 |
| Zn | 3.512561 | -0.131811 | -2.344006 |
| Zn | 4.283223 | 3.252763 | -2.412673 |
| Zn | 7.091822 | 4.845423 | -3.098855 |
| Zn | -2.069545 | -10.926811 | 1.458931 |
| Zn | -2.435657 | -9.173927 | -1.334856 |
| Zn | -1.088007 | -6.723136 | 0.639534 |
| Zn | 2.221159 | -5.598536 | 1.061143 |
| Zn | 4.687649 | -1.802937 | 0.551865 |
| Zn | 6.445528 | 1.168475 | -0.424028 |
| Zn | 9.362690 | 2.604641 | 2.290501 |
| Zn | 9.060542 | 3.000650 | -1.186035 |
| Zn | 0.499271 | -9.206899 | 2.480069 |
| Zn | 1.239754 | -5.998770 | 4.123335 |
| Zn | 4.321627 | -7.332144 | 4.339509 |
| Zn | 4.978924 | -4.462070 | 2.674019 |
| Zn | 6.799342 | -1.449773 | 3.525252 |
| Zn | 8.755600 | -0.595108 | 1.105605 |
| Zn | -6.763967 | 0.683683 | -6.293402 |
| Zn | -4.618559 | 3.136661 | -6.814827 |
| Zn | -2.916430 | 6.016553 | -5.690472 |
| Zn | -0.825949 | 6.969525 | -8.307389 |
| Zn | 2.104362 | 7.590076 | -6.690999 |
| Zn | -5.573069 | -3.905846 | -5.924747 |
| Zn | -4.908972 | -2.926207 | -2.867809 |
| Zn | -4.927483 | 0.343914 | -3.511158 |
| Zn | -3.324704 | 1.482354 | -0.776600 |
| Zn | -2.275165 | 3.712596 | -3.118035 |
| Zn | 0.765273 | 3.033391 | -2.192276 |
| Zn | 1.102911 | 5.006987 | -4.903099 |
| Zn | 5.120315 | 6.481582 | -5.353943 |
| Zn | -5.602708 | -8.499213 | -0.199794 |
| Zn | -4.391396 | -5.253559 | 0.524597 |
| Zn | -2.503917 | -2.680025 | -0.295374 |
| Zn | -0.960492 | -0.307795 | -2.346768 |
| Zn | -0.842336 | 0.098450 | 1.415103 |
| Zn | 2.079991 | 1.351637 | 0.416495 |
| Zn | 3.682288 | 4.482958 | 0.653467 |
| Zn | 6.130297 | 6.695719 | -0.491023 |


| Zn | -3.693356 | -8.219723 | 2.717618 |
| :---: | :---: | :---: | :---: |
| n | -2.983712 | -5.821883 | 4.947953 |
| Zn | -1.312330 | -4.098966 | 2.815491 |
| Zn | 2.639202 | -2.953450 | 4.589780 |
| Zn | 1.335754 | -2.426059 | 1.407370 |
| Zn | 3.608460 | 0.002442 | 3.447064 |
| Zn | 5.534660 | 2.606642 | 2.414694 |
| Zn | 7.552107 | 1.502671 | 5.065466 |
| Zn | -0.886769 | -8.348052 | 5.558859 |
| Zn | 0.178761 | -6.814079 | 8.266627 |
| Zn | 3.129524 | -5.622397 | 7.250743 |
| Zn | 3.771113 | -2.129554 | 7.637401 |
| Zn | 4.562555 | 0.881604 | 6.439536 |
| Zn | -7.510505 | 0.543946 | -1.466135 |
| Zn | -8.649579 | 2.347396 | -4.061932 |
| Zn | -5.660630 | 3.815446 | -3.587779 |
| Zn | -4.946971 | 7.080665 | -3.000491 |
| Zn | -0.907334 | 7.345173 | -3.156927 |
| Zn | 1.659116 | 9.229497 | -3.906969 |
| Zn | -7.229055 | -3.368239 | 1.121646 |
| Zn | -8.074964 | -2.655814 | -1.954516 |
| Zn | -5.057797 | -0.781934 | 0.648404 |
| Zn | -5.826050 | 3.690440 | -0.379346 |
| Zn | -3.051171 | 5.580586 | $-0.475828$ |
| Zn | 0.584189 | 5.786365 | -0.520439 |
| Zn | 3.195031 | 8.610148 | 0.434668 |
| Zn | 3.229591 | 6.675396 | -2.596051 |
| Zn | -6.867232 | -6.637330 | 2.826341 |
| Zn | -5.470433 | -3.676283 | 4.200714 |
| Zn | -3.175927 | -1.275110 | 3.500281 |
| Zn | -2.513736 | 2.207595 | 4.962003 |
| Zn | -2.276095 | 3.395612 | 1.992625 |
| Zn | 0.962337 | 3.837002 | 2.785885 |
| Zn | 4.207705 | 6.811772 | 3.108403 |
| Zn | 7.338813 | 5.499475 | 2.644241 |
| Zn | -5.140389 | -3.274990 | 7.516734 |
| Zn | -0.410493 | -2.002943 | 5.682021 |
| Zn | 0.487468 | 0.917139 | 4.301535 |
| Zn | 1.450611 | 2.272907 | 7.141629 |
| Zn | 3.729553 | 4.017592 | 4.954840 |
| Zn | 6.898050 | 4.652024 | 5.851612 |
| Zn | -1.892978 | -4.290696 | 7.647503 |
| Zn | 0.840372 | -0.903239 | 8.454877 |
| Zn | -8.476037 | 1.888361 | 1.499074 |
| Zn | -7.248503 | 6.735894 | -0.400550 |
| Zn | -4.809335 | 7.857683 | 1.494984 |
| Zn | -2.628759 | 9.168092 | -0.868023 |
| Zn | -9.263691 | -1.214082 | 2.654865 |
| Zn | -5.682657 | 1.427253 | 3.095780 |
| Zn | -8.574545 | 5.168038 | 2.310168 |
| Zn | -5.398420 | 4.809350 | 2.996504 |
| Zn | -2.029927 | 6.646207 | 3.261709 |
| Zn | -0.196127 | 8.536436 | 1.290890 |
| Zn | -7.093057 | -0.793349 | 5.102899 |
| Zn | -6.082275 | 1.258110 | 7.644107 |
| Zn | -4.975635 | 4.022282 | 6.253155 |
| Zn | -2.568762 | 4.949021 | 8.297970 |
| Zn | -0.452454 | 4.787846 | 5.769586 |
| Zn | -3.382117 | -0.470051 | 7.006210 |
| Zn | -1.571354 | 1.769817 | 8.789662 |


| $\mathrm{Zn}_{190}(1.8 \mathrm{~nm})$ |  |  |  | Zn | 9.089386 | -0.010345 | 3.422931 | Zn | -8.616283 | -4.616380 | 4.079636 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | x | y | z |  |  |  |  |  |  |  |  |
| Zn | 4.450044 | -5.057248 | -9.209072 |  |  |  |  |  |  |  |  |
| Zn | 2.864786 | -7.929026 | -8.896859 | Zn | 4.580075 | -6.233755 | 7.355495 | Zn | -6.678780 | -4.397538 | 1.452267 |
| Zn | 5.361033 | -8.845652 | -7.025274 | Zn | 8.896541 | -2.438177 | 5.897602 | Zn | -7.257902 | -1.793742 | 5.072854 |
| Zn | -1.079610 | -5.308266 | -7.524607 | Zn | -6.958414 | 4.770494 | -8.039016 | Zn | -3.822424 | -1.092417 | 1.893876 |
| Zn | -0.810370 | -2.321019 | -9.591908 | Zn | -8.859511 | -0.571491 | -4.334994 | Zn | -3.132477 | 0.770523 | 4.919202 |
| Zn | 1.654838 | 0.063464 | -10.060566 | Zn | -6.792056 | 0.122504 | -7.110698 | Zn | -4.181296 | 2.503035 | 2.119131 |
| Zn | 2.284018 | 2.448649 | -12.261581 | Zn | -4.819627 | 1.960859 | -4.178173 | Zn | -0.743744 | 3.213980 | 2.260630 |
| Zn | 5.016076 | 3.240839 | -10.679064 | Zn | -4.499392 | 2.652093 | -7.482034 | Zn | 0.784348 | 6.343001 | 3.423380 |
| Zn | -1.325635 | -9.923702 | -6.597350 | Zn | -1.848382 | 4.261966 | -6.213351 | Zn | 5.307843 | 6.369320 | 4.605867 |
| Zn | 1.002284 | -7.661194 | -6.150716 | Zn | -0.792507 | 6.748901 | -8.158198 | Zn | 5.598770 | 9.199051 | 2.565925 |
| Zn | 2.460978 | -4.803094 | -5.004898 | Zn | 1.746367 | 7.071082 | -6.139404 | Zn | -5.930726 | -6.182034 | 5.375930 |
| Zn | 2.063283 | -3.294843 | -7.841893 | Zn | -9.273110 | -5.529918 | $-2.369146$ | Zn | -4.349143 | -3.707978 | 3.980693 |
| Zn | 5.380471 | -2.212699 | -6.233826 | Zn | -7.398412 | -3.878958 | -4.476671 | Zn | -4.038832 | -1.757279 | 6.722550 |
| Zn | 7.355282 | -0.151963 | -7.904553 | Zn | -6.513602 | -1.482931 | -1.871654 | Zn | -0.988756 | -2.657711 | 6.868008 |
| Zn | 8.350857 | 0.979528 | -4.817667 | Zn | -4.694252 | -1.271169 | -4.754081 | Zn | -0.198904 | 1.212928 | 6.957787 |
| Zn | -1.238976 | -10.220410 | -3.185370 | Zn | -2.126552 | -1.130908 | -2.287531 | Zn | 1.688724 | 1.776205 | 3.885054 |
| Zn | 1.760508 | -9.645098 | -1.492276 | Zn | -1.463628 | 1.617103 | -4.238048 | Zn | 2.405494 | 4.394213 | 5.671904 |
| Zn | 3.338928 | -9.361877 | -4.453671 | Zn | 0.110566 | 5.611943 | -3.618422 | Zn | 5.404882 | 6.160143 | 7.890483 |
| Zn | 4.779514 | -6.940011 | -2.899440 | Zn | 2.543611 | 3.796997 | -6.995229 | Zn | -1.710834 | -4.634859 | 9.542286 |
| Zn | 5.627591 | -5.650107 | -6.084335 | Zn | 3.571284 | 4.794098 | -2.627378 | Zn | 1.147215 | -3.352905 | 10.548429 |
| Zn | 8.195711 | -5.500010 | -3.716619 | Zn | 5.816925 | 8.554957 | -3.679310 | Zn | 2.780945 | -0.234966 | 10.178417 |
| Zn | 7.674563 | -2.175302 | -3.960205 | Zn | -6.973385 | -7.074396 | -0.478136 | Zn | 3.242251 | 3.070337 | 10.345670 |
| Zn | 10.779027 | 0.294239 | -2.503225 | Zn | -3.878667 | -6.969218 | -1.453601 | Zn | 4.909255 | 2.822938 | 7.516917 |
| Zn | 2.243863 | -9.713462 | 2.230611 | Zn | -3.980089 | -4.538882 | -3.950216 | Zn | -11.784289 | 1.454074 | 1.354398 |
| Zn | 4.709236 | -8.557258 | 0.219988 | Zn | -4.065198 | -3.236666 | -0.651136 | Zn | -9.811695 | 4.085397 | 0.708522 |
| Zn | 7.758556 | -8.381775 | -1.715645 | Zn | -0.895857 | 0.277793 | 0.502965 | Zn | -8.476921 | 5.837612 | -1.934739 |
| Zn | 8.775429 | -6.305825 | 0.732260 | Zn | 1.975568 | 0.720428 | -1.295551 | Zn | -7.492227 | 8.345176 | -0.049601 |
| Zn | 10.771597 | -5.113271 | -1.654778 | Zn | 1.399689 | 3.826880 | -0.166226 | Zn | -4.350647 | 7.939103 | -1.182530 |
| Zn | 9.338714 | -2.228310 | $-0.912634$ | Zn | 3.613020 | 4.735789 | 2.074189 | Zn | -2.911961 | 10.499533 | 0.125112 |
| Zn | -5.426676 | -1.476261 | -9.621989 | Zn | 6.367537 | 6.361022 | -1.190687 | Zn | -9.977004 | 0.020663 | 3.690169 |
| Zn | -2.863851 | 0.542508 | -9.461031 | Zn | 8.222838 | 8.112481 | 0.892039 | Zn | -9.587019 | 1.692456 | 6.584481 |
| Zn | -0.160581 | 2.267636 | -8.402387 | Zn | -5.532172 | -7.838072 | 2.601909 | Zn | -9.000477 | 4.648792 | 4.883135 |
| Zn | 0.493951 | 4.717113 | -10.452040 | Zn | -2.923357 | -5.791671 | 1.688084 | Zn | -6.098536 | 4.646855 | 6.600620 |
| Zn | 3.516254 | 5.878810 | -9.502212 | Zn | -1.449080 | -5.501806 | 5.105808 | Zn | -7.135500 | 5.803332 | 2.111829 |
| Zn | -6.317887 | -5.824949 | -7.041661 | Zn | -0.753446 | -3.231414 | 1.478089 | Zn | -4.333392 | 5.353830 | 3.937319 |
| Zn | -4.021014 | -3.506124 | -7.575792 | Zn | -0.230835 | -1.039358 | 3.987479 | Zn | -1.551434 | 8.749749 | 2.488471 |
| Zn | -1.704016 | -0.968346 | -6.446578 | Zn | 3.172855 | -1.913273 | 0.807426 | Zn | 1.509076 | 10.042549 | 1.614121 |
| Zn | 1.545751 | -0.361260 | -6.372276 | Zn | 4.258736 | 0.993896 | 1.884128 | Zn | -8.574700 | -1.206972 | 8.051097 |
| Zn | 1.739801 | 2.316556 | -4.257350 | Zn | 6.505220 | 3.901114 | 1.023538 | Zn | -5.762211 | 0.402014 | 8.883009 |
| Zn | 4.224741 | 1.220662 | -8.125879 | Zn | 6.363870 | 3.103895 | 4.599035 | Zn | -6.407040 | 1.431918 | 5.728273 |
| Zn | 5.337489 | 4.363663 | -5.324241 | Zn | 8.431919 | 5.694273 | 3.070691 | Zn | -3.494820 | 2.972666 | 8.085935 |
| Zn | 5.088044 | 7.347315 | -6.753077 | Zn | 0.234960 | -6.675898 | 7.664889 | Zn | -1.621674 | 4.060110 | 5.359470 |
| Zn | -4.490194 | -9.373229 | -3.654624 | Zn | 2.156428 | -3.953643 | 7.525331 | Zn | -0.915448 | 7.210820 | 6.382361 |
| Zn | -3.416012 | -7.333155 | -5.971999 | Zn | 2.716487 | -2.555679 | 4.352046 | Zn | 2.691403 | 8.650052 | 4.658746 |
| Zn | -0.954001 | -6.959686 | -3.515270 | Zn | 3.154715 | 0.128636 | 6.552882 | Zn | -3.616755 | -1.938830 | 10.283134 |
| Zn | -0.885411 | -3.733176 | -4.656402 | Zn | 5.767471 | -0.208781 | 4.420050 | Zn | -0.672389 | -0.458308 | 9.761820 |
| Zn | 0.690267 | -2.990107 | -1.538127 | Zn | 7.352494 | 0.108137 | 7.528336 | Zn | -1.558612 | 2.550654 | 10.839504 |
| Zn | 2.842485 | -1.838691 | -3.785506 | Zn | 9.238606 | 2.379023 | 5.851645 | Zn | 0.595110 | 4.040158 | 8.708765 |
| Zn | 5.260985 | -1.550453 | -1.678470 | Zn | 4.980054 | -2.126014 | 8.268121 | Zn | 2.211296 | 7.184498 | 7.731935 |
| Zn | 4.957952 | 1.066423 | -4.708087 | Zn | -9.108759 | 2.526696 | -7.135584 | Zn | -5.138395 | 7.507194 | 7.949527 |
| Zn | 5.106135 | 2.082264 | $-1.578683$ | Zn | -6.809979 | 5.879823 | -4.912633 | Zn | -3.692202 | 8.320953 | 5.049552 |
| Zn | 7.761076 | 3.792869 | -2.974032 | Zn | -10.541954 | -0.144764 | -1.354873 | Zn | -2.369317 | 6.042631 | 9.057175 |
| Zn | -2.360779 | -9.626533 | -0.087948 | Zn | -7.917083 | 1.722178 | $-0.763729$ |  |  |  |  |
| Zn | -3.586328 | -10.427171 | 2.920574 | Zn | -8.057609 | 2.738362 | -3.925526 |  |  |  |  |
| Zn | -0.087155 | -7.535068 | 0.978058 | Zn | -5.673286 | 4.499917 | -0.561554 |  |  |  |  |
| Zn | 1.324043 | -5.236735 | 3.163414 | Zn | -3.966206 | 5.139063 | -3.560925 |  |  |  |  |
| Zn | 1.620025 | -6.081950 | -1.772289 | Zn | -1.924441 | 8.134883 | $-3.641160$ |  |  |  |  |
| Zn | 3.586111 | -5.315712 | 0.779513 | Zn | 0.555264 | 10.056500 | $-5.096507$ |  |  |  |  |
| Zn | 6.492450 | -4.463554 | -0.834217 | Zn | -0.045509 | 10.067929 | -1.440676 |  |  |  |  |
| Zn | 7.379464 | 0.025925 | 0.532531 | Zn | -9.258022 | -2.953853 | -0.296107 |  |  |  |  |
| Zn | 9.348628 | 2.417353 | $-0.409776$ | Zn | -10.984246 | -2.938881 | 2.528757 |  |  |  |  |
| Zn | 10.585884 | 3.157351 | 2.645091 | Zn | -6.978710 | -0.928733 | 1.697959 |  |  |  |  |
| Zn | -1.021614 | -8.699994 | 3.992876 | Zn | -7.411557 | 2.246765 | 2.793276 |  |  |  |  |
| Zn | 1.191608 | -9.576041 | 6.262756 | Zn | -4.360626 | 1.200829 | -1.086430 |  |  |  |  |
| Zn | 3.317263 | -7.565430 | 4.609075 | Zn | -1.792670 | 3.401391 | -1.556463 |  |  |  |  |
| Zn | 5.873762 | -3.797724 | 5.488831 | Zn | -2.689696 | 5.850891 | 0.912398 |  |  |  |  |
| Zn | 6.037166 | -6.457828 | 2.940483 | Zn | 0.296122 | 6.756735 | -0.224049 |  |  |  |  |
| Zn | 6.286563 | -2.882449 | 2.163915 | Zn | 2.725249 | 8.245365 | -2.792678 |  |  |  |  |
| Zn | 9.454774 | -3.367571 | 2.388231 | Zn | 3.609480 | 7.628059 | 0.396067 |  |  |  |  |



## B. 4 Sulphur Quantum Dots



Figure B.4: Equilibrium structures of S quantum dots of sizes 1.0, 1.2, 1.4, 1.8 and 2.0 nm . The structures were obtained from geometry optimization using PM7 as implemented in MOPAC.

## B.4.1 Cartesian coordinates (Angstroms) of equilibrium sulphur quantum dots

|  | $\mathrm{S}_{38}(1.0 \mathrm{~nm})$ |  |  |
| :---: | :---: | :---: | :---: |
|  | X | y | Z |
| S | -2.350686 | -2.999812 | -5 |
| S | 56609 | 0.054883 | -3 |
| S | -2.347042 | -4.509889 | -2.22 |
| S | -1.174186 | -3.450986 | -3.55 |
| S | 37 | -3. | -2.581361 |
| S | 17 | -1. | -3.050974 |
| S | 4.504577 | 1.254414 | -1.7902 |
| S | 0.199805 | -4.037109 | 0.705408 |
| S | 67 | -3 | -0.738990 |
| S | 4.678764 | -1.865693 | -0.727 |
| S | 5.772360 | -0.2861 | -1. |
| S | -4.9 | -2.662744 |  |
| S | -3.760358 | -1.697809 | -4 |
| S | -3.806411 | 1.290394 | -1 |
| S | -0.484792 | 1.408525 | -2. |
| S | 0.728337 | 2.621046 |  |
| S | -3.792671 | -3.229489 | -1 |
| S | -3.663602 | $-1.830050$ | 0.012065 |
| S | -2.507146 | -0.208431 | -0 |
| S | 2.510792 | 0.212778 | 0.6 |
| S | 3.671445 | 1.838621 | -0.00 |
| S | 3.799969 | 3.23033 | 1.5 |
| S | -0.728923 | -2.641470 |  |
| S | 0.481649 | -1.415925 | 2.839899 |
| S | 3.803782 | -1.295133 | 1.0 |
| S | 3.748398 | 1.687301 |  |
| S | 4.937622 | 2.658205 | 3.190248 |
| S | -5.757991 | 0.311221 | 1.404400 |
| S | -4.670367 | 1.882492 | 0.702 |
| S | -1.470969 | 3.745401 | 0.74204 |
| S | -0.193445 | 4.027017 | -0.701719 |
| S | -4.491130 | $-1.229350$ | 1.7968 |
| S | -1.647607 | 1.343729 | 3.05259 |
| S | -0.890087 | 3.174857 | 2.583246 |
| S | 1.167163 | 3.442003 | 3.552131 |
| S | 2.346593 | 4.506776 | 2.230198 |
| S | -0.560001 | -0.060610 | 3.789121 |
| S | 2.332398 | 2.985666 | 5.182259 |


| S | -0.179098 | 0.792456 | -0.053683 |
| :--- | ---: | ---: | ---: |
| S | 3.109678 | 2.728657 | -1.405280 |
| S | 3.257914 | 3.189062 | 0.640345 |
| S | -3.047230 | -4.751884 | 3.009713 |
| S | -1.725869 | -5.377130 | 4.144640 |
| S | 0.034851 | -4.861194 | 4.436751 |
| S | 2.399616 | -1.141383 | 1.595960 |
| S | 2.816509 | 0.531255 | 2.778939 |
| S | 4.210303 | 1.599889 | 1.595148 |
| S | 1.066793 | -3.518437 | 3.663770 |
| S | -2.844584 | 2.530557 | -1.857881 |
| S | -0.493611 | 7.863802 | -1.332503 |
| S | -5.186367 | -0.725652 | -0.023395 |
| S | -4.600513 | 0.734672 | 1.370060 |
| S | -3.723104 | 4.120271 | -1.274342 |
| S | -3.776776 | 5.070297 | 0.318275 |
| S | -0.121571 | 6.109836 | -0.739507 |
| S | 1.250609 | 6.128275 | 0.857653 |
| S | -5.904495 | -0.962715 | 4.106119 |
| S | -4.354826 | -0.509524 | 3.159590 |
| S | -3.050443 | 0.886572 | 4.284560 |
| S | -0.987779 | 2.083744 | 1.246292 |
| S | 0.439397 | 2.872239 | 2.482828 |
| S | 1.626692 | 4.245511 | 1.402809 |
| S | -1.890191 | -1.800789 | 6.095939 |
| S | -1.453677 | -0.528371 | 4.759026 |
| S | 0.072225 | 0.790256 | 5.355531 |
| S | 1.110495 | 1.284315 | 3.730804 |
| S | -5.875069 | 2.125572 | 1.566294 |
| S | -2.906228 | 4.801515 | 1.925157 |
| S | -3.944626 | 1.712969 | 5.707427 |
| S | -2.648986 | -0.775051 | -2.813242 |
| S | -1.836633 | 1.169835 | -2.916052 |
| S | -1.682423 | 1.849773 | -4.921243 |
|  |  |  |  |


|  | $\mathrm{S}_{104}$ |  | $(1.4 \mathrm{~nm})$ |
| :--- | ---: | :---: | :---: |
|  | X | y | Z |
| S | 0.334411 | -2.653798 | -7.017321 |
| S | 0.478079 | -4.641201 | -3.928979 |
| S | 1.284745 | -3.298521 | -5.413022 |
| S | 2.643878 | -2.208994 | -4.332576 |
| S | 0.559920 | -7.485439 | -2.068174 |
| S | 1.497897 | -5.729500 | -2.353385 |
| S | 3.001321 | -4.446909 | -1.759176 |
| S | 4.124973 | -3.191408 | -3.086919 |
| S | 5.908703 | -2.629627 | -2.339395 |
| S | -2.311400 | 0.101822 | -7.735747 |
| S | -3.969460 | -3.640652 | -6.912633 |
| S | -2.718004 | -2.707880 | -5.842320 |
| S | -1.283395 | -1.454433 | -6.892218 |
| S | 0.873408 | 1.932069 | -5.586746 |
| S | 2.032878 | 3.560631 | -5.512628 |
| S | 2.793224 | 3.850286 | -3.674648 |
| S | -2.455219 | -7.255387 | -5.196917 |
| S | -2.461208 | -6.360699 | -3.571726 |
| S | -1.463540 | -4.589152 | -3.391191 |
| S | 0.839344 | -1.918380 | -1.550772 |
| S | 1.810936 | -0.854965 | -3.000139 |
| S | 1.631303 | 3.740346 | 3.808016 |
| S | 3.343243 | 4.858357 | 3.814680 |
| S | 4.295343 | 4.899752 | 5.586917 |
| S | -3.524010 | -3.437233 | 3.356005 |
| S | -2.711726 | -2.347070 | 4.838737 |
| S | -1.475547 | -0.860485 | 4.328654 |
| S | 1.976777 | 2.396078 | 7.601963 |
| S | 5.462236 | 3.201861 | 6.055001 |
| S | 7.221328 | 3.630980 | 6.530643 |
| S | 0.077827 | 4.755057 | 4.663812 |
| S | 0.040983 | 2.798296 | 7.726672 |
| S | -0.830908 | 3.349205 | 5.994291 |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

