BAYESIAN ESTIMATORS OF NORMAL LINEAR REGRESSION MODEL IN THE PRESENCE OF MULTICOLLINEARITY

BY

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ABSTRACT

Multicollinearity arises in econometrics when the regressor is linearly related to other regressors in a Normal Linear Regression Model (NLRM). A major drawback of the classical approach to the estimation of NLRM is that it is indeterminate in the presence of extreme perfect multicollinearity. The use of out-of-sample information by the Bayesian approach to resolve this problem has not been fully explored in existing literature on the subject. Therefore, the Bayesian technique was employed to derive estimators for a NLRM and investigate the sensitivity of the estimators to various degrees of collinearity among the regressors.

The NLRM $y = X\theta + \varepsilon$, where$y$ is (N x 1) vector of the response variable, X is a (N x k) matrix of regressors, θ is (k x 1) vector of parameters and ε is a (N x 1) vector of normally distributed random error with zero mean and variance σ^2 was specified. Six cases of collinearity: case I- High Positive Collinearity (HPC) (0.50≤ HPC ≤0.99); case II- Moderate Positive Collinearity (MPC) (0.30≤ MPC ≤0.49); case III- Low Positive Collinearity (LPC) (0.01≤ LPC ≤0.29); case IV- High Negative Collinearity (HNC) (-0.99≤ HNC ≤-0.50); case V- Moderate Negative Collinearity (MNC) (-0.49≤ MNC ≤-0.30); case VI- Low Negative Collinearity (LNC) (-0.29≤ LNC ≤-0.01) and No Collinearity (NC) were investigated. Two Bayesian out-of-sample priors: Bayesian Informative Prior (BIP) with Normal-Gamma prior and Bayesian Non-informative Prior (BNIP) with a local uniform prior were derived and their estimates compared with classical method, namely, Likelihood Based (LB) method for all the cases of collinearity considered. Data were simulated for all the cases of collinearity for sample sizes 10, 30, 70, 100, 200 and 300. The performances were judged using Standard Error (SE) and Confidence Interval (CI). Therefore, the estimator with the minimum SE and compact CI were considered the most efficient estimator.

The derived Bayesian estimators were P($\theta|y$) ∝ t($\theta, h^{-1}Q, v$) for BIP and P($\theta|h$) ∝ $N(\theta, h^{-1}Q)$ for BNIP, where h, Q and v are precision, un-scaled variance-covariance matrix and degree of freedom, respectively. The SE and CI of BIP, BNIP and LB for HPC were [0.3843, (4.4636≤ CI ≤ 5.9949)] [2.1099, (-0.490≤ CI ≤7.9250)] and [2.1729, (-0.6213≤ CI ≤ 8.0553)]; for MPC were [0.3870, (4.3608≤ CI ≤ 5.8822)], [1.1111, (2.0341≤ CI ≤6.4023)] and [1.1278, (1.9665≤ CI ≤6.4700)]; LPC were

[0.3963, (4.4893≤ CI ≤ 6.0686)], [0.9032, (2.8449≤ CI ≤6.4475)] and [0.9301, (2.7892≤ CI ≤6.5033)]; HNC were [0.008, (9.9985≤ CI ≤10.0015)], [1.6369, (7.1784≤ CI ≤13.7071)] and [1.6856, (7.0774≤ CI ≤13.8081)]; MNC were [0.0009, (9.9983≤ CI ≤10.0017)], [0.4748, (7.5869≤ CI ≤9.4810)] and [0.4890, (7.5576≤ CI ≤9.5103)]; LNC were [0.6201, (1.3167≤ CI ≤3.7879)], [0.7658, (0.4447≤ CI ≤3.4995)] and [0.7887, (0.3974≤ CI ≤3.5468)] and NC were [0.5350, (0.9025≤ CI ≤3.0345)], [0.6958, (-0.0468≤ CI ≤2.7286)] and 0.7166, (-0.0897≤ CI ≤2.7716)], respectively. Thus, Bayesian estimators BIP and BNIP were less sensitive with minimum values of SE and narrower CI of parameter estimates for all the cases of collinearity considered compared to LB estimator. The Bayesian estimators outperformed the LB for all the cases of collinearity considered, while BIP outperformed BNIP.