S $\quad 2.441734 \quad 1.065556-1.918870$
S $\quad 3.886525 \quad 2.412120-2.542744$ S $\quad 5.239573 \quad 2.776724-0.962503$ S $\quad-2.564271-7.492614-0.342172$
S $\quad-1.414972-7.595157-1.983717$ S $\quad 0.927981-5.401420 \quad 2.042440$ S $\quad 2.241809-2.988870-0.472832$ S $2.770397-1.600705 \quad 1.136547$ S $\quad 5.297514-1.086225-1.075794$ $\begin{array}{lllll}\mathrm{S} & 6.931902 & -0.195209 & -0.175117\end{array}$ $\begin{array}{llll}\mathrm{S} & 6.919340 & 1.722601 & -0.805809\end{array}$ S $\quad 1.906176 \quad-5.269809 \quad 3.777076$ S $\quad 2.841520 \quad-3.489962 \quad 3.930250$ S $4.064453-2.368804 \quad 2.574939$ S $\quad 5.487533-0.886915 \quad 2.957952$ $\begin{array}{lllll}\mathrm{S} & 7.057400 & -0.835719 & 1.724741\end{array}$ S $\quad-4.581294 \quad 0.107851-5.031103$ S $\quad-3.369736 \quad 1.166893-6.397930$ S $\quad-2.469585 \quad 2.540720-5.097025$ S $\quad-6.904676 \quad-5.279385-3.760192$ S $\quad-6.150388-3.608216-3.410888$ S $\quad-3.404695-1.472747-4.382068$ S $\quad-2.222607 \quad 1.017224-1.393366$ $-0.6991861 .755287-4.432080$ S $\quad 0.389076 \quad 3.953154-1.453495$ $1.957783-5.080282-2.167656$ $3.067806 \quad 5.925983-0.627051$ $-4.760609-4.492207-0.413469$ $-4.488355-3.542049-2.199151$ $\begin{array}{llll}-2.322128 & -1.160911 & 0.924491\end{array}$ $-2.202252-0.963918-1.068605$ $\begin{array}{lll}0.961365 & 1.499959 & 1.292337\end{array}$ $1.130707 \quad 2.084086-0.739589$ $\begin{array}{lll}4.304198 & 4.311089 & 2.011183\end{array}$ $4.604425 \quad 4.560392-0.035973$ $-3.139372-5.685728 \quad 0.266291$ $\begin{array}{llll}-2.422485 & -4.695642 & 1.980857\end{array}$ $-0.503136-4.1235201 .606881$ $\begin{array}{llll}0.489860 & -1.234330 & 3.804560\end{array}$ $\begin{array}{lll}1.328482 & -0.463649 & 2.012815\end{array}$ $\begin{array}{llll}5.273299 & 2.054905 & 4.225905\end{array}$ $\begin{array}{lll}5.998464 & 3.060553 & 2.568169\end{array}$ $\begin{array}{lll}7.659711 & 3.956486 & 2.734166\end{array}$ $\begin{array}{llll}2.000220 & -1.902805 & 5.032432\end{array}$ $\begin{array}{llll}3.100979 & -0.353147 & 5.888922\end{array}$ $4.8271310 .088411 \quad 4.762438$ $-7.489899-0.479841-3.047399$ $\begin{array}{llll}-5.864611 & 0.562794 & -3.397500\end{array}$ $-4.953833-2.318267-2.671762$ $\begin{array}{llll}-3.476198 & 3.605712 & -3.513745\end{array}$ $\begin{array}{llll}-2.858690 & 5.110225 & -2.328710\end{array}$ $\begin{array}{llll}-7.781056 & -2.918571 & -0.408069\end{array}$ $\begin{array}{llll}-7.585829 & -2.314732 & -2.305164\end{array}$ $\begin{array}{llll}-4.747590 & 1.345481 & 1.135682\end{array}$ $-4.096266 \quad 1.762373-0.853017$ $\begin{array}{lll}-1.781663 & 3.772348 & 0.991541\end{array}$ $-1.505728 \quad 4.570432-0.953984$ $\begin{array}{llll}1.957055 & 7.672820 & 2.090006\end{array}$ $\begin{array}{lll}2.624091 & 7.672105 & 0.229456\end{array}$ $-6.118038-3.2585750 .689796$ $\begin{array}{llll}-5.030314 & -2.493911 & 2.233260\end{array}$ $\begin{array}{lll}-4.228118 & -0.638625 & 1.598054\end{array}$ $\begin{array}{lll}-1.789582 & 1.102426 & 3.650694\end{array}$ $\begin{array}{lll}-0.959615 & 2.019796 & 1.900162\end{array}$ $2.697044 \quad 0.561307 \quad 7.612416$ $\begin{array}{lll}-5.219310 & 2.670023 & 2.576982\end{array}$ $\begin{array}{llll}-3.752832 & 3.630305 & 3.541188\end{array}$ $\begin{array}{lll}-2.430047 & 4.995420 & 2.534726\end{array}$ $\begin{array}{lll}-0.792972 & 6.057257 & 3.231034\end{array}$ $\begin{array}{llll}0.054771 & 7.772942 & 2.610622\end{array}$ $-2.436956 \quad 2.617675 \quad 4.911359$

| $\mathrm{S}_{190}(1.8 \mathrm{~nm})$ |  |  | S | 2.137167 | -8.463860 | 3.987652 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| X | y | Z | S | 8.678949 | -4.009766 | 3.517239 |
| 2.165664 | -6.428788 | -9.326861 | S | -5.208662 | 4.173459 | -6.484085 |
| 5.329255 | -5.610920 | -6.003172 | S | -7.054107 | -2.451839 | -4.346661 |
| 6.905619 | -4.896406 | -5.103533 | S | -5.995737 | -1.029455 | -5.534875 |
| -3.258890 | -4.481287 | -7.891604 | S | -5.406821 | 0.348798 | -4.093553 |
| -2.232438 | -3.874895 | -9.398107 | S | -3.598933 | 2.981822 | -6.596642 |
| -0.193887 | -3.969377 | -9.560437 | S | 2 | 3.630191 | -5.216742 |
| 0.615902 | -2.132474 | -9.563089 | S | -0.596718 | 4.385618 | -6.303 |
| 1.023472 | -1.186185 | -7.873642 | S | 1 | 6.594322 | -4.286870 |
| -0.809979 | -7.206100 | -4.123211 | S | -7.300675 | -5.888439 | -0.507484 |
| 0.652265 | -6.992707 | -5.463881 | S | -5.394182 | -3.554082 | -3.552450 |
| 2.037012 | -5.634901 | -5.206220 | S | -4.744212 | -2.259564 | 262 |
| 0.583916 | -5.635563 | -8.712712 | S | -4.01 | 70 | 555 |
| 3.630209 | -2.256609 | -5.825721 | S | -2.578117 | 2.047377 | -1.535103 |
| 2.914502 | -1.159230 | -7.299967 | S | -1.799777 | 2.586882 | -3.379113 |
| 7.563010 | 2.194020 | -3.016438 | S | 1434629 | 3.209912 | 21 |
| -0.777108 | -8.695835 | -2.793154 | S | 3.166612 | 5.582461 | -3.208151 |
| -0.199303 | -8.195379 | -0.905432 | S | 4.689660 | 6.729094 | -2.403229 |
| 3.885055 | -6.417142 | -4.709366 | S | 5.952514 | 7.471720 | -3.589349 |
| 4.495334 | -5.961911 | -2.803989 | S | -5.406996 | -6.449517 | -0.176122 |
| 7.151063 | -2.977038 | -4.708930 | S | -4.149872 | -4.890974 |  |
| 6.374892 | -2.217769 | -3.051616 | S | -2.712068 | -4.613245 | -0.737398 |
| 8.647512 | 1.717987 | $-1.379730$ | S | -2.745998 | -1.174026 | 0.568888 |
| 9.966347 | 3.070999 | -0.822402 | S | 2 | 0.386744 | -0.499384 |
| 1.694554 | -8.908073 | $-0.508937$ | S | 0.250216 | 0.173607 | -0.230419 |
| 2.963933 | -7.304272 | -0.195328 | S | 8 | 1.912717 | 0.178890 |
| 4.514323 | -7.379055 | -1.477254 | S | 5 | 4.021605 | 1.160699 |
| 6.592034 | -3.948789 | -0.159783 | S | 5.405269 | 5.183934 | 0.538492 |
| 7.216502 | -2.242887 | $-1.259938$ | S | 6.319296 | 6.162130 | 2.109876 |
| 8.540978 | -1.343538 | 0.003989 | S | -5.289184 | -5.794274 | 3.553695 |
| -4.314990 | -1.301831 | -6.796151 | S | -3.611418 | -5.233916 | 2.500126 |
| -3.684539 | -0.193133 | -8.306324 | S | -2.184083 | -4.677710 | 4.120419 |
| -2.810244 | 1.571105 | -8.081074 | S | -1.623976 | -2.537282 | 1.551036 |
| -0.951354 | 2.040359 | -8.686782 | S | 0.043461 | -1.778472 | 2.462414 |
| 0.330513 | 2.840203 | $-7.346323$ | S | 1.299694 | -1.330346 | 0.754007 |
| -3.728761 | -4.394104 | -4.523898 | S | 3.192393 | -0.486558 | 3.449301 |
| -3.470117 | -3.209698 | -6.295832 | S | 4.080695 | 0.950263 | 2.411576 |
| -1.245493 | -1.763940 | -4.185265 | S | 6.280874 | 3.285269 | 4.011592 |
| -0.312491 | -0.157437 | -4.733564 | S | 7.553979 | 4.769144 | 3.027531 |
| -0.101275 | 1.452472 | -3.641279 | S | -0.714205 | -5.981206 | 4.775826 |
| 2.012958 | 2.292283 | -6.353597 | S | 1.105481 | -5.138316 | 5.490051 |
| 3.803540 | 2.445869 | -5.895655 | S | 2.984746 | -5.216884 | 4.547691 |
| 4.890847 | 3.908053 | -6.086201 | S | 4.523556 | -4.859934 | 5.703100 |
| -4.600162 | -8.257701 | -0.415908 | S | 4.290768 | -1.743426 | 4.494223 |
| -2.176881 | -5.515598 | -3.848382 | S | 5.523529 | -1.148588 | 5.914769 |
| -1.353981 | -4.556504 | -2.155827 | S | 7.362802 | 1.533473 | 4.213165 |
| -0.257517 | -2.991904 | -2.840514 | S | 4.863052 | -5.376627 | 7.462691 |
| 1.975706 | -2.752881 | -2.961759 | S | -6.749667 | 3.341144 | -5.481851 |
| 3.151139 | -1.502157 | -4.014395 | S | -5.345978 | 7.433916 | -4.103214 |
| 4.317587 | 1.692058 | -2.186895 | S | -8.601417 | -1.931542 | -3.176133 |
| 5.813892 | 2.987697 | -2.461537 | S | -8.292400 | -0.631420 | -1.660500 |
| 8.999391 | 6.253901 | -1.141287 | S | -6.223732 | 2.497056 | -3.758945 |
| 10.179552 | 7.012060 | -2.337271 | S | -5.891151 | 3.565669 | -2.109323 |
| -4.029849 | -9.444068 | 1.021331 | S | -3.792339 | 6.485865 | -3.942621 |
| -2.191239 | -8.934887 | 1.774948 | S | -2.671338 | 6.382200 | -2.445535 |
| -0.918124 | -7.451687 | 1.076910 | S | 0.854866 | 7.915258 | -3.063654 |
| 0.230099 | -5.832034 | 1.419604 | S | 2.261244 | 9.201476 | -2.365876 |
| 2.681936 | -3.649521 | -1.303779 | S | -8.664430 | -5.210254 | 0.624919 |
| 2.910542 | -2.387888 | 0.212485 | S | -8.472462 | -3.096327 | 0.849112 |
| 4.658009 | 0.198618 | -0.944314 | S | -7.412912 | -1.500808 | 0.105295 |
| 5.276508 | 0.531098 | 0.897957 | S | -6.235694 | -0.138312 | 1.062193 |
| 9.179010 | 4.733845 | 0.114522 | S | -3.027349 | 3.498037 | -0.180447 |
| 9.366152 | 4.575653 | 2.133574 | S | -1.927461 | 3.751670 | 1.436260 |
| -2.725939 | -8.495266 | 3.776136 | S | 0.336613 | 4.832557 | -0.915504 |
| -1.371152 | -7.906335 | 5.018195 | S | 1.351991 | 5.993675 | 0.419123 |
| 2.197113 | -6.185986 | 1.485562 | S | 3.608217 | 8.197324 | -1.171582 |
|  |  | 1.45562 | S | 4.766102 | 8.845492 | 0.302287 |

[^0]
## $\mathrm{S}_{244}(2.0 \mathrm{~nm})$ <br> X y Z

S $2.165664-6.428788$-9.326861
S $\quad 5.329255-5.610920-6.003172$
$6.905619-4.896406-5.103533$
$\begin{array}{lllll}\text { S } & -3.258890 & -4.481287 & -7.891604\end{array}$
S $\quad-2.232438$-3.874895 -9.398107
S $\quad-0.193887-3.969377-9.560437$
$\begin{array}{llll}0.615902 & -2.132474 & -9.563089\end{array}$
$1.023472-1.186185-7.873642$ $-0.809979-7.206100-4.123211$ $0.652265-6.992707-5.463881$ $2.037012-5.634901-5.206220$ $\begin{array}{llll}0.583916 & -5.635563 & -8.712712\end{array}$ $3.630209-2.256609-5.825721$ $2.914502-1.159230-7.299967$
$\begin{array}{llll}7.563010 & 2.194020 & -3.016438\end{array}$
$\begin{array}{llll}-0.777108 & -8.695835 & -2.793154\end{array}$
$-0.199303-8.195379-0.905432$ $3.885055-6.417142-4.709366$ $4.495334-5.961911-2.803989$ $\begin{array}{llll}7.151063 & -2.977038 & -4.708930\end{array}$ $\begin{array}{lll}6.374892 & -2.217769 & -3.051616\end{array}$ $8.647512 \quad 1.717987-1.379730$ $\begin{array}{llll}9.966347 & 3.070999 & -0.822402\end{array}$ $\begin{array}{llll}1.694554 & -8.908073 & -0.508937\end{array}$ $2.963933-7.304272-0.195328$ $4.514323-7.379055-1.477254$ $\begin{array}{llll}6.592034 & -3.948789 & -0.159783\end{array}$ $\begin{array}{llll}7.216502 & -2.242887 & -1.259938\end{array}$ $8.540978-1.343538 \quad 0.003989$ $-4.314990-1.301831-6.796151$ $\begin{array}{llll}-3.684539 & -0.193133 & -8.306324\end{array}$ $\begin{array}{llll}-2.810244 & 1.571105 & -8.081074\end{array}$ $\begin{array}{llll}-0.951354 & 2.040359 & -8.686782\end{array}$ $\begin{array}{llll}0.330513 & 2.840203 & -7.346323\end{array}$ $-3.728761-4.394104-4.523898$ $-3.470117-3.209698$-6.295832 $-1.245493-1.763940-4.185265$ $\begin{array}{llll}-0.312491 & -0.157437 & -4.733564\end{array}$ $-0.1012751 .452472-3.641279$ $\begin{array}{lll}2.012958 & 2.292283 & -6.353597\end{array}$ $3.803540 \quad 2.445869-5.895655$ $4.890847 \quad 3.908053-6.086201$ $-4.600162-8.257701-0.415908$ $-2.176881-5.515598-3.848382$ $\begin{array}{llll}-1.353981 & -4.556504 & -2.155827\end{array}$ $-0.257517-2.991904-2.840514$ $\begin{array}{lll}1.975706 & -2.752881 & -2.961759\end{array}$ $3.151139-1.502157-4.014395$ $4.317587 \quad 1.692058-2.186895$ $\begin{array}{llll}5.813892 & 2.987697 & -2.461537\end{array}$ $8.999391 \quad 6.253901-1.141287$
$\begin{array}{llll}10.179552 & 7.012060 & -2.337271\end{array}$
$\begin{array}{llll}-4.029849 & -9.444068 & 1.021331\end{array}$ $\begin{array}{llll}-2.191239 & -8.934887 & 1.774948\end{array}$ $-0.918124-7.4516871 .076910$ $\begin{array}{llll}0.230099 & -5.832034 & 1.419604\end{array}$ 2.681936 $-3.649521-1.303779$ $\begin{array}{llll}2.910542 & -2.387888 & 0.212485\end{array}$ $\begin{array}{llll}4.658009 & 0.198618 & -0.944314\end{array}$ $\begin{array}{llll}5.276508 & 0.531098 & 0.897957\end{array}$ $\begin{array}{lll}9.179010 & 4.733845 & 0.114522\end{array}$ 9.3661524 .5756532 .133574
$\begin{array}{llll}\text { S } & -2.725939 & -8.495266 & 3.776136\end{array}$
$\begin{array}{lllll}\text { S } & -1.371152 & -7.906335 & 5.018195\end{array}$
$\begin{array}{llll}2.197113 & -6.185986 & 1.485562\end{array}$
S $\quad 2.985913-6.922739 \quad 3.289770$
$\left.\begin{array}{lrrr}\text { S } & 2.137167 & -8.463860 & 3.987652 \\ \text { S } & 8.678949 & -4.009766 & 3.517239 \\ \text { S } & -5.208662 & 4.173459 & -6.484085 \\ \text { S } & -7.054107 & -2.451839 & -4.346661 \\ \text { S } & -5.995737 & -1.029455 & -5.534875 \\ \text { S } & -5.406821 & 0.348798 & -4.093553 \\ \text { S } & -3.598933 & 2.981822 & -6.596642 \\ \text { S } & -2.120932 & 3.630191 & -5.216742 \\ \text { S } & -0.596718 & 4.385618 & -6.303191 \\ \text { S } & 1.794401 & 6.594322 & -4.286870 \\ \text { S } & -7.300675 & -5.888439 & -0.507484 \\ \text { S } & -5.394182 & -3.554082 & -3.552450 \\ \text { S } & -4.744212 & -2.259564 & -2.083262 \\ \text { S } & -4.017461 & -0.599770 & -2.951055 \\ \text { S } & -2.578117 & 2.047377 & -1.535103 \\ \text { S } & -1.799777 & 2.586882 & -3.379113 \\ \text { S } & 1.434629 & 3.209912 & -1.357521 \\ \text { S } & 3.166612 & 5.582461 & -3.208151 \\ \text { S } & 4.689660 & 6.729094 & -2.403229 \\ \text { S } & 5.952514 & 7.471720 & -3.589349 \\ \text { S } & -5.406996 & -6.449517 & -0.176122 \\ \text { S } & -4.149872 & -4.890974 & 0.511919 \\ \text { S } & -2.712068 & -4.613245 & -0.737398 \\ \text { S } & -2.745998 & -1.174026 & 0.568888 \\ \text { S } & -1.928632 & 0.386744 & -0.499384 \\ \text { S } & 0.250216 & 0.173607 & -0.230419 \\ \text { S } & 1.237028 & 1.912717 & 0.178890 \\ \text { S } & 3.950565 & 4.021605 & 1.160699 \\ \text { S } & 5.405269 & 5.183934 & 0.538492 \\ \text { S } & 6.319296 & 6.162130 & 2.109876 \\ \text { S } & -5.289184 & -5.794274 & 3.553695 \\ \text { S } & -3.611418 & -5.233916 & 2.500126 \\ \text { S } & -2.184083 & -4.677710 & 4.120419 \\ \text { S } & -1.623976 & -2.537282 & 1.551036 \\ \text { S } & 0.043461 & -1.778472 & 2.462414 \\ \text { S } & 1.299694 & -1.330346 & 0.754007 \\ \text { S } & 3.192393 & -0.486558 & 3.449301 \\ \text { S } & 4.080695 & 0.950263 & 2.411576 \\ \text { S } & 6.280874 & 3.285269 & 4.011592 \\ \text { S } & 7.553979 & 4.769144 & 3.027531 \\ \text { S } & 0.336613 & 4.351991 & 5.932557675\end{array}\right) 0.0 .4191230$
$\begin{array}{llll}\text { S } & 4.214683 & 8.286252 & 3.650172\end{array}$ $\begin{array}{llll}\mathrm{S} & 5.457360 & 8.147359 & 2.019429\end{array}$ S $\quad-3.716746-3.539801 \quad 6.903638$ $\begin{array}{llll}-2.739608 & -3.128642 & 5.357608\end{array}$ $\begin{array}{llll}-0.824466 & -2.228774 & 5.783178\end{array}$ $\begin{array}{lll}-0.396455 & -0.996412 & 4.252445\end{array}$ $\begin{array}{lll}0.028038 & 2.214456 & 5.856320\end{array}$ $\begin{array}{lll}1.544337 & 2.591602 & 4.763616\end{array}$ $\begin{array}{llll}3.343580 & 2.283043 & 5.715964\end{array}$ $\begin{array}{llll}4.902633 & 3.422716 & 5.468364\end{array}$ $\begin{array}{llll}0.712485 & -3.440026 & 6.622928\end{array}$ $\begin{array}{llll}1.254915 & -3.354966 & 8.556326\end{array}$ $\begin{array}{llll}2.192070 & -1.709652 & 9.129821\end{array}$ $4.171953-1.596282 \quad 9.020655$ $4.825447-0.366168 \quad 7.591784$ $-9.2754791 .055778-1.043523$ $\begin{array}{lll}-8.266320 & 2.425513 & 0.023189\end{array}$ $\begin{array}{llll}-7.502831 & 3.985708 & -1.017659\end{array}$ $\begin{array}{lll}-4.074395 & 7.760966 & 0.277720\end{array}$ $-2.357273 \quad 7.729302-0.948093$ $-1.113754 \quad 9.090188-0.086569$ S $\quad-10.231320-2.552910 \quad 1.804120$ $\begin{array}{lllll}\mathrm{S} & -10.017090 & -1.385927 & 3.382060\end{array}$ $\begin{array}{llll}-7.197618 & 1.658568 & 1.680461\end{array}$ $\begin{array}{lll}-7.105315 & 2.100706 & 3.659907\end{array}$ $\begin{array}{lll}-3.572943 & 7.221259 & 2.118951\end{array}$ $\begin{array}{lll}-2.321241 & 8.598139 & 3.101518\end{array}$ $\begin{array}{lll}-0.621473 & 8.454466 & 1.790111\end{array}$ $\begin{array}{llll}1.154314 & 9.293199 & 2.415128\end{array}$ $\begin{array}{lll}-9.567837 & -2.277444 & 5.119469\end{array}$ $\begin{array}{lll}-5.021276 & 0.746414 & 6.247556\end{array}$ $-5.0571671 .700025 \quad 4.410173$ $\begin{array}{llll}-2.524225 & 4.560111 & 5.849099\end{array}$ $\begin{array}{llll}-1.242080 & 5.510682 & 4.560039\end{array}$ $\begin{array}{lll}-0.032602 & 6.875843 & 5.424368\end{array}$ $\begin{array}{llll}2.483051 & 9.071250 & 3.640643\end{array}$ $\begin{array}{llll}-3.193793 & 0.578701 & 6.947478\end{array}$ $\begin{array}{lll}-2.410640 & 2.047632 & 8.021959\end{array}$ $\begin{array}{llll}\mathrm{S} & -1.093819 & 3.402316 & 7.237569\end{array}$ $\begin{array}{llll}0.237353 & 4.818707 & 7.901807\end{array}$ $\begin{array}{llll}1.313913 & 5.918031 & 6.587746\end{array}$ $\begin{array}{llll}-8.448454 & 1.423729 & 4.799135\end{array}$ $\begin{array}{lll}-2.990840 & 10.315416 & 3.402735\end{array}$ $\begin{array}{llll}-3.727301 & 5.574831 & 6.852180\end{array}$ $\begin{array}{llll}-4.208809 & -0.383999 & 1.763290\end{array}$ $-3.6047391 .249577 \quad 3.045546$ $\begin{array}{llll}-1.861455 & 2.146233 & 2.666905\end{array}$ $\begin{array}{lll}1.612203 & 3.844288 & 3.308278\end{array}$ $\begin{array}{lll}2.319564 & 5.073922 & 1.928637\end{array}$ $\begin{array}{lll}3.608217 & 8.197324 & -1.171582\end{array}$ $\begin{array}{llll}4.766102 & 8.845492 & 0.302287\end{array}$ $\begin{array}{lll}-6.659046 & -4.387678 & 3.330597\end{array}$ $\begin{array}{llll}-6.433945 & -2.582520 & 4.120997\end{array}$ $\begin{array}{llll}-7.615446 & -2.097635 & 5.640232\end{array}$ $8.488091-1.067858 \quad 1.951554$ $\begin{array}{lll}8.405742 & 0.666294 & 2.938643\end{array}$ $\begin{array}{llll}5.982561 & -3.336943 & 1.615993\end{array}$
$\begin{array}{llll}\mathrm{S} & 7.441838 & -2.720433 & 2.959398\end{array}$

## B.5.1 Cartesian coordinates (Angstroms) of equilibrium silicon quantum dots

$\mathrm{Xi}_{128}(1.6 \mathrm{~nm})$
$\begin{array}{llllll}\text { Si } & 0.678825 & -2.036475 & -7.467075\end{array}$
$\begin{array}{llllll}\text { Si } & 3.425001 & 0.685102 & -7.583179\end{array}$
Si $\quad 0.962133-4.494602-5.033891$
Si $\quad 1.965147-3.197148-6.401497$
Si $3.477724-2.064418-5.222039$
Si $4.559163-0.058377-6.404960$
$\begin{array}{lllll}\text { Si } & 5.922193 & 0.536831 & -5.068959\end{array}$
Si $0.0664426-6.989434-2.652309$
Si $1.969399-5.894418-3.771945$
$\begin{array}{llllll}\text { Si } & 3.478855 & -4.746077 & -2.597622\end{array}$
$\begin{array}{lllll}\text { Si } & 4.598667 & -3.225088 & -3.727596\end{array}$
Si $6.153479-2.059330-2.540138$
Si $6.996188-0.800648-3.998943$
$\begin{array}{lllll}\text { Si } & 3.455211 & -7.173590 & 0.086589\end{array}$
Si 4.572698 -5.961975 -1.078727
$\begin{array}{llllll}\text { Si } & 5.938426 & -4.642943 & -0.002842\end{array}$
$\begin{array}{lllll}\text { Si } & 6.999584 & -3.548282 & -1.322289\end{array}$
Si -2.045384 $0.642410-7.531821$
$\begin{array}{lllll}\text { Si } & 0.657505 & 3.303983 & -7.450149\end{array}$
Si -3.037010 -3.866291 -5.864395
Si -1.965384 -2.070317 -5.175612
Si $-0.721336-0.712759-6.573019$
$\begin{array}{lllll}\text { Si } & 0.671186 & 0.592051 & -5.157687\end{array}$
Si $\quad 2.043716 \quad 1.943365-6.567716$
$\begin{array}{lllll}\text { Si } & 3.154812 & 3.113728 & -4.954613\end{array}$
Si -2.967110 -5.247227 -4.408967
Si $-1.950894-4.730834-2.582815$
$\begin{array}{lllll}\text { Si } & -0.550631 & -3.321289 & -3.815147\end{array}$
Si 0.728696 -2.031459 -2.489100
$\begin{array}{lllllllllllll}\text { Si } & 2.033070 & -0.711961 & -3.834373\end{array}$
$\begin{array}{llllllllllll}\text { Si } & 3.360021 & 0.583480 & -2.498758\end{array}$
$\begin{array}{lllllllllllll}\text { Si } & 4.737998 & 2.002847 & -3.881678\end{array}$
Si 5.899169 3.092819 -2.322940
$\begin{array}{lllll}\text { Si } & -2.020044 & -7.138335 & 0.071086\end{array}$
Si $-0.069693-6.140617-1.256235$
$\begin{array}{lllll}\text { Si } & 0.687494 & -4.752947 & 0.072419\end{array}$
Si $2.040724-3.39575-1.203743$
Si 3.366278 -2.087925 0.118183
Si $4.698798-0.755003-1.196061$
$\begin{array}{llllll}\text { Si } & 6.106845 & 0.532337 & 0.116510\end{array}$
Si $7.074637 \quad 1.937436-1.164363$
$\begin{array}{lllll}\mathrm{Si} & 0.708332 & -7.094840 & 2.732410\end{array}$
$\begin{array}{lllll}\text { Si } & 2.081836 & -6.194308 & 1.380109\end{array}$
$\begin{array}{lllll}\text { Si } & 3.185054 & -4.599866 & 2.581666\end{array}$
Si $4.763156-3.501551 \quad 1.502089$
$\begin{array}{lllll}\text { Si } & 5.918587 & -1.962697 & 2.630703\end{array}$
Si $7.089005-0.07736521 .48983$
Si -4.453757 0.844296 -4.981269
Si $-3.145857 \quad 1.843131-6.341699$
Si $\quad$-2.044587 $3.358687-5.171791$
Si -0.523874 $4.546089-6.351599$
$\begin{array}{lllll}\text { Si } & 0.531007 & 5.840222 & -4.969303\end{array}$
Si -6.122316 -3.046443 -3.722945
Si $\quad-4.587401-2.055325-2.518089$

Si -1. $1.574270 .583409-2.488644$
$\begin{array}{lllll}\mathrm{Si} & -0.619024 & 1.921798 & -3.793290\end{array}$
$\begin{array}{lllll}\mathrm{Si} & 0.697870 & 3.228884 & -2.472263\end{array}$
$\begin{array}{lllll}\text { Si } & 2.060923 & 4.664532 & -3.851077\end{array}$
$\begin{array}{lllll}\mathrm{Si} & 3.217395 & 5.769165 & -2.322262\end{array}$
$\begin{array}{llll}\text { Si } & -4.466071 & -4.597355 & 0.293085\end{array}$
Si $-3.294876-3.428344-1.260659$
$\begin{array}{lllll}\text { Si } & -1.961450 & -2.097291 & 0.116038\end{array}$
Si $-0.630398 \quad-0.750266-1.177074$
$\begin{array}{llll}\mathrm{Si} & 0.708103 & 0.552353 & 0.148339\end{array}$
$\begin{array}{llll}\text { Si } & 2.037440 & 1.892228 & -1.143907\end{array}$
$\begin{array}{llll}\mathrm{Si} & 3.366315 & 3.234065 & 0.223774\end{array}$
$\begin{array}{lllll}\text { Si } & 4.521079 & 4.416282 & -1.338300\end{array}$
$\begin{array}{lllll}\text { Si } & -3.136161 & -5.964217 & 1.282355\end{array}$
$\begin{array}{lllll}\text { Si } & -2.007572 & -4.838292 & 2.824763\end{array}$
$\begin{array}{lllll}\text { Si } & -0.619717 & -3.427165 & 1.422129\end{array}$
$\begin{array}{lllll}\mathrm{Si} & 0.705574 & -2.115704 & 2.754102\end{array}$
$\begin{array}{llll}\mathrm{Si} & 2.041474 & -0.771219 & 1.460111\end{array}$
$\begin{array}{lllll}\text { Si } & 4.679719 & 1.856551 & 1.484282\end{array}$
$\begin{array}{llll}\mathrm{Si} & 6.174708 & 2.966903 & 2.647921\end{array}$ Si -0.504348 -6.030084 3.972243 $\begin{array}{lllll}\mathrm{Si} & 0.536086 & -4.672052 & 5.306239\end{array}$ $\begin{array}{lllll}\mathrm{Si} & 2.059532 & -3.533527 & 4.148767\end{array}$ $\begin{array}{lllll}\mathrm{Si} & 3.216791 & -2.038610 & 5.306455\end{array}$ $\begin{array}{lllll}\text { Si } & 4.538908 & -1.012206 & 3.971434\end{array}$ $\begin{array}{lllll}\mathrm{Si} & -7.005999 & 0.587481 & -2.518572\end{array}$ $\begin{array}{lllll}\mathrm{Si} & -5.844081 & 1.770575 & -3.655782\end{array}$ Si $\quad-4.6778323 .308045-2.546331$
Si $-3.156209 \quad 4.443784-3.640638$ $\begin{array}{lllll}\mathrm{Si} & -1.981797 & 5.974190 & -2.455364\end{array}$ $\begin{array}{lllll}\text { Si } & -0.707860 & 6.969905 & -3.824891\end{array}$ Si $-7.010102-2.119776 \quad 0.129457$ Si $-6.038010 \quad-0.714751-1.137045$ $\begin{array}{lllll}\text { Si } & -4.620634 & 0.562391 & 0.172082\end{array}$ $\begin{array}{lllll}\mathrm{Si} & -3.285914 & 1.898854 & -1.136869\end{array}$ Si $-1.963511 \quad 3.208809 \quad 0.172574$ Si $\quad-0.592760 \quad 4.550092-1.105565$ $\begin{array}{lllll}\mathrm{Si} & 0.777289 & 5.947553 & 0.228674\end{array}$ $\begin{array}{llll}\text { Si } & 2.140772 & 6.940120 & -1.056487\end{array}$ $\begin{array}{lllll}\text { Si } & -5.834239 & -3.290770 & 1.292878\end{array}$ Si $-4.694993-2.194075 \quad 2.861360$ $\begin{array}{lllll}\text { Si } & -3.281469 & -0.783919 & 1.471319\end{array}$ $\begin{array}{llll}\mathrm{Si} & -1.953402 & 0.511232 & 2.804394\end{array}$ $\begin{array}{llll}\mathrm{Si} & -0.649430 & 1.831746 & 1.465000\end{array}$ $\begin{array}{lllll}\mathrm{Si} & 0.628028 & 3.129134 & 2.783623\end{array}$ $\begin{array}{llll}\mathrm{Si} & 2.027167 & 4.528950 & 1.562300\end{array}$ $\begin{array}{lllll}\mathrm{Si} & 3.073579 & 5.076465 & 3.369434\end{array}$ $\begin{array}{lllll}\text { Si } & -3.116279 & -3.290187 & 3.916369\end{array}$ $\begin{array}{lllll}\mathrm{Si} & -1.937238 & -2.151627 & 5.54168\end{array}$ $\begin{array}{lllll}\mathrm{Si} & -0.585623 & -0.796471 & 4.125309\end{array}$ $\begin{array}{llll}\mathrm{Si} & 0.798885 & 0.516159 & 5.535786\end{array}$ $\begin{array}{llll}\text { Si } & 2.044328 & 1.878384 & 4.13998\end{array}$ $\begin{array}{lllll}\text { Si } & 3.107722 & 3.684763 & 4.821853\end{array}$ $\begin{array}{llll}\mathrm{Si} & -0.671297 & -3.565171 & 6.501508\end{array}$ Si 2.155088 -0.798864 6.503362 $\begin{array}{lllll}\text { Si } & -6.934684 & 3.358119 & 0.276096\end{array}$ $\begin{array}{lllll}\text { Si } & -5.858206 & 4.469463 & -1.031697\end{array}$ $\begin{array}{llll}\mathrm{Si} & -4.495283 & 5.772132 & 0.061094\end{array}$ $\begin{array}{lllll}\mathrm{Si} & -3.337132 & 6.936265 & -1.124295\end{array}$ $\begin{array}{llll}\text { Si } & -6.904581 & 0.638891 & 2.983311\end{array}$ $\begin{array}{llll}\text { Si } & -6.075712 & 1.879151 & 1.507096\end{array}$ $\begin{array}{llll}\text { Si } & -4.519332 & 3.028686 & 2.706938\end{array}$ $\begin{array}{llll}\text { Si } & -3.401467 & 4.543106 & 1.578410\end{array}$ $\begin{array}{lllll}\text { Si } & -1.887026 & 5.703458 & 2.734327\end{array}$ $\begin{array}{llll}\text { Si } & -0.641531 & 6.799552 & 1.558286\end{array}$ $\begin{array}{lllll}\text { Si } & -5.851768 & -0.710301 & 4.042364\end{array}$ Si $-4.469948 \quad 0.313165 \quad 5.385024$ $\begin{array}{lllll}\text { Si } & -3.391214 & 1.866121 & 4.197574\end{array}$ $\begin{array}{llll}\mathrm{Si} & -1.881553 & 3.015032 & 5.372508\end{array}$ $\begin{array}{lllll}\mathrm{Si} & -0.880805 & 4.309127 & 4.00452\end{array}$ $\begin{array}{lllll}\mathrm{Si} & -3.320976 & -0.878731 & 6.541677\end{array}$ $\begin{array}{lllll}\text { Si } & -0.584256 & 1.855372 & 6.432346\end{array}$ $\begin{array}{lllll}\text { Si } & 3.370890 & 0.573388 & 2.825023\end{array}$

|  | $\mathrm{Si}_{172}(1.8 \mathrm{~nm})$ |  |  | Si | 0.547849 | 3.122872 | -2.259197 |
| :--- | ---: | :---: | :---: | :---: | ---: | :---: | :---: | :---: |
|  | X | y | Z | Si | 1.925727 | 4.499554 | -3.627445 |
| Si | -2.036475 | -4.751775 | -7.467075 | Si | 3.098365 | 5.619327 | -2.140341 |
| Si | -0.670591 | -3.641489 | -8.484146 | Si | -4.020648 | -6.019859 | -1.115213 |
| Si | 0.495405 | -2.132321 | -7.664170 | Si | -3.441205 | -3.849068 | 0.436124 |
| Si | 1.308303 | -0.320471 | -8.644490 | Si | -2.105389 | -2.177816 | -0.941857 |
| Si | 2.963017 | 0.347037 | -7.494293 | Si | -0.784433 | -0.862595 | -0.950097 |
| Si | -2.066255 | -7.407551 | -4.811975 | Si | 0.553177 | 0.462818 | 0.362781 |
| Si | -1.029138 | -6.065548 | -6.112257 | Si | 1.880160 | 1.784746 | -0.953636 |
| Si | 0.516641 | -4.886483 | -4.983504 | Si | 3.207186 | 3.091522 | 0.376730 |
| Si | 1.651448 | -3.384715 | -6.157868 | Si | 4.600440 | 4.467445 | -1.034478 |
| Si | 3.149054 | -2.277703 | -5.017074 | Si | 5.763116 | 5.622363 | 0.511637 |
| Si | 4.281353 | -0.739227 | -6.196005 | Si | -3.286340 | -5.970398 | 1.533327 |
| Si | 5.631570 | 0.266206 | -4.870118 | Si | -2.167727 | -4.862344 | 3.068420 |
| Si | -0.823807 | -8.484034 | -3.609976 | Si | -0.777619 | -3.495933 | 1.677301 |
| Si | 0.578294 | -7.608369 | -2.267160 | Si | 0.560083 | -2.177752 | 2.990561 |
| Si | 1.669562 | -5.992005 | -3.469198 | Si | 1.889953 | -0.858413 | 1.671770 |
| Si | 3.208424 | -4.873880 | -2.352218 | Si | 3.237367 | 0.464493 | 2.979819 |
| Si | 4.323915 | -3.408935 | -3.544198 | Si | 4.561646 | 1.784421 | 1.661950 |
| Si | 5.884371 | -2.314206 | -2.432140 | Si | 3.042823 | -7.379527 | 0.224846 |
| Si | 6.995853 | -0.795324 | -3.604772 | Si | 4.366631 | -6.017963 | -0.795045 |

$\begin{array}{llll}\text { Si } & 5.692257 & -4.721886 & 0.194231\end{array}$ $\begin{array}{lllll}\mathrm{Si} & 7.006451 & -3.433705 & -0.884533\end{array}$ $\begin{array}{llll}\mathrm{Si} & 8.032578 & -2.404375 & 0.525679\end{array}$ Si -4.643119 -2.031799 -7.495482 $\begin{array}{lllll}\mathrm{Si} & -3.494925 & -0.845064 & -8.670204\end{array}$ $\begin{array}{lllll}\mathrm{Si} & -2.125334 & 0.430471 & -7.665666\end{array}$
$\begin{array}{lllll}\text { Si } & -0.278763 & 1.261097 & -8.585672\end{array}$
$\begin{array}{lllll}\mathrm{Si} & 0.399583 & 2.942331 & -7.480195\end{array}$
Si $-4.678967-4.676908-4.766973$
Si $-3.547954-3.593166-6.321062$
Si $-2.147044-2.217768-4.886193$
$\begin{array}{lllll}\mathrm{Si} & -0.833467 & -0.887847 & -6.153112\end{array}$
$\begin{array}{lllll}\mathrm{Si} & 0.526256 & 0.466701 & -4.858869\end{array}$
Si $1.9656231 .905909-6.258389$
$\begin{array}{llll}\mathrm{Si} & 3.090312 & 2.989387 & -4.736332\end{array}$
Si -4.673920 -7.382995 -2.104046
Si $-3.568602-6.233169-3.664588$
Si $\quad-2.139114-4.848481-2.265687$
Si $-0.820405 \quad-3.530352-3.579657$
$\mathrm{Si} \quad 0.517324-2.199444-2.274287$
Si $1.845186 \quad-0.873351-3.588779$
$\begin{array}{lllll}\text { Si } & 3.186777 & 0.444441 & -2.277432\end{array}$
$\begin{array}{lllll}\text { Si } & 4.565261 & 1.798820 & -3.640811\end{array}$
$\begin{array}{lllll}\mathrm{Si} & 5.714991 & 2.962356 & -2.168162\end{array}$
$\begin{array}{lllll}\mathrm{Si} & -3.560000 & -8.580289 & -0.924777\end{array}$
$\begin{array}{llll}\text { Si } & -2.189842 & -7.596342 & 0.371039\end{array}$
$\begin{array}{lllll}\mathrm{Si} & -0.809533 & -6.182885 & -0.950421\end{array}$
$\begin{array}{lllll}\text { Si } & 0.517755 & -4.836176 & 0.371844\end{array}$
$\begin{array}{lllll}\mathrm{Si} & 1.842103 & -3.515827 & -0.956964\end{array}$
$\begin{array}{lllll}\mathrm{Si} & 3.179647 & -2.166927 & 0.322517\end{array}$
Si $4.532522-0.880294-0.985434$
$\begin{array}{llll}\mathrm{Si} & 5.885968 & 0.454418 & 0.323796\end{array}$
$\begin{array}{lllll}\mathrm{Si} & 7.304670 & 1.762466 & -1.024822\end{array}$
$\begin{array}{lllll}\text { Si } & 8.173191 & 3.194958 & 0.244150\end{array}$
$\begin{array}{lllll}\mathrm{Si} & -0.775969 & -8.472032 & 1.711307\end{array}$
$\begin{array}{lllll}\mathrm{Si} & 0.425283 & -7.374538 & 2.943444\end{array}$
$\begin{array}{lllll}\text { Si } & 1.953584 & -6.212923 & 1.766862\end{array}$
$\begin{array}{lllll}\text { Si } & 3.100550 & -4.706803 & 2.861985\end{array}$
$\begin{array}{lllll}\text { Si } & 4.582316 & -3.573651 & 1.747037\end{array}$
$\begin{array}{lllll}\mathrm{Si} & 5.744700 & -2.072617 & 2.835437\end{array}$
$\begin{array}{lllll}\mathrm{Si} & 7.305067 & -0.840675 & 1.691479\end{array}$
$\begin{array}{llll}\mathrm{Si} & 8.297309 & 0.463800 & 3.042976\end{array}$
Si -7.334940 -2.032761 -4.799481
Si $-5.998670-1.043480-6.138146$
Si $-4.805196 \quad 0.500843-4.966902$
Si $-3.2446331 .609917-6.116585$
$\begin{array}{lllll}\mathrm{Si} & -2.155173 & 3.133180 & -4.979351\end{array}$
Si $-0.638940 \quad 4.249624-6.168157$
$\begin{array}{lllll}\text { Si } & 0.370490 & 5.600753 & -4.841873\end{array}$
Si $-7.339477-4.702854-2.139090$
Si $-6.191448-3.539184-3.634383$
Si $\quad-4.769780 \quad-2.175020 \quad-2.253780$
$\begin{array}{lllll}\text { Si } & -3.446740 & -0.868412 & -3.578875\end{array}$
Si $\quad-2.114743 \quad 0.456399-2.258056$
$\begin{array}{lllll}\mathrm{Si} & -0.783861 & 1.781470 & -3.561381\end{array}$
Si $\quad 0.547849 \quad 3.122872-2.259197$
. 25727 4.499554-3.627445
3.098365-5.619327-2.140341

Si $-4.840322-4.849068 \quad 0.436124$
Si $-3.441205-3.504740 \quad-0.941857$
Si $-2.105389-2.177816 \quad 0.367420$
$\begin{array}{llll}1 & -0.784433 & -0.862595 & -0.95009\end{array}$
$\begin{array}{llll}\mathrm{Si} & 0.553177 & 0.462818 & 0.362781\end{array}$
$\begin{array}{lllll}\text { Si } & 1.880160 & 1.784746 & -0.953636\end{array}$
$\begin{array}{llll} & 3.207186 & 3.091522 & 0.376730\end{array}$
$\begin{array}{llll}\text { Si } & 4.600440 & 4.467445 & -1.034478\end{array}$
$\begin{array}{llll}\mathrm{Si} & 5.763116 & 5.622363 & 0.511637\end{array}$
1.53332
$\begin{array}{llll}\text { Si } & -2.167727 & -4.862344 & 3.068420\end{array}$
Si $-0.777619 \quad-3.4959331 .677301$
$\begin{array}{llll}0.560083 & -2.177752 & 2.990561\end{array}$
$\begin{array}{lllll}\text { Si } & 1.889953 & -0.858413 & 1.671770\end{array}$

Si $4.561646 \quad 1.784421 \quad 1.661950$
$\begin{array}{lllll}\mathrm{Si} & 4.366631 & -6.017963 & -0.795045\end{array}$
$\begin{array}{llll}\mathrm{Si} & 5.969428 & 3.195997 & 3.070029\end{array}$ $\begin{array}{llll}\text { Si } & 7.104168 & 4.329326 & 1.543217\end{array}$ $\begin{array}{lllll}\text { Si } & -0.626325 & -6.001355 & 4.204424\end{array}$ Si $\quad 0.406331 \quad-4.689879 \quad 5.526918$ $\begin{array}{lllll}\mathrm{Si} & 1.949470 & -3.554651 & 4.336823\end{array}$ $\begin{array}{llrr}\mathrm{Si} & 3.108616 & -2.033228 & 5.438925\end{array}$ Si $4.637284-0.8995364 .326235$ $\begin{array}{lllll}\text { Si } & 5.795715 & 0.649525 & 5.519580\end{array}$ $\begin{array}{llll}\text { Si } & 7.133923 & 1.665632 & 4.187630\end{array}$ Si -8.465848 -0.759354 -3.683834 $\begin{array}{lllll}\mathrm{Si} & -7.487546 & 0.515517 & -2.294610\end{array}$ $\begin{array}{lllll}\mathrm{Si} & -5.928287 & 1.644460 & -3.463700\end{array}$ Si $-4.801502 \quad 3.145939-2.326471$ $\begin{array}{lllll}\mathrm{Si} & -3.268006 & 4.265037 & -3.454565\end{array}$ $\begin{array}{llll}\mathrm{Si} & -2.196106 & 5.836639 & -2.360569\end{array}$ $\begin{array}{llll}\mathrm{Si} & -0.661944 & 6.943690 & -3.525999\end{array}$ $\mathrm{Si} \quad 0.553369$ 8.138344 -2.421191 $\begin{array}{lllll}\mathrm{Si} & -8.504605 & -3.603161 & -0.886664\end{array}$ $\begin{array}{llll}\mathrm{Si} & -7.516158 & -2.218903 & 0.381959\end{array}$ Si -6.105816 -0.857967 -0.927960 $\begin{array}{llll}\text { Si } & -4.744697 & 0.474533 & 0.390583\end{array}$ $\begin{array}{lllll}\text { Si } & -3.421874 & 1.783106 & -0.937401\end{array}$ $\begin{array}{llll}\text { Si } & -2.074382 & 3.103863 & 0.371717\end{array}$ $\begin{array}{lllll}\mathrm{Si} & -0.769197 & 4.441487 & -0.946193\end{array}$ $\begin{array}{lllll}\text { Si } & 0.572158 & 5.787702 & 0.362715\end{array}$ $\begin{array}{lllll}\text { Si } & 1.858838 & 7.184105 & -1.039051\end{array}$ $\begin{array}{llll}\mathrm{Si} & 3.342873 & 8.002863 & 0.185436\end{array}$ $\begin{array}{lllll}\mathrm{Si} & -5.944199 & -3.334153 & 1.588793\end{array}$ $\begin{array}{lllll}\text { Si } & -4.769805 & -2.197277 & 3.067146\end{array}$ $\begin{array}{lllll}\text { Si } & -3.398526 & -0.841429 & 1.692842\end{array}$ $\begin{array}{lllll}\text { Si } & -2.057666 & 0.469217 & 3.003107\end{array}$ $\begin{array}{lllll}\text { Si } & -0.746968 & 1.795719 & 1.688362\end{array}$ $\begin{array}{lllll}\text { Si } & 0.592761 & 3.120008 & 2.997755\end{array}$ $\begin{array}{lllll}\text { Si } & 1.909677 & 4.446492 & 1.677352\end{array}$ $\begin{array}{llll}\mathrm{Si} & 3.333303 & 5.841680 & 3.074859\end{array}$ $\begin{array}{lllll}\text { Si } & 4.459508 & 6.965096 & 1.529822\end{array}$ $\begin{array}{lllll}\mathrm{Si} & -3.259762 & -3.333078 & 4.213969\end{array}$ $\begin{array}{lllll}\text { Si } & -2.120924 & -2.220044 & 5.750816\end{array}$ $\begin{array}{llll}\mathrm{Si} & -0.741742 & -0.851094 & 4.323709\end{array}$ $\begin{array}{lllll}\mathrm{Si} & 0.598277 & 0.465222 & 5.662789\end{array}$ $\begin{array}{lllll}\text { Si } & 1.912423 & 1.780282 & 4.307099\end{array}$ $\begin{array}{llll}\text { Si } & 3.287594 & 3.144181 & 5.719809\end{array}$ $\begin{array}{llll}\text { Si } & 4.423169 & 4.282819 & 4.196894\end{array}$ $\begin{array}{lllll}\mathrm{Si} & -0.579713 & -3.355139 & 6.879925\end{array}$ $\begin{array}{lllll}\mathrm{Si} & 0.673594 & -2.267442 & 8.051584\end{array}$ $\begin{array}{lllll}\mathrm{Si} & 1.951996 & -0.877612 & 7.075529\end{array}$ $\begin{array}{llll}\mathrm{Si} & 3.351461 & 0.395216 & 8.048951\end{array}$ $\begin{array}{lllll}\text { Si } & 4.437122 & 1.626706 & 6.874732\end{array}$ $\begin{array}{lllll}\text { Si } & -8.444189 & 1.886953 & -0.973681\end{array}$ $\begin{array}{llll}\text { Si } & -7.281090 & 3.007770 & 0.258410\end{array}$ $\begin{array}{lllll}\mathrm{Si} & -5.941597 & 4.321878 & -0.777189\end{array}$ $\begin{array}{lllll}\mathrm{Si} & -4.600940 & 5.635880 & 0.221462\end{array}$ $\begin{array}{lllll}\mathrm{Si} & -3.305179 & 6.982180 & -0.826179\end{array}$ $\begin{array}{lllll}\text { Si } & -2.174770 & 8.019347 & 0.492940\end{array}$ $\begin{array}{lllll}\mathrm{Si} & -8.313251 & -0.947021 & 1.842511\end{array}$ $\begin{array}{llll}\mathrm{Si} & -7.262404 & 0.367482 & 2.957642\end{array}$ $\begin{array}{llll}\text { Si } & -6.137119 & 1.887460 & 1.808572\end{array}$ $\begin{array}{lllll}\mathrm{Si} & -4.558940 & 2.994877 & 2.865662\end{array}$ $\begin{array}{lllll}\text { Si } & -3.418951 & 4.493603 & 1.734082\end{array}$ $\begin{array}{lllll}\text { Si } & -1.917116 & 5.644291 & 2.866352\end{array}$ $\begin{array}{llll}\text { Si } & -0.708761 & 7.207407 & 1.747207\end{array}$ $\begin{array}{llll}\text { Si } & 0.612718 & 8.180142 & 3.100813\end{array}$ $\begin{array}{lllll}\mathrm{Si} & -5.897406 & -0.675381 & 4.239998\end{array}$ $\begin{array}{lllll}\mathrm{Si} & -4.574539 & 0.310794 & 5.569667\end{array}$ $\begin{array}{llll}\mathrm{Si} & -3.450278 & 1.890651 & 4.417019\end{array}$ $\begin{array}{lllll}\text { Si } & -1.864420 & 2.989060 & 5.466079\end{array}$ $\begin{array}{llll}\mathrm{Si} & -0.758331 & 4.522771 & 4.361287\end{array}$ $\begin{array}{llll}\mathrm{Si} & 0.810475 & 5.662929 & 5.546021\end{array}$ $\begin{array}{lllll}\text { Si } & 1.827596 & 7.005796 & 4.217122\end{array}$ $\begin{array}{lllll}\text { Si } & -3.238184 & -0.682430 & 6.883945\end{array}$ $\begin{array}{llll}\mathrm{Si} & -2.130339 & 0.561592 & 8.057658\end{array}$ $\begin{array}{llll}\text { Si } & -0.716156 & 1.817538 & 7.094815\end{array}$ $\begin{array}{lllll}\text { Si } & 0.569167 & 3.203697 & 8.066685\end{array}$ $\begin{array}{lllll}\text { Si } & 1.790440 & 4.295695 & 6.888682\end{array}$ $\begin{array}{llll}\text { Si } & 8.078192 & 0.552396 & -2.555247\end{array}$ $\begin{array}{llllll}\mathrm{Si} & 1.940991 & -8.577776 & -0.956471\end{array}$

## B. 5 Silicon Quantum Dots



Figure B.5: Equilibrium structures of Si quantum dots of sizes 1.6, 1.8, 2.0 and 3.0 nm . The structures were obtained from geometry optimization using PM7 as implemented in MOPAC.

| $\mathrm{Si}_{244}(2.0 \mathrm{~nm})$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | X | y | Z | Si | -3.337213 | -8.337787 | -0.634911 | Si | 4.464505 | -3.511740 | 7.087568 | Si | -3.288321 | -0.601236 | 7.322160 |
| Si | 3.394125 | -4.751775 | $-7.467075$ | Si | -1.986418 | -7.323505 | 0.668382 | Si | -7.134309 | 3.569380 | -4.412760 | Si | -1.975280 | 0.669835 | 8.374955 |
| Si | 4.610993 | -3.588989 | $-8.526417$ | Si | -0.657915 | -5.941053 | -0.671968 | Si | -5.811985 | 4.504052 | -5.835625 | Si | -0.695230 | 1.957042 | 7.304770 |
| Si | 5.767047 | -2.239570 | $-7.250359$ | Si | 0.667305 | -4.629956 | 0.654182 | Si | -4.677361 | 6.092973 | $-4.759054$ | Si | 0.665284 | 3.291195 | 8.246139 |
| Si | 3.522360 | -7.156096 | $-4.419005$ | Si | 1.976886 | -3.305283 | -0.673197 | Si | -3.113254 | 7.252395 | $-5.861543$ | Si | 1.858072 | 4.530280 | 7.154902 |
| Si | 4.442187 | -5.876150 | -5.925856 | Si | 3.283410 | -1.975522 | 0.631247 | Si | -1.929043 | 8.398345 | $-4.718557$ | Si | -8.416619 | 4.654522 | 1.912477 |
| Si | 6.039052 | -4.689372 | -4.771920 | Si | 4.623759 | -0.676520 | -0.693576 | Si | -8.335013 | $-0.583790$ | -3.362819 | Si | -7.265484 | 5.821627 | 3.106970 |
| $\mathrm{Si}_{\mathrm{Si}}$ | 7.193282 | -3.151004 | $-5.905940$ | Si | 5.944189 | 0.649356 | 0.621989 | Si | -7.322088 | 0.675473 | -1.975685 | Si | -5.822951 | 7.192099 | 2.200576 |
| Si | 8.342157 | -1.946984 | -4.755317 | Si | 7.348217 | 1.941405 | -0.732390 | Si | -5.935214 | 2.005070 | -3.293810 | Si | -4.716979 | 8.465882 | 3.416373 |
| Si | 4.684050 | -8.480267 | -3.444580 | Si | 8.249766 | 3.323422 | 0.586644 | Si | -4.580607 | 3.330904 | -1.971499 | Si | -8.235224 | 1.840538 | 4.747394 |
| Si | 5.767706 | -7.151374 | $-2.283137$ | Si | -0.684299 | -8.347707 | 1.977050 | Si | -3.287051 | 4.673343 | $-3.309656$ | Si | -7.234390 | 3.177517 | 5.861969 |
| Si | 7.215084 | -5.789450 | $-3.208873$ | Si | 0.616169 | -7.344787 | 3.300573 | Si | -1.975051 | 5.982080 | -1.986858 | Si | -6.149280 | 4.727806 | 4.673057 |
| Si | 8.333513 | -4.709608 | -1.944991 | Si | 1.976192 | -5.978607 | 1.960004 | Si | -0.655766 | 7.358499 | $-3.330882$ | Si | $-4.525343$ | 5.798090 | 5.785363 |
| Si | -1.868748 | -4.529765 | $-7.169138$ | Si | 3.263315 | -4.669203 | 3.278768 | Si | 0.697590 | 8.287066 | -1.980832 | Si | -3.530021 | 7.125718 | 4.446094 |
| Si Si | -0.574322 0.713867 | -3.392716 | -8.222697 -7.356315 | Si | 4.546036 | -3.314570 | 1.937345 | Si | -8.331609 | -3.375242 | $-0.613584$ | Si | -5.685411 | 2.292738 | 7.164281 |
| Si Si | 0.713867 | -1.963196 | $-7.356315$ | Si | 5.914535 | -2.002948 | 3.251478 | Si | -7.316024 | -2.010427 | 0.655636 | Si | -4.697301 | 3.398298 | 8.504768 |
| $\stackrel{\mathrm{Si}}{\mathrm{Si}}$ | 2.001678 | -0.645219 | $-8.385180$ | Si | 7.320553 | -0.684905 | 1.959150 | Si | -5.932993 | $-0.665496$ | -0.655490 | Si | -3.601948 | 4.344680 | 7.113516 |
| Si | 3.312677 | 0.656301 | $-7.366461$ | Si | 8.320177 | 0.601135 | 3.338399 | Si | -4.628992 | 0.671129 | 0.662808 | Si | -0.714732 | 4.648812 | 4.647637 |
| Si | 4.711505 | 1.889466 | -8.399837 | Si | 1.886598 | -8.386959 | 4.668803 | Si | -3.298357 | 1.988889 | -0.662029 | Si | 0.817289 | 5.804533 | 5.794627 |
| Si | 5.831468 | 3.120456 | -7.266537 | Si | 3.094618 | -7.268402 | 5.835498 | Si | -1.986002 | 3.316030 | 0.649617 | Si | 1.882893 | 7.153816 | 4.519632 |
| Si | -1.872494 | -7.189245 | -4.522494 | Si | 4.663124 | -6.149503 | 4.727234 | Si | -0.667996 | 4.636525 | $-0.669477$ | Si | -5.875337 | -3.154359 | 7.283070 |
| Si | -0.842309 | -5.838163 | -5.829392 | Si | 5.758581 | -4.525033 | 5.777220 | Si | 0.655442 | 5.939478 | 0.663168 | Si | -4.697139 | -1.859942 | 8.280191 |
| Si <br> Si | 0.701509 | -4.673218 | $-4.674128$ | Si | 7.084638 | -3.667988 | 4.240153 | Si | 1.997010 | 7.298049 | $-0.646903$ | Si | 5.801444 | 0.835121 | 5.799047 |
| $\mathrm{Si}_{\mathrm{Si}}$ | 1.989806 | -3.416122 | -6.057828 | Si | -4.545824 | 3.433484 | -7.183497 | Si | 3.393501 | 8.398835 | 0.579354 | Si | 7.166720 | 1.823694 | 4.477711 |
| Si Si | 3.294897 | -2.034529 | -4.683499 | Si | -3.327790 | 4.724179 | -8.263550 | Si | -7.146994 | -4.520434 | 3.478189 | Si | 2.161455 | -5.818405 | 7.173969 |
| $\stackrel{\mathrm{Si}}{\mathrm{Si}}$ | 4.637862 | $-0.686733$ | $-6.024963$ | Si | -2.058491 | 5.914674 | -7.220337 | Si | -5.949600 | -3.324111 | 1.993242 | Si | 3.433661 | -4.703127 | 8.419013 |
| Si | 5.959761 | 0.632443 | -4.679144 | Si | -7.168219 | $-1.820380$ | -4.481311 | Si | -4.622867 | -1.953890 | 3.330182 | Si | -3.319017 | 1.976220 | 4.569812 |
| $\mathrm{Si}_{\mathrm{Si}}$ | 7.182765 | 2.208537 | $-5.814142$ | Si | -5.812700 | -0.841173 | -5.799000 | Si | -3.311761 | -0.656286 | 1.986227 | Si | -1.994135 | 3.300270 | 5.936949 |
| Si | 8.470619 | 3.418924 | $-4.714983$ | Si | -4.647278 | 0.707480 | -4.632040 | Si | -1.995417 | 0.657041 | 3.281028 |  |  |  |  |
| Si | -0.595464 | -8.258030 | -3.351213 | Si | -3.319116 | 1.973412 | $-5.939917$ | Si | -0.670088 | 1.984128 | 1.965888 |  |  |  |  |
| Si | 0.692928 | -7.337127 | $-1.960780$ | Si | -1.968782 | 3.341970 | -4.623011 | Si | 0.639366 | 3.308964 | 3.288073 |  |  |  |  |
| Si | 1.985088 | -5.963534 | $-3.311756$ | Si | -0.617511 | 4.678653 | -5.953129 | Si | 1.983392 | 4.603540 | 1.971653 |  |  |  |  |
| Si | 3.302556 | -4.609659 | -1.975374 | Si | 0.680184 | 5.983298 | -4.635233 | Si | 3.393507 | 5.994654 | 3.366575 |  |  |  |  |
| Si | 4.617778 | -3.319047 | -3.345404 | Si | 2.266926 | 7.175719 | $-5.778278$ | Si | 4.522203 | 7.119394 | 1.819005 |  |  |  |  |
| $\mathrm{Si}_{\mathrm{Si}}$ | 5.950710 | -2.003134 | -2.019860 | Si | 3.436713 | 8.489707 | $-4.703224$ | Si | -5.833612 | -5.793223 | 4.497060 |  |  |  |  |
| Si | 7.332269 | -0.722489 | -3.354154 | Si | -7.167161 | -4.488424 | $-1.838190$ | Si | -4.724583 | -4.689469 | 6.114904 |  |  |  |  |
| Si | 8.368759 | 0.621106 | $-2.049507$ | Si | -6.005106 | -3.331566 | -3.345478 | Si | -3.320046 | -3.281916 | 4.661773 |  |  |  |  |
| Si | 2.004022 | -8.381786 | $-0.683314$ | Si | -4.592771 | -1.961256 | -1.981127 | Si | -1.992492 | -1.964636 | 5.983158 |  |  |  |  |
| Si | 3.311943 | -7.364398 | 0.623781 | Si | -3.279571 | -0.640530 | -3.311725 | Si | -0.666676 | $-0.653774$ | 4.604345 |  |  |  |  |
| Si | 4.664899 | -5.982299 | -0.661008 | Si | -1.979920 | 0.676614 | -1.997381 | Si | 0.655545 | 0.647916 | 5.917851 |  |  |  |  |
| Si | 5.946299 | -4.642594 | 0.657789 | Si | -0.651120 | 1.995201 | -3.323916 | Si | 1.965813 | 1.986463 | 4.599089 |  |  |  |  |
| Si | 7.328624 | -3.326833 | $-0.661381$ | Si | 0.644035 | 3.304184 | -1.994891 | Si | 3.387613 | 3.367131 | 5.991481 |  |  |  |  |
| Si | 8.345176 | -1.987538 | 0.631633 | Si | 1.968104 | 4.591540 | -3.313743 | Si | 4.524703 | 4.474255 | 4.504279 |  |  |  |  |
| Si | 4.688962 | -8.287551 | 1.938599 | Si | 3.300488 | 5.932897 | $-1.990466$ | Si | -3.145981 | -5.794114 | 7.241917 |  |  |  |  |
| Si | 5.860282 | -7.225098 | 3.186502 | Si | 4.403509 | 7.116189 | $-3.555836$ | Si | -1.921614 | -4.659978 | 8.369386 |  |  |  |  |
| Si Si | 7.122737 | -5.606520 | 2.348022 | Si | -5.819495 | -5.821614 | -0.822639 | Si | -0.650210 | -3.295742 | 7.334412 |  |  |  |  |
| Si <br> Si | 8.609342 -4.473383 | -4.690535 | 3.391334 | Si | -4.622726 | -4.650488 | 0.709935 | Si | 0.674639 | -1.985275 | 8.338295 |  |  |  |  |
| Si | -4.473383 -3.364298 | -1.818158 | -7.184581 | Si | -3.285998 | -3.297332 | -0.648403 | Si | 1.992226 | -0.699644 | 7.298502 |  |  |  |  |
| Si | -3.364298 -1.958201 | -0.638473 | -8.331130 | Si | -1.983468 | -1.972046 | 0.661183 | Si | 3.340629 | 0.637653 | 8.310382 |  |  |  |  |
| Si | -0.714294 | 0.680011 2.035763 | -7.328716 | Si | -0.656241 | -0.660956 | -0.681285 | Si | 4.484457 | 1.832426 | 7.150307 |  |  |  |  |
| Si | 0.647190 | 2.035763 3.318825 | -8.347812 -7.344501 | Si | 0.661430 | 0.661431 | 0.641841 | Si | -8.491055 | 4.747565 | -3.385142 |  |  |  |  |
| Si | 1.883429 | 4.716753 | -8.383155 | Si | 1.974978 | 1.972922 | -0.691020 | Si | -7.145434 | 5.756245 | -2.240590 |  |  |  |  |
| Si | 3.130183 | 5.819384 | -7.258527 | Si | 3.296479 | 3.290467 | 0.628481 | Si | -5.841196 | 7.225610 | -3.231945 |  |  |  |  |
| Si | -4.490444 | -4.457379 | $-4.466490$ | Si | 4.648945 | 4.640252 | $-0.721122$ | Si | -4.684355 | 8.287418 | -1.949831 |  |  |  |  |
| Si | -3.374456 | -3.368630 | -6.014334 | Si | 5.811624 | 5.810418 | 0.807380 | Si | -8.337475 | 1.995676 | $-0.661839$ |  |  |  |  |
| Si | -1.957836 | -1.970743 | -4.621281 | Si | -4.382611 | -7.094197 | 3.550180 | Si | -7.363708 | 3.347284 | 0.638925 |  |  |  |  |
| Si | -0.642193 | -0.661310 | -5.950336 | Si | -3.293828 | -5.938713 | 1.988128 | Si | -5.959902 | 4.655119 | $-0.667281$ |  |  |  |  |
| Si | 0.665382 | 0.662752 | -4.643737 | Si | -1.971979 | -4.571947 | 3.315790 | Si | -4.649116 | 5.982518 | 0.640366 |  |  |  |  |
| Si | 1.984272 | 1.983697 | -5.980932 | Si | -0.654006 | -3.296782 | 1.970526 | Si | -3.293346 | 7.360114 | -0.632673 |  |  |  |  |
| Si | 3.296979 | 3.281405 | -4.658039 | Si | 0.655969 | -1.981275 | 3.294470 | Si | -1.979893 | 8.395898 | 0.660600 |  |  |  |  |
| Si | 4.677144 | 4.662782 | -6.139660 | Si | 1.970957 | -0.660573 | 1.963490 | Si | -8.362831 | -0.705914 | 1.962866 |  |  |  |  |
| Si | 5.794732 | 5.777364 | -4.535482 | Si Si | 3.295729 4.618082 | 0.657366 1.972025 | 3.279432 1.941410 | Si Si | -7.363752 -5.980524 | 0.617602 1.981901 | 3.286567 1.980540 |  |  |  |  |
| Si | -4.490147 | -7.172338 | -1.834015 | Si | 6.005346 | 3.349194 | 3.368845 | Si | -4.664044 | 3.305139 | 3.281848 |  |  |  |  |
| Si | -3.375693 | -6.015082 | -3.380759 | Si | 7.166411 | 4.504063 | 1.852664 | Si | -3.320595 | 4.613802 | 1.959168 |  |  |  |  |
| Si | -1.960630 | -4.611245 | -1.987722 | Si | -3.445583 | -8.481153 | 4.722625 | Si | -1.996004 | 5.951904 | 3.314624 |  |  |  |  |
| Si | -0.634106 | -3.309288 | -3.322593 | Si | -2.273968 | -7.149856 | 5.764124 | Si | -0.702738 | 7.328850 | 1.963947 |  |  |  |  |
| Si | 0.671178 | -1.985442 | -1.991832 | Si | -0.676786 | -5.963697 | 4.636851 | Si | 0.591590 | 8.232809 | 3.377969 |  |  |  |  |
| Si | 1.987616 | -0.684327 | -3.370974 | Si | 0.638563 | -4.656416 | 5.960366 | Si | -8.331039 | $-3.376714$ | 4.724159 |  |  |  |  |
| Si | 3.297842 | 0.635586 | $-2.014220$ | Si | 1.957197 | -3.325613 | 4.581980 | Si | -7.200648 | -2.113921 | 5.890465 |  |  |  |  |
| Si | 4.604526 | 1.951744 | -3.332202 | Si | 3.314346 | -1.996511 | 5.925338 | Si | -5.977096 | $-0.638144$ | 4.670723 |  |  |  |  |
| Si | 5.954614 | 3.295193 | -1.999190 | Si | 4.634703 | $-0.710311$ | 4.610806 | Si | -4.636923 | 0.677900 | 5.957987 |  |  |  |  |
| Si | 7.1355 | 4.4195 | $-3.535750$ | Si | 4.634703 | -0.71031 | 4.610806 | Si | -4.646923 | 0.67900 | 5.957987 |  |  |  |  |

## B.6.1 Cartesian coordinates (Angstroms) of equilibrium $\mathbf{Z n S}$ quantum dots

| $\mathrm{Zn}_{19} \mathrm{~S}_{19}(1.0 \mathrm{~nm})$ |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | X | y | Z | S |  | 3.701666 | 1.003959 |  | -3.651792 | -0.819028 | -3.452976 |
| S | -2.680919 | -2.264999 | -4.161032 | S | 3.576479 |  |  | Zn |  |  |  |
| S | 0.296397 | 0.611870 | -4.710072 | S | -3.079024 | -4.303706 | 2.826396 | S | -1.996966 | 0.569422 | -2.047157 |
| S | -1.825081 | -4.958336 | $-1.837858$ | Zn | -1.567582 | -4.368927 | 1.102402 | Zn | -0.644470 | 1.795592 | -3.500248 |
| Zn | -0.840470 | -3.402812 | -3.064184 | S | 0.882515 | $-1.652500$ | 3.647517 | S | 0.746921 | 3.153665 | -2.016054 |
| S | 0.825196 | -2.174364 | -2.100975 | Zn | 1.913315 | -0.453843 | 1.939796 | Zn | 2.196579 | 4.575116 | -3.076298 |
| Zn | 1.681124 | -0.373144 | -3.324087 | S | 4.646442 | 1.283756 | 3.966238 | S | 3.083397 | 6.230938 | -1.819169 |
| S | 3.290616 | 0.818163 | $-2.123186$ | Zn | 4.404783 | 1.736869 | 1.759671 | S | -4.859296 | -4.463807 | 0.830419 |
| S | 0.874690 | -4.891125 | 0.883489 | Zn | 1.435044 | -3.667058 | 2.852219 | Zn | -3.671810 | -3.425245 | $-0.846480$ |
| Zn | 1.873136 | -3.412069 | $-0.379669$ | S | -3.530538 | 3.711704 | -2.007436 | S | -1.968708 | -2.066133 | 0.611023 |
| S | 3.580818 | -2.152532 | 0.723541 | S | -4.861152 | 0.751731 | 0.819644 | Zn | -0.671991 | -0.692600 | $-0.677757$ |
| Zn | 2.841431 | -0.257686 | $-0.220887$ | Zn | -3.634534 | 1.737664 | -0.773531 | S | 0.681553 | 0.661145 | 0.693322 |
| S | -4.215999 | -2.320498 | -2.653386 | S | -0.3754 | 5.051858 | 1.000493 | Zn | 2.184357 | 1.911765 | -0.563790 |
| Zn | -3.035081 | -0.382111 | -2.813235 | Zn | -0.081799 | 4.724113 | -1.158579 | S | 3.079404 | 3.516343 | 0.937294 |
| S | -1.767756 | 1.025221 | -1.759025 | S | -4.79 | -3.341266 | 2.1 | Zn | 4.308950 | 4.747266 | -0.518810 |
| Zn | -0.279760 | 2.115636 | $-3.199673$ | Zn | -4.33 | -1.508804 | 0.798890 | Zn | -3.214394 | -5.754422 | 1.722347 |
| S | 0.772309 | 3.866289 | $-2.285845$ | S | -2.121410 | 0.552467 | 4.031256 | S | -1.845877 | -4.746674 | 3.486337 |
| Zn | -3.061812 | -3.445993 | $-0.812979$ | Zn | , | 1.775330 | 3.127173 | Zn | -0.663647 | -3.600467 | 1.790486 |
| S | -2.105024 | $-2.168263$ | 0.812944 | S | 1.004398 | 3.315691 | 4.018756 | S | 0.698385 | $-2.156794$ | 3.207753 |
| Zn | -0.596237 | -0.826748 | -0.621040 | Zn | 1.527764 | 4.152244 | 1.873554 | Zn | 2.105122 | -0.755768 | 1.872654 |
| S | 0.773591 | 0.438073 | 0.758106 | Zn | -1.373790 | -1.400797 | 3.152220 | S | 3.029507 | 0.719845 | 3.427114 |
| Zn | 2.597876 | 2.875080 | -1.335987 | Zn | 2.739060 | 2.108316 | 4.424067 | Zn | 4.509247 | 2.533946 | 2.648784 |
| S | 3.734420 | 3.453768 | 0.630578 | Zn | -2.174772 | 3.614902 | 1.437616 | S | 6.664271 | 2.231902 | 2.268822 |
|  | -0.386334 | -3.406082 | 1.872594 | Zn | 569 | 66 | 0282 | Zn | 2.225630 | -3.127503 | 4.613212 |
| Zn | 0.705018 | -2.136512 | 3.574583 | $\mathrm{Zn}_{52} \mathrm{~S}_{52}(1.4 \mathrm{~nm})$ |  |  |  | S | 3.134989 | $-1.843360$ | 6.240927 |
| Zn | 2.754470 | -1.682324 | 2.762575 |  |  |  |  | Zn | 4.359009 | -0.608666 | 4.693192 |
| S | 3.606220 | 0.457194 | 3.611954 |  | X | y | Z | S | 1 | 1.251464 | -1.726815 |
| Zn | 4.101579 | 1.746739 | 1.872696 | S | -0.335373 | -2.712679 | -7.485542 | Zn | -5.983719 | 1.752016 | -2.812097 |
| S | -4.706121 | 0.632904 | 0.242192 | S | 0.390677 | -4.693930 | -4.862874 | S | -4.753033 | 3.560812 | -1.686529 |
| Z | -3.195905 | 2.167318 | -0.279301 | Zn | 1.444209 | -2.999087 | -6.019525 | Zn | -3.321948 | 4.012 | -3.210991 |
| S | -2.286145 | 3.905556 | 0.780644 | S | 3.203983 | -2.081183 | -5.063055 | S | -0.975372 | 6.493216 | -0.799868 |
| Zn | -0.290485 | 4.208603 | -0.295503 | S | -0.513078 | -7.440280 | -2.851307 | S | -7.770094 | -1.588957 | 1.374281 |
| Zn | -3.326241 | -0.359140 | 1.634717 | Zn | 1.245100 | -6.003153 | -3.177693 | Zn | -7.342384 | -0.351430 | $-0.353078$ |
| S | -2.137139 | 0.825190 | 3.255365 | S | 3.044415 | -5.155663 | $-2.213966$ | S | -4.766682 | 0.293620 | 0.332385 |
| Zn | -1.335393 | 2.890554 | 2.595942 | Zn | 4.082489 | -3.522746 | $-3.484606$ | Zn | -3.417042 | 1.821421 | -0.734938 |
| S | 0.624972 | 3.445324 | 3.752186 | S | 5.825307 | -2.696223 | -2.462210 | S | 8 | 3.158827 | 0.668997 |
| Zn | 1.632129 | 4.145541 | 1.645517 | S | -2.013888 | 1.060956 | -7.449845 | Zn | -0.812319 | 4.291431 | -0.868174 |
| Zn | -0.225546 | -0.242207 | 2.808408 | Zn | -1.746163 | -3.839205 | -5.998591 | S | 1.219719 | 7.158760 | 1.721442 |
| Zn | 1.863374 | 1.736024 | 4.119810 | S | -1.839498 | -1.787934 | $-4.539996$ | Zn | 1.3944 | 6.826689 | $-0.523396$ |
|  |  |  |  | Zn | -0.995464 | -0.654318 | -6.549379 | Zn | -5.987159 | -2.773561 | 1.789423 |
| $\mathrm{Zn}_{31} \mathrm{~S}_{31}(1.2 \mathrm{~nm})$ |  |  |  | S | 0.766670 | 0.567639 | $-4.864878$ | S | -4.705551 | -1.713387 | 3.612190 |
|  |  |  |  | Zn | 2.217766 | 2.062771 | -5.320006 | Zn | -3.376832 | -0.757638 | 1.858597 |
|  | 60 |  |  | S | 3.598642 | 3.558784 | -4.613709 | S | -2.016345 | 0.639728 | 3.214709 |
| S | 3.605832 | -1.933996 | -2.793934 | Zn | -2.116249 |  |  | Zn | -0.535276 | 2.041471 | 2.068987 |
| S | 3.632944 | -3776379 | -1.853517 |  | -2.186249 |  |  | S | 0.508607 | 3.529016 | 3.468861 |
| S | -1.872624 | -2.550859 | -4.042484 | S | -1.821557 | -4.492536 | -1.735783 | Zn | 2.040564 | 5.191596 | 2.466654 |
|  | 0.830832 | 0.422135 | -4.431769 | Zn | -0.537355 | -3.075358 | -3.061545 | S | 3.538078 | 5.615298 | 4.136938 |
| S | -2.190777 | -5.197045 | $-1.029794$ | S |  |  |  | Zn | -3.304950 | -3.278399 | 4.003501 |
| Zn | -1.066877 | -3.697899 | $-2.316337$ | Zn | 2.240996 | -0.719115 | -3.454065 | S | -0.845523 | $-0.697925$ | 6.539136 |
| S | 0.529881 | -2.196832 | $-1.328496$ | S | 3.540938 | 0.666296 | -1.968 | Zn | -0.786247 | -0.910622 | 4.338128 |
| Zn | 1.318609 | -1.332826 | -3.229933 | Zn |  |  |  | S | 1.383742 | 1.856691 | 7.005704 |
|  | 4.138465 | 1.795449 | -2.428993 | S | 6.007386 | 3.673541 | -1.5 | Zn | 2.094486 | 2.478583 | 4.972683 |
| S | 2.654400 | 1.753574 | -4.162679 | S | -2.177783 | -7.498932 | 0.91 | S | 4.072716 | 3.704964 | 4.717958 |
| S | 0.691800 | -4.984885 | 1.232690 | Zn |  | -6.5 |  | Zn | 1.526067 | -0.421804 | 6.773233 |
| Zn | 1.845235 | -3.879001 | $-0.398510$ | S | 0.756229 | -4.991860 | 0.585271 | Zn | -5.779978 | 1.610554 | 1.723672 |
|  | 3.979893 | $-0.366072$ | 0.836690 | Zn | 2.188523 | -3.586317 | $-0.746523$ | S | -5.100814 | 3.100152 | 3.163829 |
| SZnZnS | 3.892742 | -0.120557 | $-1.354902$ | S | 3.504555 | -2.116286 | 0.649797 | Zn | -3.513921 | 4.308704 | 1.993332 |
|  | 3.933378 | -3.026497 | 1.809829 | Zn | 5.073840 | -1.152408 | -1.083473 |  | -2.464141 | 5.971708 | 3.133006 |
|  | -1.597418 | 3.837438 | -2.690294 | S | 6.868555 | -0.117152 |  | Zn | -1.000236 | 6.673496 | 1.606358 |
| S | -4.941116 | -2.148988 | -1.497662 | Zn | 6.615728 | 2.149952 | $-0.068052$ | Zn | -3.465280 | 1.989886 | 4.360060 |
| ZnSZ | -3.232152 | -1.347750 | -2.742631 | Zn | 2.222029 | -5.377412 | 2.089116 | Z | -2.389983 | 3.187428 | 5.967200 |
|  | -2.046184 | 0.529328 | -1.963869 | S | 3.607283 | -4.669465 | 3.580927 | Zn | -1.260940 | 4.530600 | 4.469691 |
|  | -0.630551 | 1.766621 | -3.309693 | Zn | 4.726019 | -2.998472 | 2.332526 | Zn | $-0.867348$ | 1.701890 | 6.626861 |
| S | 1.893434 | 5.082888 | -2.126957 | S | 6.056198 | -1.680479 | 3.617342 |  |  |  |  |
| Zn | 1.340151 | 4.117598 | -4.175458 | Zn | 6.643905 | -0.116427 | 2.124034 |  |  |  |  |
| Zn | -4.106892 | -4.139050 | $-1.149268$ | S | -4.805374 | 0.865738 | $-4.513385$ |  |  |  |  |
| S | -1.910547 | -1.807252 | 1.027333 | Zn | -3.124126 | 1.792802 | -5.722850 |  |  |  |  |
| Zn | -0.739587 | -0.437388 | $-0.299120$ | S | -1.786042 | 3.538061 | -4.616594 |  |  |  |  |
| S | 0.553310 | 1.071400 | 0.953437 | Zn | -4.106901 | -4.210234 | -4.294908 |  |  |  |  |
| Zn | 3.198644 | 3.478427 | -1.194365 | S | -4.834349 | -2.674673 | -2.714800 |  |  |  |  |

## B. 6 ZnS Quantum Dots


1.0 nm

1.2 nm

1.8 nm

2.2 nm

Figure B.6: Equilibrium structures of ZnS quantum dots of sizes 1.0, 1.2, 1.4, 1.6, 1.8, 2.0 and 2.2 nm . The structures were obtained from geometry optimization using PM7 as implemented in MOPAC.

| $\mathrm{Zn}_{95} \mathrm{~S}_{95}(1.8 \mathrm{~nm})$ |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | X | y | Z |  |  |  |  |  |  |  |  |
| S | 3.745444 | -5.487903 | -5.841981 | S | 8.958777 | 0.334880 | 3.844293 | S | -8.146016 | -3.275495 | 1.907366 |
| S | 1.770894 | -8.141490 | -3.343905 | Zn | 69 | -3.950789 | 4.322517 | Zn | -5.980503 | -3.818789 | 2.460696 |
| S | 5.502625 | -4.669484 | -5.093786 | Zn | 8.270984 | -1.958672 | 3.116305 | S | -4.582472 | $-2.082168$ | 3.29 |
| S | -2.172881 | $-4.741965$ | $-7.555176$ | S | -3.423926 | 1.816298 | -8.093079 | Zn | -3.186480 | -0.693638 | 2.060805 |
| Zn | -0.702933 | -3.230349 | -8.108517 | S | -7.603 | -2.048299 | -4.594108 | S | -1.926873 | 0.697313 | 3.409491 |
| S | 1.543856 | -3.140020 | $-8.384123$ | Zn | -5.937738 | -1.032418 | -5.750276 | Zn | -0.648817 | 2.027565 | 1.990 |
| Zn | 1.582866 | -0.904997 | -8.231444 | S | -4.836210 | 0.598175 | -4.600007 | S | 0.656165 | 3.412401 | 3.357 |
| S | 2.954428 | 0.673766 | $-7.705829$ | Zn | -3.930892 | 2.423436 | -5.922216 | Zn | 2.010411 | 4.673583 | 1.804 |
| S | -2.023790 | -7.554238 | $-4.717571$ | S | -2.176414 | 3.334510 | -4.616811 | S | 3.451547 | 6.048590 | 3.318670 |
| Zn | -1.124610 | -5.846855 | -5.886502 | Zn | -0.548230 | 4.652526 | -5.580434 | Zn | 4.547124 | 6.941548 | 1.475289 |
| S | 0.521612 | -4.674353 | -4.829457 | S | 0.580992 | 6.2 | -4. | S | -4.626023 | -4.875370 | 5.610204 |
| Zn | 2.248312 | -3.658505 | -6.274263 | S | -7.551831 | -4.718080 | -1.893441 | Zn | -3.123076 | -3.339448 | 4.756 |
| S | 3.110902 | -1.986775 | $-4.866326$ | Zn | -6.194829 | -3.535606 | -3.424508 | S | -1.814257 | $-2.011781$ | 6.06 |
| Zn | 4.157459 | -0.360838 | -6.049033 | S | -4.710852 | -2.092145 | $-2.001156$ | Zn | -0.495813 | $-0.734060$ | 4.649686 |
| S | 5.889033 | 0.663131 | -5.176675 | Zn | -3.400885 | -0.653289 | -3.315040 | S | 0.770396 | 0.734548 | 6.014936 |
| Zn | -0.476699 | -8.046498 | -3.249554 | S | -2.000658 | 0.619127 | -1.990576 | Zn | 1.863709 | 1.849767 | 4.4 |
| S | -0.845518 | -8.127015 | -0.879270 | Zn | -0.705895 | 2.111202 | -3.277854 | S | 0 | 3.945470 | 6.561523 |
| Zn | 2.358040 | -5.993943 | -3.965965 |  |  |  | -2.071263 | Zn | 4.405146 | 4.357613 | 4.376974 |
| S | 3.283514 | -4.666673 | $-2.218032$ | Zn | 6276 | 4.855591 | -3.522369 | Zn | -0.553769 | -3.641496 | 7.1 |
| Zn | 4.722846 | -3.203604 | -3.466990 |  |  | 6.030843 | $-2.541169$ | S | -0.891924 | -3.527857 | 9.357890 |
| S | 6.134372 | -1.961740 | $-2.195874$ | Zn | 3.022118 | 7.847786 | -1.639687 | Zn | 0.451874 | -1.926141 | 9.677727 |
| Zn | 7.008647 | -0.344963 | -3.480328 | Zn | 1824 | -5.859421 | $-0.969973$ | S | 1.997364 | -0.385499 | 9.425018 |
| S | 8.691878 | 1.054612 | $-2.862237$ | S | -4.652795 | $-4.740027$ | 0.637709 | Zn | 4.350702 | 1.848059 | 6.667109 |
| Zn | 2.262428 | -7.880605 | -1.228324 | Zn | -3.302170 | -3.383713 | -0.631527 | Zn | -8.153888 | 1.740693 | $-0.778484$ |
| S | 3.578041 | -7.360232 | 0.371182 | S | -1.923759 | -2.013237 | 0.626493 | S | -7.593548 | 3.178457 | 0.74 |
| Zn | 4.659324 | -5.540691 | $-0.580995$ | Zn | -0.556587 | $-0.586732$ | $-0.616800$ | Zn | -5.894767 | 4.338038 | $-0.332466$ |
| S | 6.311235 | -4.517006 | 0.482796 | S | 0.780733 | 0.725623 | 0.678714 | S | -4.899200 | 5.956993 | 0.751152 |
| Zn | 7.265451 | -2.561791 | -0.196026 | Zn | 2.054779 | 2.115903 | -0.819653 | Zn | -3.237508 | 7.264732 | -0.228 |
| S | 8.972524 | -1.552390 | 0.812176 | , | 3.424616 | 3.36908 | 0.536374 | S | -2.919119 | 9.061079 | 0.999238 |
| S | -4.847395 | $-1.961516$ | $-7.488962$ | Zn | 4.854801 | 4.707132 | -0.962081 | Zn | -7.963997 | -1.066642 | 2.098956 |
| Zn | -3.370968 | -0.432918 | -8.009983 | S | 7 | 5.774135 | 0.612429 | S | -7.381683 | 0.454851 | 3.534347 |
| S | -0.998348 | -0.806443 | -8.130513 | S | -5.458451 | -5.666169 | 3.876816 | Zn | -6.184954 | 1.908749 | 2.132923 |
| Zn | -1.279174 | 2.230251 | -7.921345 | Zn | -3.634220 | -6.143595 | 2.378243 | S | -4.813670 | 3.221286 | 3.352851 |
| S | 0.325129 | 3.553299 | -7.432711 | S | -1.91237 | -4.772138 | 3.212384 | Zn | -3.371898 | 4.681554 | 1.979497 |
| S | -4.831961 | -4.780912 | $-4.730380$ | Zn | -0.538768 | -3.368705 | 1.969652 | S | -2.121948 | 6.219977 | 3.335425 |
| Zn | -3.628632 | -3.510689 | -6.176864 | S | 0.785729 | -2.070100 | 3.308117 | Zn | -1.465088 | 8.260732 | 2.432571 |
| S | -2.166124 | -2.045541 | $-4.726781$ | Zn | 2.132936 | $-0.778086$ | 1.953857 | S | 0.633468 | 8.883056 | 3.030809 |
| Zn | -0.832887 | -0.609704 | -5.839397 | S | 3.402200 | 0.593421 | 3.375650 | Zn | -5.573263 | -0.527919 | 4.603268 |
| S | 0.512434 | 0.751369 | -4.763001 | Zn | 4.634569 | 1.858984 | 1.833306 | S | -4.525911 | 0.607662 | 6.205961 |
| Zn | 1.831116 | 2.183029 | -6.242243 | S | . 058545 | 3.128739 | 3.458482 | Zn | -3.346180 | 2.056858 | 4.773101 |
| S | 3.245269 | 3.404533 | -4.987744 | Zn | 7.193775 | 4.030353 | 1.643042 | S | -2.413873 | 3.793400 | 5.958725 |
| Zn | 7.849006 | 4.593021 | -5.893718 | Zn | -0.257035 | $-5.943312$ | 4.298210 | Zn | -0.811851 | 4.846444 | 4.652875 |
| S | -4.765243 | -7.554106 | $-2.006854$ | S | 0.770440 | $-4.967866$ | 5.987126 | S | 0.688062 | 6.118384 | 5.900537 |
| Zn | $-3.546400$ | -6.209929 | $-3.516493$ | Zn | 2.148570 | -3.577329 | 4.681690 | Zn | 1.592187 | 7.182191 | 4.101707 |
| S | -2.074189 | -4.726060 | $-2.091156$ | S | 3.346233 | -2.287791 | 6.190511 | Zn | $-2.691699$ | -0.286169 | 7.256026 |
| Zn | -0.728634 | -3.355713 | -3.428727 | Zn | 4.6913 | -1.071229 | 4.797072 | S | -1.787124 | 0.956396 | 8.883414 |
| S | 0.561132 | -2.000832 | -2.079458 | S | 5.904045 | 0.440479 | 6.251519 | Zn | 0.572024 | 1.322914 | 9.257056 |
| Zn | 1.879319 | -0.634305 | -3.438251 | n | 7.056107 | 1.265747 | 4.410788 | S | 0.561063 | 3.546603 | 8.73539 |
| , | 3.248323 | 0.698937 | $-2.205723$ | Zn | 2.113112 | -0.742599 | 7.097787 | Zn | 1.644753 | 4.377958 | 6.961387 |
| Zn | 4.524042 | 2.079168 | -3.657870 | S | $-5.797520$ | 3.834440 | -5.413836 | Zn | -5.638128 | 4.951611 | 4.618427 |
| S | 6.051394 | 3.249689 | $-2.341483$ | S | -5.027841 | 5.587109 | $-4.608161$ | Zn | -3.920167 | 6.898955 | 4.530668 |
| Zn | 7.904927 | 2.911865 | $-3.877346$ | Zn | -8.083929 | -0.558430 | $-3.093903$ | Zn | -1.502987 | 2.972347 | 7.744146 |
| Zn | -3.236172 | -8.092680 | $-0.546046$ | S | -8.353282 | 1.665898 | -3.004438 | S | 6.012679 | -2.160943 | 3.400600 |
| S | -3.144365 | -8.272866 | 1.714762 | Zn | -6.244100 | 2.355922 | -3.593245 | Zn | 8.966665 | 0.693717 | 1.568154 |
| Zn | -0.633084 | -5.839126 | $-0.759575$ | S | -4.806410 | 3.188487 | -1.936679 | Zn | 1.282823 | 8.386096 | 0.816248 |
| S | 0.729468 | -4.726935 | 0.546999 | Zn | -3.388466 | 4.823749 | $-3.154299$ | S | 3.649441 | 8.662035 | 0.476395 |
| Zn | 2.026478 | -3.314668 | -0.804534 | S | -2.119910 | 6.170202 | -1.874535 | S | 3.404555 | -4.807497 | 3.218543 |
| S | 3.366880 | $-2.050813$ | 0.531692 | Zn | -0.207683 | 7.249535 | -2.719323 | Zn | 4.794132 | -3.450689 | 2.008180 |
| Zn | 4.593506 | -0.641182 | -0.875460 | S | 0.899444 | 8.784153 | $-1.590524$ | Zn | -0.654462 | 4.739645 | -0.632704 |
| S | 5.917099 | 0.601799 | 0.487220 | Zn | -8.018044 | -3.281106 | $-0.336315$ | S | 0.561398 | 6.007461 | 0.78408 |
| Zn | 7.293015 | 1.852351 | -0.906497 | S | -8.061471 | $-0.772296$ | $-0.521003$ |  |  |  |  |
| S | 8.800175 | 2.779307 | 0.690126 | Zn | -5.752574 | -0.742216 | $-0.567435$ |  |  |  |  |
| Zn | -0.914246 | -8.131923 | 1.760217 | S | -4.652338 | 0.631293 | 0.771994 |  |  |  |  |
| S | 0.720251 | $-7.602323$ | 3.051864 | Zn | -3.324019 | 1.977527 | $-0.654533$ |  |  |  |  |
| Zn | 2.206466 | -6.1566 | 1.883849 | S | -2.024881 | 3.307234 | 0.681308 |  |  |  |  |



## B.7. 1 Cartesian coordinates (Angstroms) of equilibrium CdS quantum dots

|  | $\mathrm{Cd}_{19} \mathrm{~S}_{19}(1.0$ |  |  |
| :--- | :---: | :---: | :---: |
|  | $\mathrm{nm})$ |  |  |
|  | X | y | Z |
| S | -1.604550 | -1.666159 | -4.289204 |
| S | -0.168635 | -0.214354 | -4.794729 |
| S | -2.086553 | -5.311817 | -1.900860 |
| Cd | -0.642976 | -3.590751 | -2.601041 |
| S | 1.172676 | -2.192317 | -1.620241 |
| Cd | 1.904421 | -0.726212 | -3.479746 |
| S | 3.694502 | 0.721596 | -2.497154 |
| S | 0.857558 | -5.414099 | 0.466009 |
| Cd | 2.625541 | -3.993630 | 0.045212 |
| S | 4.213420 | -2.290817 | 0.951199 |
| Cd | 4.464051 | -0.828486 | -0.856089 |
| S | -5.262795 | -2.079342 | -1.841908 |
| Cd | -3.531337 | -0.705727 | -2.667858 |
| S | -2.191946 | 1.145593 | -1.658527 |
| Cd | -0.740683 | 1.884862 | -3.537850 |
| S | 0.681088 | 3.693139 | -2.639218 |
| Cd | -4.031256 | -4.054216 | -1.428652 |
| S | -2.203529 | -2.188273 | 0.328419 |
| Cd | -0.414058 | -0.435941 | -0.304533 |
| S | 0.995857 | 1.032970 | 0.967955 |
| Cd | 2.338475 | 2.337019 | -0.934352 |
| S | 3.569428 | 4.116780 | 0.275553 |
| Cd | -0.685966 | -3.848106 | 1.424515 |
| S | 0.632084 | -2.524353 | 3.326712 |
| Cd | 2.469494 | -0.940151 | 2.403131 |
| S | 3.638739 | 0.444650 | 4.089169 |
| Cd | 5.194244 | 2.106381 | 2.026783 |
| S | -5.443098 | 0.950421 | 0.530916 |
| Cd | -3.979483 | 2.623370 | -0.100051 |
| S | -2.276286 | 4.235622 | 0.830002 |
| Cd | -0.806460 | 4.479642 | -0.979017 |
| Cd | -3.877782 | -0.667120 | 1.331984 |
| S | -2.510662 | 0.754094 | 3.285958 |
| Cd | -0.971915 | 2.458622 | 2.279829 |
| S | 0.673123 | 3.849606 | 3.744387 |
| Cd | 2.153823 | 4.634883 | 1.957695 |
| Cd | -0.991600 | -0.938608 | 3.827096 |
| Cd | 2.097583 | 2.018900 | 4.552455 |
|  |  |  |  |
|  |  |  |  |


$\begin{array}{llll}\text { S } & 6.725341 & 0.975311 & -5.389468\end{array}$ $\begin{array}{llll}\mathrm{Cd} & -1.074761 & -9.314834 & -3.159485\end{array}$ S $1.128224-7.977848$-2.580052 $\begin{array}{lllll}C d & 2.479023 & -6.436705 & -4.056678\end{array}$ $\begin{array}{llllll}\mathrm{S} & 3.830673 & -5.022488 & -2.147166\end{array}$ $\begin{array}{llll}\text { Cd } & 5.198788 & -3.546048 & -3.716821\end{array}$ S $6.729424-2.103669-2.310863$ $\begin{array}{lllll}C d & 7.828618 & -0.321755 & -3.665958\end{array}$ $\begin{array}{lllll}\text { S } & 9.579019 & 1.176468 & -2.503367\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & 2.562790 & -8.694584 & -0.878887\end{array}$ $\begin{array}{lllll}\mathrm{S} & 3.977350 & -7.959965 & 0.825061\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & 5.268412 & -6.068507 & -0.410150\end{array}$ $\begin{array}{lllll}\text { S } & 6.896814 & -4.912039 & 0.954617\end{array}$ Cd 8.019900 -3.070674 -0.309034 $\begin{array}{lllll}\mathrm{S} & 9.577288 & -1.497539 & 0.686169\end{array}$ $\begin{array}{lllll}\text { S } & -4.996103 & -2.384641 & -8.286291\end{array}$ Cd $\quad-3.208167$-1.140097 -9.190626 $\begin{array}{lllll}\text { S } & -2.534820 & 1.072585 & -8.080872\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -0.796593 & 2.474831 & -8.862670\end{array}$ S $\quad 0.788486 \quad 3.974711-8.154592$ S $\quad-5.298730-5.329812-5.234657$ $\begin{array}{lllll}\text { Cd } & -3.693345 & -4.000190 & -6.598816\end{array}$ S $\quad-2.297559-2.235046-5.162985$ Cd $-0.841221-0.630478-6.431120$ S $\quad 0.744552 \quad 0.800332-5.244447$ $\begin{array}{lllll}\text { Cd } & 2.295625 & 2.387976 & -6.707716\end{array}$ $\begin{array}{lllll}\mathrm{S} & 3.820241 & 3.796363 & -5.368398\end{array}$ Cd $6.147441 \quad 3.649554-7.413666$ S -5.425533 -8.082710 -2.000206 Cd $-3.971921-6.718190-3.673231$ S $\quad-2.247630 \quad-5.208530-2.297227$ Cd $\quad-0.700348$-3.690133 -3.768026 $\begin{array}{lllll}\mathrm{S} & 0.729496 & -2.245072 & -2.257496\end{array}$ $\begin{array}{lllll}C d & 2.144200 & -0.766612 & -3.779244\end{array}$ S $\quad 3.605798 \quad 0.693682-2.316607$ $\begin{array}{lllll}\mathrm{Cd} & 4.955920 & 2.316464 & -3.815077\end{array}$ $\begin{array}{lllll}\mathrm{S} & 6.485460 & 3.137014 & -1.854840\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & 8.520828 & 3.313947 & -2.959848\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -3.809422 & -8.809871 & -0.381822\end{array}$ $\begin{array}{lllll}\text { S } & -1.323939 & -8.996648 & -0.660159\end{array}$ $\mathrm{Cd}-0.648754-6.472766-0.871316$ S $0.778190 \quad-5.2183050 .700523$ $\begin{array}{lllll}C d & 2.225045 & -3.634959 & -0.752281\end{array}$ $\begin{array}{lllll}S & 3.709308 & -2.246539 & 0.782738\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & 5.040201 & -0.653262 & -0.799814\end{array}$ S $\quad 6.489145 \quad 0.303994 \quad 1.012723$ $\begin{array}{lllll}\mathrm{Cd} & 8.669273 & 0.967640 & -0.143202\end{array}$ $\begin{array}{llll}S & 9.517156 & 2.615054 & 1.779704\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -1.172796 & -8.686059 & 2.440354\end{array}$ $\begin{array}{lllll}\mathrm{S} & 0.659330 & -8.025823 & 3.782095\end{array}$ $\begin{array}{lllll}C d & 2.253604 & -6.649386 & 2.300596\end{array}$ $\begin{array}{lllll}\text { S } & 3.579348 & -5.046621 & 3.716118\end{array}$ $\begin{array}{lllll}C d & 5.229101 & -3.543660 & 2.396166\end{array}$ $\begin{array}{lllll}S & 6.712741 & -2.216795 & 4.020336\end{array}$ $\begin{array}{llll}\mathrm{Cd} & 8.527756 & -1.041773 & 2.850650\end{array}$ $\begin{array}{lllll}\text { S } & 9.491182 & 1.364730 & 3.492187\end{array}$ $\begin{array}{lllll}\text { Cd } & 4.664587 & -6.060949 & 5.729596\end{array}$ $\begin{array}{lllll} & 7.414439 & -3.493306 & 6.083982\end{array}$ $\begin{array}{lllll}S & -5.333087 & 4.455013 & -7.075541\end{array}$ S $\quad-8.004567 \quad-2.080006 \quad-5.310824$ $\begin{array}{lllll}\mathrm{Cd} & -6.154381 & -0.941346 & -6.615882\end{array}$ S $\quad-5.288535 \quad 0.817660$-5.084624 $\begin{array}{lllll}\mathrm{Cd} & -3.982671 & 2.482815 & -6.599326\end{array}$ S $\quad-2.147420 \quad 3.760713-5.129208$ Cd $\quad-0.467126 \quad 5.252004-6.240238$ $\begin{array}{lllll}S & 0.900083 & 6.877473 & -5.175185\end{array}$ S -8.287149 -4.962396 -2.214097 Cd $\quad-6.639500$-3.729552 -3.857380 S -5.090273 $-2.277675-2.249370$ $\begin{array}{lllll}C d & -3.668137 & -0.655877 & -3.681891\end{array}$ $\begin{array}{lllll}\text { S } & -2.194776 & 0.771297 & -2.229243\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -0.697760 & 2.277096 & -3.675106\end{array}$ $\begin{array}{lllll}\mathrm{S} & 0.818660 & 3.761029 & -2.336779\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & 2.438553 & 5.328260 & -3.794650\end{array}$ S $4.014078 \quad 6.686868$-2.378391 $\begin{array}{llll}\mathrm{Cd} & 5.942975 & 7.922085 & -4.133042\end{array}$ $\begin{array}{llll}\mathrm{Cd} & -6.647293 & -6.243125 & -0.853144\end{array}$ S $\quad-5.308163-5.041104 \quad 0.873500$ $\begin{array}{lllll}\mathrm{Cd} & -3.628914 & -3.730364 & -0.746984\end{array}$ $\begin{array}{lllll}\text { S } & -2.211942 & -2.222504 & 0.688112\end{array}$ Cd $\quad-0.725908 \quad-0.728601 \quad-0.799581$ $\begin{array}{llll}S & 0.745776 & 0.731004 & 0.648154\end{array}$ $\begin{array}{llll}\text { Cd } & 2.185377 & 2.265798 & -0.854750\end{array}$ $\begin{array}{lllll}\text { S } & 3.767603 & 3.623637 & 0.611312\end{array}$ S $\quad-2.313402 \quad 3.891840 \quad 6.573708$ $\begin{array}{llll}\text { Cd } & -0.767231 & 5.132878 & 4.911146\end{array}$
$\begin{array}{llll}\text { Cd } & 5.277252 & 5.184341 & -0.975387\end{array}$ $\begin{array}{lllll}\mathrm{S} & 6.952025 & 6.445818 & 0.683627\end{array}$ S $\quad-3569477-8.924405 \quad 2165275$ $\begin{array}{lllll}C d & -4.302004 & -6.608879 & 2.698255\end{array}$ $\begin{array}{lllll}\mathrm{S} & -2.132283 & -5.177791 & 3.626763\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -0.768246 & -3.717999 & 2.128880\end{array}$ $\begin{array}{lllll}\text { S } & 0.663866 & -2.186225 & 3.606110\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & 2.181506 & -0.738299 & 2.138505\end{array}$ $\begin{array}{lllll}\mathrm{S} & 3.591556 & 0.693211 & 3.723646\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & 5.028953 & 2.040259 & 2.086485\end{array}$ $\begin{array}{lllll}\mathrm{S} & 6.419503 & 3.623764 & 3.716989\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & 7.905608 & 4.581610 & 1.874950\end{array}$ Cd $-0.479143-6.152608 \quad 5.180855$ S $\quad 0.775755 \quad-4.988693 \quad 6.901127$ $\begin{array}{lllll}\mathrm{Cd} & 2.129611 & -3.372101 & 5.248263\end{array}$ $\begin{array}{lllll}\mathrm{S} & 3.600160 & -2.190665 & 6.848737\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & 4.983028 & -0.681524 & 5.312518\end{array}$ $\begin{array}{llll}\text { S } & 6.652088 & 0.646009 & 6.795459\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & 7.679859 & 2.006053 & 5.130291\end{array}$ $\begin{array}{llll}\mathrm{Cd} & 5.196888 & -4.090927 & 7.593042\end{array}$ $\begin{array}{llll}\text { S } & -9.071610 & 2.060155 & -3.498033\end{array}$ $\begin{array}{lllll}\mathrm{S} & -5.915283 & 5.053836 & -5.165781\end{array}$ Cd -8.668978 -0.403348 -3.753724 S $-8.705992-0.656397-1.086821$ $\begin{array}{lllll}\mathrm{Cd} & -6.853465 & 2.977766 & -4.159812\end{array}$ $\begin{array}{llll}\mathrm{S} & -5.217412 & 3.638213 & -2.115307\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -3.687806 & 5.180883 & -3.638808\end{array}$ $\begin{array}{lllll}\text { S } & -2.296781 & 6.670584 & -2.248456\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -0.334536 & 8.061840 & -3.315486\end{array}$ $\begin{array}{llll}\text { S } & 0.661537 & 9.474194 & -1.719237\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -9.075634 & -3.255410 & -0.749193\end{array}$ $\begin{array}{lllll}\text { S } & -9.333596 & -3.067758 & 1.721485\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -6.278512 & -0.708727 & -0.740194\end{array}$ $\begin{array}{lllll}\mathrm{S} & -5.173997 & 0.807981 & 0.802581\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -3.641443 & 2.245542 & -0.722605\end{array}$ $\begin{array}{lllll}\text { S } & -2.237192 & 3.735789 & 0.731887\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -0.742733 & 5.136425 & -0.806561\end{array}$ $\begin{array}{llll}\mathrm{S} & 0.911384 & 6.506218 & 0.489682\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & 2.571779 & 8.210671 & -0.992194\end{array}$ $\begin{array}{llll}\mathrm{S} & 3.494420 & 9.284109 & 1.353911\end{array}$ S -7.205940 -5.325164 4.524945 $\begin{array}{lllll}\mathrm{Cd} & -7.044335 & -3.927178 & 2.540732\end{array}$ $\begin{array}{lllll}\mathrm{S} & -5.325327 & -2.107631 & 3.574681\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -3.671699 & -0.757701 & 2.164073\end{array}$ $\begin{array}{lllll}\text { S } & -2.278041 & 0.792347 & 3.591257\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -0.746331 & 2.206871 & 2.084580\end{array}$ $\begin{array}{lllll}\mathrm{S} & 0.753440 & 3.616568 & 3.582575\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & 2.174832 & 5.023126 & 1.989785\end{array}$ $\begin{array}{lllll}S & 3.740484 & 6.393531 & 3.656071\end{array}$ $\begin{array}{llll}\mathrm{Cd} & 5.196292 & 7.592346 & 1.981376\end{array}$ $\begin{array}{llll}\text { S } & -5.241176 & -5.854324 & 4.962238\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -3.875349 & -3.674166 & 5.017769\end{array}$ S $\quad-2.407305 \quad-2.230479 \quad 6.408396$ $\begin{array}{lllll}\mathrm{Cd} & -0.796659 & -0.649753 & 4.994441\end{array}$ $\begin{array}{llll}\text { S } & 0.573895 & 0.744943 & 6.580028\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & 2.071638 & 2.153063 & 5.176813\end{array}$ S $\quad 3.515640 \quad 3.665654 \quad 6.797021$ $\begin{array}{lllll}\text { Cd } & 4.809222 & 4.844473 & 5.187862\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -0.682911 & -3.304248 & 7.909250\end{array}$ $\begin{array}{lllll}\mathrm{S} & -0.240587 & -1.425966 & 9.483469\end{array}$ $\begin{array}{llll}\mathrm{Cd} & 2.028412 & -0.481611 & 8.384450\end{array}$ $\begin{array}{llll}\mathrm{S} & 3.638290 & 0.644660 & 9.708779\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & 4.873377 & 1.882195 & 8.133671\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -8.882754 & 2.286570 & -1.058443\end{array}$ $\begin{array}{lllll}\mathrm{S} & -8.234297 & 3.747940 & 0.652739\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -6.323854 & 5.144202 & -0.499392\end{array}$ $\begin{array}{llll}\mathrm{S} & -5.297313 & 6.789978 & 0.870784\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -3.505646 & 8.050734 & -0.433996\end{array}$ $\begin{array}{llll}\text { S } & -2.333251 & 9.653367 & 0.857210\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -9.027839 & -0.758045 & 2.227639\end{array}$ $\begin{array}{lllll}\text { S } & -8.219499 & 0.846045 & 3.745140\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -6.786158 & 2.379473 & 2.233360\end{array}$ $\begin{array}{lllll}\mathrm{S} & -5.213612 & 3.737219 & 3.619857\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -3.672045 & 5.313847 & 2.241335\end{array}$ $\begin{array}{lllll}\mathrm{S} & -1.996860 & 6.805362 & 3.389675\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -0.372913 & 8.510070 & 1.465216\end{array}$ $\begin{array}{lllll}\text { S } & 1.859268 & 9.466292 & 2.696991\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -6.358537 & -0.374083 & 5.021248\end{array}$ $\begin{array}{lllll}\text { Cd } & 1.629213 & 5.268449 & 7.841182\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -6.119737 & 5.104402 & 5.557577\end{array}$ $\begin{array}{llll}\mathrm{S} & -5.212988 & 0.939547 & 6.732018\end{array}$ $\begin{array}{lllll}\mathrm{Cd} & -3.596863 & 2.347514 & 5.140150\end{array}$ $\begin{array}{lllll}\text { S } & 0.554422 & 6.947347 & 6.344526\end{array}$

## B. 7 CdS Quantum Dots



Figure B.7: Equilibrium structures of CdS quantum dots of sizes 1.0, 1.2, 1.4, 1.8 and 2.2 nm . The structures were obtained from geometry optimization using PM7 as implemented in MOPAC.

| $\mathrm{Cd}_{159} \mathrm{~S}_{159}$ |  |  | S | -2.199291 | -2.274633 | -5.138716 |  | 7.833357 | 8.053492 | -0.601450 | S | 3.568643 | 0.714703 | 9.446451 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | X | ) |  |  |  |  |  |  |  |  |  |  |  |  |
| S | 0.208606 | -5.475552-10.867429 | Cd | -0.885541 | -1.007174 | -5.13895480 |  | -6.807478 | -9.126954 | 1.751914 | Cd | 5.344778 | 2.342934 | 7.923191 |
| S | 3.148362 | $-2.453242-11.028980$ | S | 0.720469 | 0.681802 | -5.279413 | ca | -5.329297 | -8.138596 | 3.554740 | S | 5.965711 | 3.656039 | 9.941393 |
| Cd | 0.392281 | -8.441842 <br> -76361.936217 | Cd | 2.164986 | 2.118717 | $-6.705726$ | Cd | -3.796550 | -6.541168 | 1.988246 | Cd | 2.224395 | -3.795165 | 10.323444 |
| Cd | 1.620091 | $-6.763361-9.052108$ | Ca | 3.753617 | 3.740803 | -5.265274 | Ca | -2.204573 | -5.192880 | 3.678060 | Cd | 5.220445 | -0.701422 | 10.346440 |
| S | 3.477956 | -5.345169 -8.065812 | Cd | 5.391531 | 5.397473 | -6.499800 | Cd | -0.743684 | -3.757788 | 2.162845 | ca | -11.135814 | 2.937331 | -2.245681 |
| Cd | 4.581213 | $-3.478526-9.405316$ | S | 6.841414 | 6.915844 | -5.080633 | ca | 0.782798 | -2.254985 | 3.684161 | Cd | -9.632873 | 4.390473 | -3.269291 |
| S | 6.465089 | $\begin{array}{r}-2.202910 \\ -11175075 \\ \hline\end{array}$ | Cd | -5.973057 | -9.887425 | -1.819136 | Cd | 2.208132 | -0.763510 | 2.210492 | S | -8.426205 | 6.296389 | -2.146087 |
| $\mathrm{S}_{\mathrm{Cd}}$ | 0.807713 | -11.175075-4.599894 | S | -5.042750 | -7.760756 | -1.618342 | S | 3.700698 | 0.737654 | 3.729870 | Cd | -6.833771 | 7.356273 | -3.513471 |
| Cd | 1.685099 | -9.715717 -6.158200 | Cd | -3.925377 | -6.781392 | -3.640292 | Cd | 5.110899 | 2.244723 | 2.249374 | S | -5.542156 | 9.240594 | $-2.310943$ |
| Cd | 4.778526 | -6.616091 -6.3882066 | C | -2.307674 | -5.200951 | -2.174891 | S | 6.538107 | 3.753482 | 3.765427 | Cd | -4.074828 | 9.918363 | -3.994992 |
| S | 6.554207 | -5.225156 -5.225420 | Cd | -0.005992 | -3.785858 | -3.588527 | Cd | 7.920059 | 5.317366 | 2.358202 | S | -11.257777 | 0.254822 | 0.496873 |
| Cd | 7.625992 | $-3.471377-6.681654$ | S | 0.749487 | -2.209867 | -2.202989 | S | 8.762359 | 7.521640 | 3.065565 | Cd | -9.999181 | 2.377183 | 0.035632 |
| S | 9.435900 | -2.150578-5.421916 | Cd | 2.126805 | -0.769752 | $-3.772230$ | Cd | -3.343108 | -9.540614 | 4.483987 | S | -8.170720 | 3.755499 | 1.052237 |
| S | 3.992839 | -11.183456 -1.985467 | S | 3.652289 | 0.739425 | -2.267833 | S | -2.099632 | -8.418184 | 6.376909 | Cd | -6.802507 | 4.857392 | $-0.658697$ |
| Cd | 4.986202 | -9.594906 --3.321286 | Cd | 5.081025 | 2.205642 | -3.745968 | Cd | -0.653793 | -6.803496 | 4.918607 | S | -5.242313 | 6.483070 | 0.748450 |
| S | 6.781686 | -8.214515 -2.058454 | S | 6.685573 | 3.660526 | -2.284002 | S | 0.83271 | -5.276199 | 6.513265 | Cd | -3.864103 | 7.830630 | -0.851798 |
| Cd | 7.760072 | -6.636066 -3.458194 | Cd | 8.129617 | 5.395507 | -3.612698 | Cd | 2.320898 | -3.734814 | 5.062217 | S | -2.277077 | 9.459922 | 0.539722 |
| S | 9.596769 | $-5.275524-2.248712$ | S | 9.631189 | 6.908871 | -1.977659 | S | 3.783385 | -2.238618 | 6.657720 | Cd | -0.916967 | 11.718153 | -0.817007 |
| Cd | 10.206905 | -3.775669 -3.912784 | S | -5.667780 | -10.949627 | 0.320792 | Cd | 5.185973 | -0.754216 | 5.128978 | S | -11.121092 | -1.925061 | 4.180901 |
| S | -0.663822 | -0.834395-11.2 | Cd | -3.206369 | -10.3061 | 0.582753 | S | 6.808318 | 0.775985 | 6.592330 | Cd | -9.924405 | -0.902018 | 2.362636 |
| S | 0.820823 | 0.657171-11.410895 | S | -2.013955 | 7.989838 | 1.164896 | Cd | 8.198060 | 2.359205 | 5.165987 | S | -8.083600 | 0.664599 | 3.712036 |
| S | 3.723545 | 3.635304-10.876182 | Cd | -0.913605 | -6.781441 | $-0.700619$ | c | 10.338061 | 3.562225 | 5.202533 | Cd | -6.611467 | 2.098299 | 2.256744 |
| Cd | -1.906920 | -5.727811 -9.73 | S | 0.741166 | -5.169451 | 0.724503 | Cd | 2.503092 | -6.617979 | 7.810855 | S | -5.206975 | 3.722831 | 3.813578 |
| S | -1.722171 | $-4.885677-7.598764$ | Cd | 2.176145 | -3.715218 | -0.72772 | S | 3.83302 | -5.307312 | 9.64192 | Cd | -3.7284 | 5.023972 | 2.297577 |
| Cd | 0.669802 | $-3.093383-10.220281$ | S | 3.710299 | -2.218673 | 0.772256 | Cd | 5.312974 | -3.851075 | 8.036932 | S | -2.239162 | 6.640697 | 3.782641 |
| S | 1.141660 | -1.950539-7.993842 | Cd | 5.045809 | -0.768627 | -0.790901 | S | 6.821985 | -2.228044 | 9.686819 | Cd | -0.963543 | 7.858337 | 2.036730 |
| Cd | 2.804335 | $-0.163883-10.086201$ | S | 6.523871 | 0.742533 | 0.745796 | Cd | 8.070846 | -0.999281 | 7.908387 | S | 0.639602 | 9.484604 | 3.666994 |
| S | 3.926602 | 0.960177-8.16 | Cd | 7.964757 | 2.110062 | -0.727681 | S | -11.10509 | 0.179049 | -5.53121 | Cd | 2.0657 | 10.5691 | 2.162596 |
| Cd | 5.377768 | 2.767804 -9.366026 | S | . 603607 | 3.647902 | 0.984068 | Cd | -9.333221 | 1.588366 | -6.643828 | Cd | -9.539973 | -3.352148 | 5.077353 |
| S | 7.005541 | $4.042202-8.035548$ | Cd | 10.395174 | 5.293769 | $-0.484672$ | c | -8.241614 | 3.414300 | -5.313599 | S | -8.130441 | -2.068547 | 6.853958 |
| Cd | -4.057321 | $-9.896332-6.552707$ | S | -2.343215 | -11.071509 | 3.001988 | Cd | -6.543007 | 4.598303 | -6.598273 | Cd | -6.672338 | -0.693969 | 5.230017 |
| S | -2.396005 | $\begin{aligned} & -8.364011\end{aligned}-4.782814$ | Cd | -0.081597 | -9.991280 | 2.416713 | S | -5.314612 | 6.36954 | -5.338288 | S | -5.10693 | 0.767344 | 6.740821 |
| Cd | -0.999606 | $-7.004027-6.222290$ | S | 1.023601 | -8.189778 | 3.763018 | Cd | -3.533409 | 7.454049 | -6.823950 | Cd | -3.672750 | 2.225711 | 5.177450 |
| S | 0.688039 | $\begin{aligned} & -5.285411\end{aligned}-5.015816$ | $\mathrm{Cd}^{\text {d }}$ | 2.229781 | -6.646335 | 2.112159 | S | -2.376001 | 9.323079 | $-5.493276$ | S | -2.215528 | 3.732382 | 6.720792 |
| Cd | 1.936141 | $-3.760377-6.493794$ | S | 3.806978 | -5.241136 | 3.729030 | S | -11.195046 | -1.014343 | -1.197461 | Cd | -0.686371 | 5.121710 | 5.253931 |
| S | 3.682087 | -2.218559 -5.197619 | Cd | 5.153519 | -3.722403 | 2.247330 | Cd | -10.39694 | 0.47543 | -3.05300 | S | 0.75030 | 6.62956 | 6.852065 |
| Cd | 5.065852 | $-0.714975-6.718023$ | S | 6.699631 | -2.247803 | 3.751010 | S | -8.053689 | 1.104171 | -1.945800 | Cd | 2.226105 | 8.034179 | 5.400537 |
| S | 6.645279 | $\begin{array}{llll}0.788721 & -5.228599\end{array}$ | Cd | 7.970683 | -0.841979 | 2.063591 | Cd | -6.603540 | 1.938699 | $-3.679425$ | S | 4.180222 | 8.821372 | 6.615448 |
| Cd | 7.974496 | $2.574107-6.408355$ | S | 9.731394 | 0.629063 | 3.293895 | S | -5.181829 | 3.632477 | -2.224282 | Cd | -6.56131 | -3.499577 | 7.798761 |
| S | 9.740721 | $3.881385-5.07$ | Cd | 11.012687 | 3.090962 | 3.060838 | Cd | -3.69529 | 4.897529 | $-3.750030$ | S | -5.16068 | -2.325629 | 9.646750 |
| S | -1.346166 | -11.105097 -0.9 | Cd | 2.760878 | -9.600784 | 5.242912 | S | -2.276641 | 6.517390 | $-2.305458$ | Cd | -3.669723 | -0.845907 | 8.107748 |
| Cd | -0.791049 | -9.832734 -3.35318 | S | 3.990222 | -8.230040 | 6.903612 | Cd | -0.858147 | 7.935763 | -3.846399 | S | -2.165463 | 0.656059 | 9.667699 |
| S | 0.750064 | -8.104364 --2.023702 | Cd | 5.372438 | -6.727239 | 5.278036 | S | 0.499014 | 9.528981 | -2.223340 | Cd | -0.795682 | 1.994700 | 8.017204 |
| Cd | 2.125328 | -6.709716-3.485220 | S | 6.857827 | -5.272175 | 6.792118 | Cd | 2.073717 | 10.3871 | -3.743836 | S | 0.894032 | 3.719627 | 9.153205 |
| S | 3.736335 | -5.164120 -2.147213 | Cd | 8.144461 | -3.867721 | 5.249938 | S | -11.165433 | -4.531365 | 1.031408 | Cd | 3.315649 | 5.733133 | 8.478714 |
| Cd | 5.009176 | -3.679639 -3.670776 | S | 9.798568 | -2.246431 | 6.693201 | Cd | -9.841131 | -3.424777 | $-0.668310$ | S | 4.204655 | 4.565654 | 10.610981 |
| S | 6.591215 | -2.196349 -2.252500 | Cd | 10.458702 | -0.723708 | 5.006990 | S | -8.117414 | -2.046103 | 0.831338 | Cd | -3.67232 | -4.0482 | 10.168621 |
| Cd | 7.995883 | -0.716718 -3.737439 | S | -8.140352 | 0.294306 | -8.349555 | Cd | -6.793830 | -0.742814 | -0.816433 | Cd | -0.148507 | -0.355429 | 11.634533 |
| S | 9.578282 | 0.633189 -2.068061 | Cd | -6.441149 | 1.590647 | -9.470992 | S | -5.153873 | 0.722833 | 0.746416 | Cd | 2.582816 | 2.676550 | 10.634181 |
| Cd | 10.397254 | 2.251402-3.541979 | S | -5.195846 | 3.411270 | -8.282703 | Cd | -3.718709 | 2.116619 | $-0.745067$ | S | -10.994854 | 3.438764 | 3.417913 |
| S | 0.364257 | -11.219467 0.257537 | Cd | -3.280975 | 4.494202 | $-9.646193$ | S | -2.233956 | 3.665372 | 0.769766 | Cd | $-9.6708$ | 5.24301 | 2.779335 |
| Cd | 2.271862 | -9.922740 -0.851037 | S | -2.166598 | 6.410214 | -8.451274 | Cd | -0.824883 | 4.982284 | -0.833169 | S | -8.245886 | 6.908053 | 3.924497 |
| S | 3.682593 | $-8.1411580 .691776$ | Cd | -10.050157 | -1.982381 | -5.817030 | S | 0.656017 | 6.545111 | 0.684741 | Cd | -6.666340 | 7.737287 | 2.362527 |
| Cd | 5.179099 | $-6.734842-0.667986$ | S | -7.955989 | -1.819165 | -4.821236 | Cd | 1.984277 | 7.885092 | -0.927136 | S | -5.382334 | 9.635905 | 3.672426 |
| S | 6.686756 | -5.167050 0.774971 | Cd | -6.452170 | -0.974779 | -6.608305 | S | 3.546221 | 9.518674 | 0.721865 | Cd | -3.877645 | 10.279773 | 2.052023 |
| Cd | 8.014942 | -3.725463 -0.816831 | S | -5.065271 | 0.683323 | -5.189601 | Cd | 5.130412 | 10.608895 | -0.754485 | Cd | -9.447086 | 2.348162 | 4.949539 |
| S | 9.600333 | $-2.1961610 .569087$ | Cd | -3.532648 | 1.944593 | -6.675534 | Cd | -9.749878 | -6.189353 | 1.836969 | S | -8.260056 | 3.768481 | 6.720049 |
| Cd | 11.510048 | 0.1383860 .286778 | S | -2.152556 | 3.666513 | -5.259262 | S | -8.203164 | -5.086881 | 3.733286 | Cd | -6.742168 | 5.271174 | 5.312294 |
| S | 3.498930 | -11.006566 3.521633 | Cd | -0.649841 | 5.019488 | -6.829238 | Cd | -6.701675 | -3.539539 | 2.183428 | S | -5.293875 | 6.831392 | 6.772755 |
| Cd | 4.980568 | $-9.4695692 .388556$ | S | 0.734084 | 6.616903 | -5.275577 | S | -5.146959 | -2.162588 | 3.757886 | Cd | -3.938988 | 8.050345 | 5.168846 |
| S | 6.766764 | -8.267282 | Cd | 2.476143 | 7.908338 | -6.535343 | Cd | -3710215 | -0.770013 | 2.174770 | S | -2.402344 | 9.843507 | 6.698034 |
| Cd | 7.885017 | $\begin{array}{lll}-6.646275 & 2.450495\end{array}$ | S | 3.736004 | 9.763866 | -5.214133 | S | -2.223569 | 0.759504 | 3.703260 | Cd | -0.960406 | 10.556118 | 5.113646 |
| S | 9.706518 | -5.321292 3.744579 | Cd | -10.022836 | -6.399033 | $-4.027086$ | Cd | -0.739130 | 2.190578 | 2.231431 | Cd | -6.620854 | 2.459495 | 7.838455 |
| Cd | 10.352579 | -3.777327 2.12 | S | -8.407050 | -4.837155 | -2.259500 | S | 0.748565 | 3.730833 | 3.717603 | S | -5.316886 | 3.834080 | 9.636498 |
| S | -5.182979 | 0.135302-11.165709 | Cd | -6.668964 | -3.794833 | -3.694948 | Cd | 2.212838 | 5.124061 | 2.167363 | Cd | -3.848713 | 5.298194 | 8.034371 |
| S | -2.229468 | $3.025239-11.164455$ | S | -5.193485 | -2.178664 | -2.212341 | S | 3.624365 | 6.732424 | 3.585524 | S | -2.231206 | 6.815929 | 9.646307 |
| Cd | -5.600329 | -2.002103-10.130670 | Cd | -3.630962 | -0.852385 | -3.649568 | Cd | 5.291111 | 8.147243 | 2.410935 | Cd | -1.028486 | 8.041278 | 7.834567 |
| S | -4.712279 | $-2.118247-7.998377$ | S | -2.172038 | 0.742054 | -2.211656 | S | 7.598068 | 8.978710 | 2.105303 | Cd | -3.769284 | 2.256875 | 10.322308 |
| Cd | -2.785580 | 0.447767-10 | Cd | -0.718464 | 2.088370 | -3.792153 | Cd | -6.578914 | -6.452319 | 4.843934 | Cd | -0.586489 | 5.220781 | 10.132783 |
| S | -1.758647 | $1.039570-8.086735$ | S | 0.733481 | 3.647278 | -2.277864 | S | -5.168893 | -5.274208 | 6.590054 | S | 0.617823 | -2.278152 | 9.524318 |
| Cd | 0.044669 | 2.702472 -10.144826 | Cd | 2.192144 | 5.040922 | -3.802787 | Cd | -3.664529 | -3.708553 | 5.028781 | Cd | 2.130673 | 2.116652 | -0.765491 |
| S | 1.066473 | 3.891368-8.207812 | S | 3.620620 | 6.665837 | -2.334373 | S | -2.198046 | -2.267179 | 6.616104 | S | 0.719192 | 0.716748 | 0.776664 |
| Cd | 2.887849 | 5.360753 -9.479719 | Cd | 5.245193 | 8.166537 | -3.719127 | Cd | -0.788861 | -0.827976 | 5.040843 | Cd | -0.711847 | -3.882886 | 7.881208 |
| S | 4.046574 | 7.014252-8.083303 | S | 6.710569 | 9.836259 | -2.152155 | S | 0.711689 | 0.676984 | 6.540454 |  |  |  |  |
| Cd | -7.417002 | $-4.775792-7.534859$ | S | -8.536683 | -7.980882 | 0.565763 | Cd | 2.217636 | 2.218171 | 5.168352 |  |  |  |  |
| S | -5.425697 | $\begin{aligned} & -5.334820\end{aligned}-5.228959$ | Cd | -7.080616 | -6.345289 | -0.877949 | S | 3.641360 | 3.704785 | 6.664708 |  |  |  |  |
| Cd |  | -4.023395 -6.381511 | S | -5.320795 | -5.056776 | 0.726211 | Cd | 5.008729 | 5.208978 | 5.128859 |  |  |  |  |
|  |  |  | Cd | -3.767795 | -3.698389 | $-0.773323$ | S | 4.997120 | 7.010464 | 7.234800 |  |  |  |  |
| S | 3.602515 | 3.6164550 .715407 |  | 215073 | -227204 | 0.754284 | Cd | -3395511 | -6814867 |  |  |  |  |  |
| Cd | 5.013371 | $4.993994-0.907258$ |  | -2.215073 | -2.22724 |  |  |  |  | .53161 |  |  |  |  |
|  | 6.306207 | 6.4225550 .581219 | Cd | -0.766319 | -0.800510 | -0.737969 | S | -2.078081 | -5.561967 | 9.375095 |  |  |  |  |

## Appendix C

## Vertical excitation energies from different methods for various atomic clusters

## C. 1 Excited State Energies (eV) for $\mathbf{S i}_{3}$

Table C.1: First Eight Lowest Excited State Energies (eV) for $\mathrm{Si}_{3}$

| EOM-CCSD | oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: | :---: |
| 1.406 | 1.310 | -0.047 | 1.267 | 1.447 |
| 1.478 | 1.342 | 2.258 | 1.343 | 1.653 |
| 1.981 | 1.497 | 2.797 | 1.896 | 2.118 |
| 2.197 | 2.316 | 3.292 | 2.142 | 2.307 |
| 2.583 | 2.458 | 3.630 | 2.475 | 2.787 |
| 2.998 | 2.538 | 3.875 | 3.026 | 3.230 |
| 3.494 | 3.211 | 4.274 | 3.364 | 3.965 |
| 3.619 | 3.342 | 4.341 | 3.535 | 3.995 |

## C. 2 Excited State Energies (eV) for $\mathbf{Z n}_{4}$

Table C.2: First Eight Lowest Excited State Energies (eV) for $\mathbf{Z n}_{4}$

| EOM-CCSD | oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: | :---: |
| 3.696 | 3.180 | 1.963 | 3.475 | 3.699 |
| 3.714 | 3.341 | 2.705 | 3.645 | 3.745 |
| 3.880 | 3.415 | 2.760 | 3.772 | 3.871 |
| 4.143 | 3.419 | 2.921 | 3.793 | 4.188 |
| 4.231 | 3.865 | 3.084 | 3.829 | 4.267 |
| 4.234 | 3.970 | 3.342 | 3.849 | 4.297 |
| 4.314 | 4.268 | 4.568 | 4.387 | 4.344 |
| 4.684 | 4.432 | 4.713 | 4.389 | 4.610 |

## C. 3 Excited State Energies (eV) for $\mathbf{Z n}_{6}$

Table C.3: First Eight Lowest Excited State Energies (eV) for $\mathrm{Zn}_{6}$

| oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: |
| 2.143 | 1.261 | 3.217 | 3.106 |
| 2.433 | 1.613 | 3.292 | 3.155 |
| 2.527 | 1.755 | 3.411 | 3.179 |
| 2.694 | 1.758 | 3.469 | 3.349 |
| 2.833 | 1.912 | 3.478 | 3.397 |
| 2.951 | 2.045 | 3.592 | 3.407 |
| 2.968 | 2.141 | 3.638 | 3.428 |
| 3.084 | 2.171 | 3.676 | 3.546 |

## C. 4 Excited State Energies (eV) for $\mathbf{Z n}_{8}$

Table C.4: First Eight Lowest Excited State Energies (eV) for $\mathrm{Zn}_{8}$

| oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: |
| 2.060 | 0.985 | 2.569 | 2.207 |
| 2.131 | 1.493 | 2.613 | 2.341 |
| 2.480 | 2.758 | 2.678 | 2.342 |
| 2.506 | 2.772 | 2.853 | 2.714 |
| 2.571 | 3.008 | 2.962 | 2.769 |
| 2.576 | 3.376 | 3.015 | 2.825 |
| 2.721 | 3.396 | 3.026 | 2.842 |
| 2.816 | 3.691 | 3.123 | 2.852 |
|  |  |  |  |

## C. 5 Excited State Energies (eV) for $\mathbf{Z n}_{16}$

Table C.5: First Eight Lowest Excited State Energies (eV) for $\mathrm{Zn}_{16}$

| oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: |
| 1.116 | 0.388 | 1.599 | 1.355 |
| 1.124 | 0.524 | 1.672 | 1.510 |
| 1.153 | 1.490 | 1.800 | 1.613 |
| 1.211 | 1.655 | 1.818 | 1.770 |
| 1.448 | 1.687 | 1.965 | 1.781 |
| 1.508 | 1.913 | 1.991 | 1.897 |
| 1.821 | 1.972 | 2.046 | 1.905 |
| 1.892 | 1.977 | 2.069 | 1.919 |

## C. 6 Excited State Energies (eV) for $\mathbf{Z n}_{24}$

Table C.6: First Eight Lowest Excited State Energies (eV) for $\mathrm{Zn}_{24}$

| oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: |
| 0.844 | 0.421 | 1.420 | 1.420 |
| 1.083 | 0.765 | 1.440 | 1.440 |
| 1.224 | 0.916 | 1.583 | 1.583 |
| 1.311 | 1.092 | 1.608 | 1.608 |
| 1.486 | 1.244 | 1.624 | 1.624 |
| 1.489 | 1.256 | 1.660 | 1.660 |
| 1.527 | 1.515 | 1.691 | 1.691 |
| 1.593 | 1.592 | 1.710 | 1.710 |

## C. 7 Excited State Energies (eV) for $\mathbf{C d}_{4}$

Table C.7: First Eight Lowest Excited State Energies (eV) for $\mathrm{Cd}_{4}$

| oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: |
| 3.042 | 5.586 | 3.251 | 3.840 |
| 3.251 | 5.639 | 3.519 | 4.004 |
| 3.372 | 5.652 | 3.534 | 4.021 |
| 3.655 | 5.670 | 3.572 | 4.255 |
| 3.732 | 5.816 | 3.697 | 4.445 |
| 3.854 | 5.830 | 3.743 | 4.447 |
| 4.207 | 5.835 | 4.226 | 4.550 |
| 4.513 | 5.835 | 4.243 | 4.828 |

## C. 8 Excited State Energies (eV) for $\mathbf{C d}_{6}$

Table C.8: First Eight Lowest Excited State Energies (eV) for $\mathrm{Cd}_{6}$

| oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: |
| 2.849 | 5.290 | 3.000 | 3.600 |
| 3.091 | 5.393 | 3.162 | 3.620 |
| 3.157 | 5.497 | 3.201 | 3.679 |
| 3.218 | 5.517 | 3.225 | 3.706 |
| 3.225 | 5.558 | 3.275 | 3.869 |
| 3.444 | 5.580 | 3.318 | 3.900 |
| 3.555 | 5.613 | 3.384 | 3.904 |
| 3.746 | 5.706 | 3.407 | 3.961 |

## C. 9 Excited State Energies (eV) for $\mathbf{C d}_{8}$

Table C.9: First Eight Lowest Excited State Energies (eV) for $\mathrm{Cd}_{8}$

| oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: |
| 2.724 | 5.220 | 2.679 | 3.100 |
| 2.776 | 5.295 | 2.813 | 3.181 |
| 2.809 | 5.394 | 2.867 | 3.246 |
| 2.932 | 5.401 | 2.937 | 3.327 |
| 3.109 | 5.407 | 2.939 | 3.356 |
| 3.156 | 5.436 | 2.992 | 3.387 |
| 3.252 | 5.448 | 3.038 | 3.395 |
| 3.311 | 5.479 | 3.058 | 3.450 |

## C. 10 Excited State Energies (eV) for $\mathbf{C d}_{16}$

Table C.10: First Eight Lowest Excited State Energies (eV) for $\mathrm{Cd}_{16}$

| oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: |
| 1.390 | 5.064 | 1.767 | 1.919 |
| 1.495 | 5.087 | 1.866 | 2.011 |
| 1.585 | 5.208 | 1.880 | 2.040 |
| 1.799 | 5.217 | 1.939 | 2.074 |
| 1.805 | 5.232 | 1.991 | 2.115 |
| 1.888 | 5.261 | 2.013 | 2.238 |
| 1.948 | 5.273 | 2.122 | 2.277 |
| 1.992 | 5.280 | 2.193 | 2.285 |

## C. 11 Excited State Energies (eV) for $S_{3}$

Table C.11: First Eight Lowest Excited State Energies (eV) for $\mathrm{S}_{3}$

| EOM-CCSD | oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: | :---: |
| 1.779 | 0.672 | 0.424 | 1.620 | 10.066 |
| 1.786 | 1.106 | 0.588 | 1.662 | 10.311 |
| 3.372 | 2.752 | 2.637 | 3.621 | 10.319 |
| 5.198 | 4.370 | 4.293 | 4.941 | 10.354 |
| 5.352 | 4.658 | 4.310 | 5.225 | 10.747 |
| 5.449 | 4.894 | 4.909 | 5.345 | 11.232 |
| 5.540 | 5.114 | 5.304 | 5.371 | 1.801 |
| 5.747 | 5.403 | 5.810 | 6.125 | 1.865 |
|  |  |  |  |  |

## C. 12 Excited State Energies (eV) for $\mathbf{S}_{5}$

Table C.12: First Eight Lowest Excited State Energies (eV) for $\mathrm{S}_{5}$

| EOM-CCSD | oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: | :---: |
| 2.827 | 3.178 | 4.561 | 2.621 | 2.842 |
| 3.499 | 3.624 | 5.395 | 3.180 | 3.539 |
| 3.713 | 4.366 | 5.471 | 3.345 | 3.810 |
| 4.018 | 4.384 | 5.714 | 3.560 | 4.080 |
| 4.040 | 4.561 | 5.938 | 3.692 | 4.084 |
| 4.192 | 4.744 | 6.050 | 3.891 | 4.243 |
| 4.432 | 4.833 | 6.272 | 4.086 | 4.332 |
| 4.515 | 5.182 | 6.374 | 4.102 | 4.341 |
|  |  |  |  |  |

## C. 13 Excited State Energies (eV) for $\mathbf{S}_{5}$

Table C.13: First Eight Lowest Excited State Energies (eV) for $\mathrm{S}_{5}$

| EOM-CCSD | oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: | :---: |
| 2.827 | 3.178 | 4.561 | 2.621 | 2.842 |
| 3.499 | 3.624 | 5.395 | 3.180 | 3.539 |
| 3.713 | 4.366 | 5.471 | 3.345 | 3.810 |
| 4.018 | 4.384 | 5.714 | 3.560 | 4.080 |
| 4.040 | 4.561 | 5.938 | 3.692 | 4.084 |
| 4.192 | 4.744 | 6.050 | 3.891 | 4.243 |
| 4.432 | 4.833 | 6.272 | 4.086 | 4.332 |
| 4.515 | 5.182 | 6.374 | 4.102 | 4.341 |
|  |  |  |  |  |

## C. 14 Excited State Energies (eV) for $S_{6}$

Table C.14: First Eight Lowest Excited State Energies (eV) for $\mathrm{S}_{6}$

| oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: |
| 4.039 | 5.406 | 3.405 | 3.952 |
| 4.039 | 5.436 | 3.406 | 3.952 |
| 4.113 | 5.436 | 3.781 | 4.440 |
| 4.113 | 5.631 | 3.901 | 4.484 |
| 4.244 | 5.631 | 3.901 | 4.508 |
| 4.513 | 5.913 | 4.044 | 4.508 |
| 4.582 | 6.299 | 4.099 | 4.544 |
| 4.584 | 6.502 | 4.110 | 4.600 |

## C. 15 Excited State Energies (eV) for $\mathbf{S}_{10}$

Table C.15: First Eight Lowest Excited State Energies(eV) for $\mathrm{S}_{10}$

| oeINDO | INDO/s | TDDFT | CIS(D) |
| ---: | ---: | ---: | ---: |
| 3.346 | 5.004 | 3.242 | 3.943 |
| 3.399 | 5.072 | 3.434 | 4.014 |
| 3.552 | 5.078 | 3.558 | 4.171 |
| 3.798 | 5.255 | 3.670 | 4.198 |
| 3.976 | 5.266 | 3.775 | 4.357 |
| 4.011 | 5.398 | 3.885 | 4.551 |
| 4.131 | 5.609 | 3.905 | 4.618 |
| 4.322 | 5.616 | 3.969 | 4.675 |

## C. 16 Excited State Energies (eV) for $\mathbf{S}_{20}$

Table C.16: First Eight Lowest Excited State Energies (eV) for $\mathrm{S}_{20}$

| oeINDO | INDO/s | TDDFT |
| :---: | :---: | :---: |
| 3.431 | 5.065 | 3.490 |
| 3.442 | 5.096 | 3.519 |
| 3.512 | 5.131 | 3.623 |
| 3.573 | 5.145 | 3.648 |
| 3.610 | 5.172 | 3.691 |
| 3.660 | 5.189 | 3.727 |
| 3.712 | 5.221 | 3.761 |
| 3.781 | 5.268 | 3.777 |

## C. 17 Excited State Energies (eV) for (ZnS) $)_{2}$

Table C.17: First Eight Lowest Excited State Energies (eV) for $(\mathrm{ZnS})_{2}$ cluster

| oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: |
| 0.695 | -1.116 | 1.147 | 1.645 |
| 2.222 | -0.575 | 1.752 | 2.005 |
| 2.851 | 1.935 | 2.209 | 2.933 |
| 3.040 | 2.462 | 2.884 | 3.497 |
| 3.127 | 3.161 | 3.516 | 3.635 |
| 3.492 | 3.302 | 3.518 | 3.912 |
| 3.586 | 3.390 | 3.925 | 4.147 |
| 3.748 | 3.916 | 3.969 | 4.521 |
|  |  |  |  |

## C. 18 Excited State Energies (eV) for (ZnS) ${ }_{3}$

Table C.18: First Eight Lowest Excited State Energies (eV) for $(\mathrm{ZnS})_{3}$ cluster

| oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: |
| 2.852 | 4.295 | 3.372 | 4.042 |
| 2.853 | 4.296 | 3.372 | 4.043 |
| 4.268 | 5.144 | 4.107 | 4.814 |
| 4.393 | 5.876 | 4.444 | 4.894 |
| 4.395 | 5.878 | 4.447 | 4.895 |
| 4.590 | 6.079 | 4.483 | 5.012 |
| 4.604 | 6.079 | 4.484 | 5.021 |
| 4.754 | 6.462 | 4.592 | 5.187 |
|  |  |  |  |

## C. 19 Excited State Energies (eV) for (ZnS) 4

Table C.19: First Eight Lowest Excited State Energies (eV) for $(\mathrm{ZnS})_{4}$ cluster

| oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: |
| 2.527 | 3.795 | 3.612 | 4.016 |
| 3.377 | 4.229 | 3.777 | 4.510 |
| 3.377 | 4.229 | 4.076 | 4.546 |
| 4.123 | 4.784 | 4.076 | 4.546 |
| 4.338 | 5.860 | 4.246 | 4.945 |
| 4.354 | 5.860 | 4.246 | 4.945 |
| 4.399 | 5.914 | 4.554 | 5.129 |
| 4.676 | 6.615 | 4.691 | 5.131 |
|  |  |  |  |

## C. 20 Excited State Energies (eV) for (ZnS) $)_{10}$

Table C.20: First Eight Lowest Excited State Energies (eV) for $(\mathrm{ZnS})_{10}$ cluster

| oeINDO | INDO/s | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: |
| 1.545 | 3.408 | 1.635 | 1.935 |
| 1.746 | 3.477 | 1.870 | 2.143 |
| 1.778 | 3.657 | 1.887 | 2.279 |
| 2.121 | 3.760 | 2.265 | 2.618 |
| 2.137 | 3.837 | 2.268 | 2.677 |
| 2.280 | 4.091 | 2.432 | 2.934 |
| 2.398 | 4.698 | 2.520 | 3.012 |
| 2.432 | 4.731 | 2.621 | 3.035 |

## C. 21 Excited State Energies (eV) for (ZnS) $)_{19}$

Table C.21: First Eight Lowest Excited State Energies (eV) for $(\mathrm{ZnS})_{19}$ cluster

| oeINDO | INDO/s | TDDFT |
| :---: | :---: | :---: |
| 2.685 | 4.492 | 2.421 |
| 2.729 | 4.982 | 2.734 |
| 2.944 | 5.494 | 2.885 |
| 3.005 | 5.750 | 2.916 |
| 3.046 | 5.765 | 3.019 |
| 3.099 | 5.913 | 3.090 |
| 3.179 | 5.984 | 3.107 |
| 3.244 | 6.010 | 3.227 |

## C. 22 Excited State Energies (eV) for (CdS) ${ }_{2}$

Table C.22: First Eight Lowest Excited State Energies(eV) for $(\mathrm{CdS})_{2}$ cluster

| oeINDO | INDO/s | TDDFT |
| :---: | :---: | :---: |
| 1.009 | -2.484 | 0.940 |
| 2.618 | -2.036 | 1.854 |
| 3.220 | 0.776 | 2.486 |
| 3.328 | 1.166 | 2.542 |
| 3.614 | 1.446 | 3.367 |
| 3.916 | 1.539 | 3.542 |
| 4.059 | 1.841 | 3.759 |
| 4.226 | 1.871 | 4.011 |

## C. 23 Excited State Energies (eV) for (CdS) ${ }_{3}$

Table C.23: First Eight Lowest Excited State Energies (eV) for (CdS) $)_{3}$ cluster

| oeINDO | INDO/s | TDDFT |
| :---: | :---: | :---: |
| 2.908 | 3.178 | 3.412 |
| 2.909 | 3.435 | 3.412 |
| 4.083 | 3.436 | 4.311 |
| 4.086 | 3.958 | 4.314 |
| 4.539 | 3.959 | 4.430 |
| 4.665 | 4.102 | 4.773 |
| 4.666 | 4.318 | 4.841 |
| 4.868 | 4.493 | 4.841 |

## C. 24 Excited State Energies (eV) for (CdS) $)_{10}$

Table C.24: First Eight Lowest Excited State Energies (eV) for (CdS) ${ }_{10}$ cluster

| oeINDO | INDO/s | TDDFT |
| :---: | :---: | :---: |
| 1.800 | 2.570 | 1.573 |
| 2.180 | 2.749 | 1.857 |
| 2.244 | 2.879 | 1.890 |
| 2.451 | 3.127 | 2.134 |
| 2.466 | 3.171 | 2.303 |
| 2.697 | 3.208 | 2.338 |
| 2.716 | 3.289 | 2.357 |
| 2.854 | 3.383 | 2.448 |

## C. 25 Excited State Energies (eV) for (CdS) $)_{19}$

Table C.25: First Eight Lowest Excited State Energies (eV) for (CdS) ${ }_{19}$ cluster

| oeINDO | INDO/s | TDDFT |
| :---: | :---: | :---: |
| 2.570 | 1.039 | 3.605 |
| 2.837 | 1.258 | 3.845 |
| 2.912 | 1.669 | 3.904 |
| 3.002 | 1.920 | 4.208 |
| 3.097 | 2.015 | 4.274 |
| 3.201 | 2.094 | 4.555 |
| 3.247 | 2.267 | 4.876 |
| 3.297 | 2.366 | 4.983 |

## Appendix D

## Published Paper from the Thesis

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Research paper
Efficient determination of excitation energies and absorption spectra for quantum dots and large systems from ab initio data

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HIGHLIGHTS

- Computing excited states of big systems is costly; a new method is proposed for this.
- This method parameterizes a Hamiltonian using ab inito excitation energies of tiatomics.
- The new method is applied to $\mathrm{Si}_{2}$ with EOM-CCSD energies and the INDO Hamiltonian.
- The parameterized Hamiltonian is transferable with mean error of 0.3 eV .
- Excitation energies and UV-VIS spectra are predicted for large clusters up to Sizro.

ARTICLEINFO

## Keywards

Excitation energy
Alsorption spectra
ZOM-CCSD
TDDFT
CIS(D)

ABSTRACT
A semi-empirical Hamiltonian with the INDO approximation and CIS was parameterized to reproduce EOMCCSD excitation energies for diatomic silicon $\mathrm{Si}_{2}$ with different inter-atomic separations. The model Hamiltonian is transferable, producing excitation energies for clusters, $\mathrm{Si}_{n}(n=3,4,5$ ) with mean absolute errors (MAEs) of $0.26,0.18$, and 0.09 eV , respectively, from the EOM-CCSD energies. The absorption spectra are also in qualitative agreement with the EOM-CCSD spectra. For clusters $\mathrm{Si}_{19}$ and $\mathrm{Si}_{40}$ comparison with TDDFT gave MAEs of ( $n=124,147,172,779$ ) using the model Hamiltonian.

## 1. Introduction

Nanoscience and nanotechnology have opened up a large vista of possibilities in the development of novel materials and new products. Certain nanomaterials, called quantum dots, are being used in photovoltaic cells, light-emitting diodes, sensors, detectors, and for biomedical imaging. These nanosized materials have characteristics which are typically different from those of the corresponding bulk compounds and have the advantage that their properties can be tuned by changing their sizes and/or shapes. The electronic and optical behavior of quantum dots depend on their electronic excitation energies and ultra-violet-visible (UV-VIS) absorption spectra. Both experimental and theoretical approaches are employed in the study of the excitation energies and absorption spectra of materials. However, the experimental approach is sometimes limited by high infrastructure costs and the sophistication involved in the experiments. Hence, simple, reliable, and accurate theoretical methods are sought for in order to augment
experiments and make predictions useful for experimentalists [1]. Theoretical methods can give insight and provide deep understanding of processes in these nanomaterials and, thus, complement experimental work.

Theoretical methods that can be used in the calculation of excitation energies and absorption spectra include the GW [2] method, solutions of the Bethe-Salpeter equation (BSE) [3-5], Quantum Monte Carlo (QMC), the equation-of-motion coupled-cluster with single and double excitations (EOM-CCSD) method [6], the complete active-space secondorder perturbation theory (CASPT2) [7], the configuration interaction with singles (CIS) and doubles [CIS(D)] excitations methods [8], timedependent density functional theory (TDDFT) [9,10] and others. Although these methods can give excitation energies and absorption spectra which are accurate to different degrees, their use is limited in practice by the size of the system [11,12]. They become very computationally expensive and sometimes prohibitive as the system size increases.

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The $a b$ initio EOM-CCSD method has been found to give accurate excitation energies [13] and, in fact, in a previous study, [14] the method produced the most accurate excitation energies in comparison to experimental results. However, EOM-CCSD calculations are computeintensive and can only be used in practice for the accurate determination of the excited state properties of systems with only a few ( $\$ 50$ ) electrons. A pragmatic alternative to EOM-CCSD for obtaining excitation energies is the TDDFT method. TDDFT is being used extensively for calculations of excitation energies and absorption spectra and can routinely handle moderately large systems with up to approximately 100 atoms ( $\sim 200-500$ electrons). It serves as a useful compromise between accuracy and computational expense and is typically accurate within a mean absolute error of $0.3-0.5 \mathrm{eV}[12]$. In this work, our goal is to describe a procedure that can give excitation energies with near-EOM-CCSD accuracy and which can be used in practice for systems with well over 1000 electrons.

This work is divided as follows. In the next section, we describe our approach. This is followed by Section 3 where we apply the method and present our results. We end by making theoretical predictions for large clusters and giving our conclusion and suggestions for further work.

## 2. Theory

Our new method consists in determining the parameters of a semiempirical Hamiltonian in order to reproduce the $a b$ initio EOM-CCSD excitation energies for diatomics at different atom-atom separations. This model Hamiltonian can then be used for systems that are more complex than diatomics, for example, quantum dots. Our approach is different from other methods which parameterize the energies of structures that are more complex than diatomics. Later on, it is seen that our parameterization of diatomics does indeed give models which are 'transferable', i.e., the same model/parameters obtained for diatomics give good results for more complex structures.

The semi-empirical Hamiltonian that we have chosen to parameterize is Zeiner's Intermediate Neglect of Differential Overlap (ZINDO) [15] model Hamiltonian. The ZINDO Hamiltonian systematically omits certain integrals from the Hartree-Fock (HF) method while some other integrals are parameterized. In addition, only valence electrons are considered. Although these approximations drastically reduce the number of integrals and, thus, allow semi-empirical methods to handle large systems, the same approximations generally make these methods less accurate [16]. On the other hand, although based on HF, some electron correlation effects are incorporated into these Hamiltonians through model parameters. A variation of ZINDO, which we denote as $\mathrm{ZINDO} / \mathrm{S}$ is the INDO approach which was parameterized to give good excitation energies when the HF-INDO process is followed by CIS calculations. The theory behind ZINDO and ZINDO/S, like other semi-empirical methods can be found, for example, in Refs. [17-19] and will not be discussed in details here. However, for completeness, and to establish some notation, we give highlights.

In the ZINDO approach, as in the HF formalism, we seek to solve the following secular equation:
$F C=E S C$
where $F$ is the Fock matrix, $C$ the basis vector and $E$ is the orbital energy. In the INDO approximation, of which ZINDO is a variant, the overlap matrix $S$ in the equation above is taken to be a unit matrix following from the neglect of differential overlap (NDO). Thus, in ZINDO, we solve:
$F C=B C$
Within INDO, the Fock matrix elements can be divided into three categories. For the unrestricted Fock matrix, we have:

1. The diagonal Fock matrix elements (atoms $A=B$ and indices $\mu=2$ ):

where

$$
\begin{equation*}
H_{\mu \mu}^{\infty n}=U_{\psi H}+\sum_{A \neq B}\left[P_{A B H}-Z_{B}\right] \gamma_{A B} \tag{4}
\end{equation*}
$$

$$
\begin{equation*}
P_{D D}=\sum_{k \boldsymbol{a} D} P_{k k} \tag{5}
\end{equation*}
$$

$$
\begin{equation*}
P_{m n}=\sum_{i=1} C_{m i} C_{n i} \tag{6}
\end{equation*}
$$

$U_{\mu \mu}=\langle\mu|-\frac{1}{2} \nabla^{2}-\frac{Z}{\mid r_{i}-R_{A}}|\mu\rangle$
2. The off-diagonal one center matrix elements (atoms $A=B$ and indices $\mu \neq v$ ):
$F_{\mu \nu}^{u}=H_{i \nu \gamma}^{\text {core }}+\sum_{\lambda o n A} \sum_{\sigma v N}\left[P_{\lambda \sigma}\langle\mu \nu \mid \lambda \sigma\rangle-P_{\lambda \sigma}^{u}\langle\mu \lambda \mid \nu \sigma\rangle\right]$
$H_{\mu \nu}^{\omega w}=0$
3. The off-diagonal two center matrix elements (atoms $A \neq B$ and indices $\mu \neq v$ )

$$
\begin{equation*}
F_{\mu \nu}^{u}=H_{\mu \nu}^{\text {MrN }}-P_{\mu \nu}^{u} \gamma_{A B} \tag{10}
\end{equation*}
$$

$$
\begin{equation*}
H_{\mu \nu}^{\infty n}=\frac{1}{2}\left(\beta_{\mu}+\beta_{\eta}\right) S_{\mu \nu} \tag{11}
\end{equation*}
$$

In all the equations above, the superscript $u$ represents the spin component: $u=\alpha$ ('spin up') or $\beta$ ('spin down'). For the restricted Hartree Fock case, $P_{\mu \nu}^{u}=\frac{1}{2} P_{\mu v}$ for both values of $u$. The important terms and parameters are: the one-electron one center integral $U_{\psi w}$, the twocenter electron-nuclear attraction $V_{A B}$, and the two-center two-electron repulsion integral $\langle\mu v \mid \lambda \sigma\rangle$. Details on previous determination of these parameters and terms can be found in Refs. [18,20], for example.

Finally, after solving Eq. (2), the GS approach is used as a post-Hartree-Fock-like method in order to calculate excitation energies; UV-VIS absorption spectra can also be computed. In CIS, the wavefunction is expanded as a linear combination of the ground state and singly excited Slater determinants. Details of the CIS method can be found in Ref. [21], for example.

In principle, another semi-empirical Hamiltonian which can be used is the popular tight-binding method or its density functional theory (DFT)-based method, the tight-binding DFT (DFTB) approach [22]. Unfortunately, however, it is not straightforward to obtain the absorption spectra from DFTB.

## 3. Computational details

We tested our method with silicon. First, accurate $a b$ initio excitation energies for $\mathrm{Si}_{2}$ were obtained at each of eight different Si -Si separations from 1.6 to $3.0 \AA$ in steps of $0.2 \AA$. These energies were obtained using the EOM-CCSD method with the triple-zeta valence basis functions having two sets of polarization functions (the "def2-tzvpp" basis functions of Ref. [23]). At each geometry, the eight lowest vertical excitation energies from the ground state were calculated, yielding a
total of 64 excitation energies for all eight geometries. For inter-atomic separations between 1.6 and $3.0 \AA$, the ground state of $\mathrm{Si}_{2}$ is the triplet state. This was confirmed by carrying out CCSD/def2-tzvpp calculations for both the singlet and triplet states. All energy calculations were performed with the ORCA4.0 program [24] unless otherwise stated.

Having obtained these 'benchmark' or 'training' energies, we proceeded to parameterize the ZINDO/S Hamiltonian so as to reproduce the EOM-CCSD energies. For this purpose, we minimized the following quantity with respect to the parameters of the model Hamiltonian:
$\chi^{2}=\frac{1}{N_{\text {data }}} \sum_{i=1}^{N \text { dan }} \| E_{i}^{\text {fit }}-E_{l}^{d}|-\epsilon|$
where $N_{\text {duta }}=64$ is the number of excitation energies that were fitted, $E_{i}^{i}$ is the $i$ th $a b$ initio excitation energy in the training data set and $E_{i}^{\text {fit }}$ the $i$ th excitation energy from the ZINDO/GS result. The constant $\epsilon=0.1 \mathrm{eV}$ is a practical difference in energy that we expected for each excitation energy in the parameterization although a value of 0 eV will be ideal.

The simplex method was used for the optimization with the ZINDO/ S parameters as the initial guess. Eight parameters were optimized, namely, $U_{p p}, \zeta_{s}, \zeta_{p}, \beta_{s}, \beta_{p}, \gamma_{s s}, F_{p p}^{2}$ and $G_{i p}$. The parameters $\gamma_{s p}$, and $\gamma_{p p}$ were not directly optimized but were set equal to $\gamma_{g}$ while $U_{n}$ was initially fixed at zero. At the end of the parameterization, the mean absolute error (MAE) calculated as
$\mathrm{MAE}=\frac{1}{N_{\text {data }}} \sum_{i=1}^{N_{\text {dan }}}\left|E_{i}^{\text {fit }}-E_{\mid}^{\text {it }}\right|$
was determined. A value of 0.21 eV was obtained for the training dataset and, in Fig. 1, we show a scatter plot which compares the excitation energies resulting from our optimized parameters in INDO/CIS with those of EOM-CCSD. All the points in the plot would lie on the diagonal line if the MAE were zero. We also compare the original ZINDO/S with EOM-CCSD. Our new parameters perform better than the original ZINDO/S parameters which gave an MAE of 0.67 eV .

After the optimization of excitation energies, the best values of $U_{x}$ and $U_{p p}$ which reproduce the $a b$ initio first ionization potential $\left(\mathrm{IP}_{\mathrm{at}}\right)$ of the silicon atom were determined. $\mathbb{P}_{\text {al }}$ was calculated using the "deltaSCF" approach, namely: $I P_{1 i}=E(N-1)-E(N)$ where, $E(X)$ is the $a b$ initio (CCSD) energy of the X-electron system. For the silicon atom, we obtained $I P_{\mathrm{a}}=8.069 \mathrm{eV}$ at the CCSD/def2-tzvpp level which is in agreement with the experimental value [25] of 8.1517 eV . By varying $U_{p p}$ and keeping $\left(U_{p p}-U_{n}\right)$ fixed at the value obtained in the previous


Fig. 1. Scatter plot of oelNDO (blue circles) and original $\mathbb{Z N D O} / 5$ (red pluses) excitation energies against EOM-CCSD energies for diatomics. All the points would lie on the diagonal green line if the MAE were zero. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article)

Table 1
Our parameters (oelNDO), optimized for diatomics and the original $7 \mathrm{INDO} / \mathrm{S}$ parameters.

| Parameters | oelnDO | ZINDO/S |
| :---: | :---: | :---: |
| $U_{\text {ss }}$ | -25.4244 eV | -36.235929 eV |
| $U_{p p}$ | -13.6400 eV | -28.594917 eV |
| $\zeta$ | 1.430753 bohrs | 1.52 bohrs |
| $\zeta_{p}$ | 1.411963 bohrs | 1.52 bohrs |
| $\beta_{s}$ | 29.06189 eV | 1.0 eV |
| $\beta_{p}$ | 9.662134 eV | 13.0 eV |
| $\gamma_{s s}$ | 2.311795 eV | 7.57 eV |
| $\gamma_{\text {p }}$ | 2.311795 eV | 7.57 eV |
| $\gamma_{p p}$ | 2.311795 eV | 7.57 eV |
| $F_{p p}^{2}$ | 1.750254 eV | 2.527 eV |
| $G_{p p}$ | 3.132682 eV | 4.8122 eV |

fit to excitation energies, we found the values of $U_{p p}$ and $U_{s s}$ which best reproduced the $\operatorname{CCSD~IP}_{a 1}$ without changing the energies obtained in the previous optimization of the excitation energies. Our INDO parameters, optimized for excitation (oe) energies are listed in Table 1 as "oeINDO" along with the original ZINDO/S parameters.

## 4. Results and discussions

The key question for our approach is whether the parameters we obtained are 'transferable' or not, i.e., whether the same model parameters which we obtained for diatomics will lead to accurate excitation energies for structures more complex than diatomics which were not included in the training set. We verified the transferability of our model by computing the excitation energies for different $\mathrm{Si}_{n}$ complexes with $n=3,4,5,19,40$. The equilibrium geometries of $\mathrm{Si}_{n}$ ( $n=3,4,5,19,40$ ) have been reported in literature [26-28] and we used these as well as perturbations around the $n=3,4,5$ structures. For $\mathrm{Si}_{3}, \mathrm{Si}_{4}$ and $\mathrm{Si}_{5}$, the equilibrium structures are isosceles triangle, planar rhombus and squashed trigonal bi-pyramid, respectively, and all have singlet ground state configurations [27,28]. For the larger clusters ( $\mathrm{Si}_{19}$ and $\mathrm{Si}_{40}$ ) for which EOM-CCSD calculations were prohibitive, we have used the TDDFT approach with the 'B3LYP' exchange correlation functional [29] as well as the CIS and CIS(D) approaches but only after estimating their accuracies for smaller silicon clusters.

### 4.1. Excitation of the equilibriom $\mathrm{Si}_{3}$ structure

We have determined the excitation energies for the eight lowest lying states of the equilibrium $\mathrm{Si}_{3}$ structure which has sides 2.292 , $2.292,2.960 \AA$. The first excitation energy from oeINDO is 1.3 eV while EOM-CCSD/def2-tzvpp gives 1.4 eV . The absorption spectra for this structure is shown in Fig. 2 in which the relative intensity, scaled such that the largest intensity has a value of unity, is plotted. There is reasonable agreement in the results although the highest peak of oelNDO is red-shifted from that of EOM-CCSD by about 0.5 eV . The MAE for the eight excitations is 0.22 eV from the EOM-CCSD results. In comparison, TDDFT/B3LYP and CIS(D) give MAEs of 0.17 and 0.39 eV from the EOM-CCSD energies for this cluster.

### 4.2. Excitation of the equilibrium $\mathrm{Si}_{4}$ structure

For the $\mathrm{Si}_{4}$ equilibrium structure, there is good agreement between the oeINDO absorption spectra and the EOM-CCSD ones: the MAE of oeINDO energies relative to EOM-CCSD is only 0.18 eV and the absorption spectra, shown in Fig. 3, has the highest peak of oeINDO blueshifted from the EOM-CCSD one by only about 0.10 eV .


Fig. 2. Computed UV-VIS absorption spectra of the $\mathrm{Si}_{3}$ equilibrium structure.


Fig. 3. Computed UV-VIS absorption spectra of the $\mathrm{Si}_{4}$ equilibrium structure.

### 4.3. Excitation of the equibibrium $\mathrm{Si}_{5}$ structure

The excitation energies and the UV-VIS spectra of the equilibrium $\mathrm{Si}_{5}$ cluster are also well predicted by oelNDO. The spectra is shown in Fig. 4 together with the EOM-CCSD, TDDFT, and GS(D) spectra. The position of the highest peak of oeINDO deviates by only 0.09 eV from that of the EOM-CCSD. In addition, the MAEs of oeINDO excitation energies from the EOM-CCSD, TDDFT, and GSD energies are 0.09,


Fig. 4. Computed UV-VIS absorption spectra of the $\mathrm{Si}_{5}$ equilibrium structure.


Fig. 5. Computed UV-VIS absorption spectra of the $\mathrm{Si}_{19}$ equilibrium structure.
0.04 , and 0.16 eV , respectively.

### 4.4. Excitations of the equibibrium $\mathrm{Si}_{19}$ and $\mathrm{Si}_{40}$ structures

For the $\mathrm{Si}_{n}(n=19,40)$ equilibrium structures, we did not have enough computational resources to carry out EOM-CCSD calculations, thus, we compared with TDDFT/B3LYP and CIS(D) excitation energies. The MAE for the oeINDO excitation energies for $\mathrm{Si}_{19}$ compared to the TDDFT/B3LYP and CIS(D) energies are 0.32 eV and 0.36 eV respectively. For $\mathrm{Si}_{40}$, the corresponding MAEs are 0.12 eV and 0.03 eV when oeINDO is benchmarked against TDDFT/B3LYP and CIS(D), respectively. For all ( $n=3,4,5,19,40$ ) equilibrium structures, the MAE of our oeINDO with respect to the TDDFT/B3LYP energies is 0.15 eV and 0.25 eV with respect to the CIS(D) excitation energies. The corresponding results for ZINDO/S are 1.23 and 1.13 eV , respectively. We present the UV-VIS absorption spectra for $\mathrm{Si}_{19}$ and $\mathrm{Si}_{40}$, in Figs. 5 and 6, respectively. The oeINDO results compare favorably well with the TDDFT and CIS(D) spectra. The highest peak of the oeINDO spectra for $\mathrm{Si}_{19}$ matches well with that of TDDFT and CIS(D) while the $\mathrm{Si}_{40}$ spectra reproduces the $\mathrm{GS}(\mathrm{D})$ pattern in the same energy range.

We have summarized the results of the eight lowest lying excitation energies for each of the $\mathrm{Si}_{3}, \mathrm{Si}_{4}, \mathrm{Si}_{5}, \mathrm{Si}_{19}$, and $\mathrm{Si}_{40}$ equilibrium structures in Table 2. For a total of 24 excitation energies, eight for each of these $\left(\mathrm{Si}_{3}, \mathrm{Si}_{4}\right.$, and $\left.\mathrm{Si}_{5}\right)$ equilibrium structures, the MAE of our oeINDO energies with respect to the EOM-CCSD energies is 0.16 eV . This compares favorably with the TDDFT/B3LYP and CIS(D) energies which have MAEs of 0.09 eV and 0.11 eV , respectively, with respect to EOM-OCSD


Fig. 6. Computed UV-VIS absorption spectra of $\mathrm{Si}_{40}$ equilibrium structure.

Table 2
MAE in eV of oeINDO and ZINDO/S from the EOM-CCSD, TDDFT, and CIS(D) $a b$ initio excitation energies of $\mathrm{Si}_{n}$ equilibrium geometries ( $n=3,4,5,19,40$ ). The EOM-CCSD calculations for $\mathrm{Si}_{19}$ and $\mathrm{Si}_{40}$ were beyond our computational capacity.

| Methods | EOM-CCSD | TDDFT | CIS(D) |
| :---: | :---: | :---: | :---: |
| $\mathrm{Si}_{3}$ |  |  |  |
| oelNDO | 0.22 | 0.17 | 0.39 |
| ZINDO/S | 0.91 | 0.98 | 0.74 |
| $\mathrm{Si}_{4}$ |  |  |  |
| oelNDO | 0.18 | 0.24 | 0.31 |
| ZINDO/S | 1.38 | 1.71 | 1.25 |
| $\mathrm{Si}_{5}$ |  |  |  |
| oelNDO | 0.09 | 0.04 | 0.16 |
| ZINDO/S | 1.47 | 1.52 | 1.39 |
| $\mathrm{Si}_{19}$ |  |  |  |
| oelNDO | - | 0.32 | 0.36 |
| ZINDO/S | - | 1.42 | 1.46 |
| $\mathrm{Si}_{40}$ |  |  |  |
| oelNDO | - | 0.12 | 0.03 |
| ZINDO/S | - | 0.68 | 080 |

excitation energies. The original ZINDO/S energies have an MAE of 1.28 eV from EOM-CCSD results.

### 4.5. Excitations of equilibrium and non-equilibrium $\mathrm{Si}_{n}$ structures

We have also included structures that are perturbed from the equilibrium $\mathrm{Si}_{n}(n=3,4,5)$ geometries in our evaluation of the accuracy and transferability of our oelNDO model. The ones we examined were such that the equilibrium inter-atomic distances were scaled by the factors $0.9,1.1$ and 1.2 , respectively. This procedure led to 12 structures in all, including the equilibrium geometries. The MAEs of the excitation energies of these structures from the EOM-OCSD energies are presented in Table 3. A scatter plot which compares the oelNDO energies with the EOM-CCSD ones for these small clusters is also shown in Fig. 7. The TDDFT/B3LYP and GS(D) energies are also compared with the EOM-CCSD energies. Clearly, from the table and figure, the oeINDO model parameters are transferable with an accuracy between 0.01 and 0.5 eV . This accuracy is similar to the TDDFT/B3LYP one which typically range from 0.01 to 0.3 eV .

### 4.6. Excitations of large silicon clusters

Lastly, we carried out calculations and predicted the spectra for large $\mathrm{Si}_{n}$ ( $n=124,147,172,779$ ) clusters using the oelNDO model. Fig. 8 shows the results for the absorption spectra of these clusters. The largest (smallest) one has a diameter of $3.0(1.6) \mathrm{nm}$. The absorption edges and excitation energies results agree with the report by Jackson et al. which predict that large Si clusters are more metallic than bulk silicon [30]. In addition, the absorption edge and the first excitation energy of about 0.3 eV for the $\mathrm{Si}_{147}$ equilibrium cluster determined by

Table 3
MAEs of oeINDO, ZINDO/S and TDDFT/B3LYP excitation energies (eV) with respect to EOM-CCSD energies for different (equilibrium and non-equilibrium) $\mathrm{Si}_{n}(n=3,4,5)$ clusters. The data used consisted of 32 excitation energies for each cluster size.

| Methods | $\mathrm{Si}_{3}$ | $\mathrm{Si}_{4}$ | $\mathrm{Si}_{5}$ |
| :---: | :---: | :---: | :---: |
| oeINDO | 0.25 | 0.27 | 0.11 |
| ZINDO/S | 0.87 | 1.32 | 1.31 |
| TDDFT/B3LYP | 0.06 | 0.11 | 0.08 |
| CIS(D) | 0.13 | 0.15 | 0.06 |



Fig. 7. Scatter plot of oeINDO (black circles), original ZINDO/S (pink pluses), CIS(D) (blue asterisks) and TDDFT (green square) excitation energies against EOM-CCSD energies for equilibrium and non-equilibrium $\mathrm{Si}_{n}$ ( $n=3,4,5$ ) clusters. The data used consisted of 32 excitation energies for each cluster size. All the points would lie on the diagonal line if the MAE were zero. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)


Fig. 8. Computed UV-VIS absorption spectra of $\mathrm{Si}_{n}(n=124,147,172,779)$ geometries.
oeINDO agrees with the electronic gap predicted in the same report [30].

## 5. Conclusion

We have presented a method that can give excitation energies of clusters which are accurate to better than 0.5 eV . This method consists in parameterizing a semi-empirical Hamiltonian using high-level $a b$ onitio data of diatomics. In this case, the reference $a b$ initio data was at the EOM-CCSD level although, in principle, $a b$ initio data from other methods such as GW or multireference EOM-OCSD can also be used. We parameterized the ZINDO/S Hamiltonian and demonstrated that the parameters obtained for diatomics are transferable to more complex systems. In particular, we obtained excitation energies with accuracies of 0.5 eV and less as well as UV-VIS spectra in qualitative agreement with the EOM-CCSD ones. However, unlike EOM-OCSD which is computationally prohibitive, oeINDO is not compute intensive and we were able to carry out calculations for quantum dots silicon clusters with 779 atoms (more than 10,000 electrons). Our results for large clusters show that these clusters become more metallic when compared to bulk silicon which has a band gap of approximately 1.2 eV .

It is worth mentioning the calculation times in order to give a good idea of the usefulness of this method. We achieved an accuracy close to EOM-CCSD accuracy using less than a hundredth of the time and resources. The oeINDO results are also reasonably close to TDDFT results and sometimes better but with calculations that take not more than a tenth of the TDDFT calculation time, depending on the size of the system.

We note here that we have experimented with other semi-mpirical methods, namely PM3 [31] and AM1 [32]. These model Hamiltonians gave small MAEs for small ( $n=3,4,5$ ) clusters but large MAEs for the larger ( $n=19,40$ ) clusters. The reason for this poor performance for large clusters is possibly due to the form of the two-electron two-center integrals in these models. We plan to investigate this further in future work.

The oeINDO model, like the ZINDO/S and other INDO semi-empirical models neglect three- and four-center terms. Future work can investigate improvement of the accuracy of oeINDO through the inclusion of explicit three-center terms into the model Hamiltonian; we expect the four-center terms to be much less important.

## Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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