The derived Bayesian estimators for normal linear regression model provided better estimates than the classical method at various degrees of multicollinearity. They are also less sensitive to the problem of collinearity and capable of handling perfect correlation.

Keywords: Likelihood based method, Informative prior, Confidence interval, Collinearity.

Word count: 488

DEDICATION

The work is dedicated to the glory of THE ALMIGHTY GOD.

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In all your ways acknowledge Him, and He shall direct your paths. Proverbs 3:6. I acknowledge the Almighty God, my Father, my Maker, the King of kings, Lord of lords, the Invisible and the Giver of life for the success of this programme.

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CERTIFICATION

I certify that this work was carried out by Mr. O. O. OJO in the department of Statistics, University of Ibadan.

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INTRODUCTION

1.1 General Background of the Study

Regression model is one of the most important models used in econometric modelling. It is also termed as the most important component of other econometric models, including the Seemingly Unrelated Regression (SUR) model, Simultaneous equation model, Vector Autoregressive (VAR) model (Hill et al (1997)). Regression models describe the relationship (linear or non-linear) between a dependent variable called Y and other variable called independent variable X which is used to predict the values of dependent variable, if the case is a simple regression model; but a relationship between a dependent variable Y and two or more independent variables $X's$, if the case is a multiple regression model.

Estimation of regression models with collinear regressors will have effect on calculations regarding the individual parameters which may not give valid results about any individual predictor and will make it very difficult to separate the effect of individual independent variables on dependent variable. For the case of perfect multicollinearity, it becomes a more serious problem in the sense that, the design matrix X will have less than a full rank while the moment matrix $X^{'}X$ cannot be inverted which in turn will make the popular method of estimation, Ordinary Least Squares (OLS) estimator not to exist.

Basic assumptions of Classical linear regression model require among others (latter discussed in chapter three) that the regressors (explanatory variables) be not highly correlated among themselves, i.e. they should be orthogonal. The violation of this assumption is referred to as Multicollinearity. The cause, effect, diagnostics and possible remedies are discussed in chapter two.

Various estimation methods are employed in literature for estimating parameters of linear regression models such as the Ordinary Least Squares (OLS), Generalized Least Squares (GLS), Maximum Likelihood Method (MLM), Method of Moments (MM) etc which are referred to as the Classical estimators and the Bayesian approach. Dreze (1962) argued that classical inferences have shortcomings in that; the available information on parameters is ignored. The classical estimation methods have gained a lot of attention in literature and have been so much applied in research activities; while research on the Bayesian method and its applications has only of recent been on the increase.

Most statistical works are done using the classical approach, and it entails a random sampling of observations drawn from a distribution with an unknown parameter. The parameter is assumed to be fixed but unknown. It does not allow any probability distribution to be associated with it. The only probability that can be considered is the probability distribution of the random sample of size *n* given the parameter. This explains how the random sample varies over all possible random samples, given the fixed but unknown parameter value.

However, Bayesian inference entails the rules of probability and probabilities of events are numbers between 0 and 1; where 0 means impossibility or failure while 1 means certainty.

Given two events, M and N, defined on a sample space, the conditional probability that M occurs given N has occurred is defined as:

$$P\,(M|N) = \frac{P(M \cap N)}{P(N)} \qquad (1.1)$$

In equation (1.1), $P(M \cap N)$ is the probability that both M and N occur while $P\,(M|N)$ is the probability that M occurs given N has occurred.

Interchanging the roles of M and N yields;

$$P(M \cap N) = P\,(N|M)P(M) \qquad (1.2)$$

Substituting equation (1.2) into (1.1) gives;

$$P\,(M|N) = \frac{P\,(N|M)P(M)}{P(N)} \qquad (1.3)$$

In the field of Econometrics, data is used to learn about economic phenomenon through the econometrics models containing parameters.

Suppose we want to make inference on a parameter, say θ of a particular model and also learn about the data say, y. In Bayesian point of view, we replace M by θ and N by y and write equation (1.3) as:

$$P\,(\theta|y) = \frac{P\,(y|\theta)P(\theta)}{P(y)} \qquad (1.4)$$

$P(\theta|y)$ is the basic interest in Econometrics which means that if given the data, what do you know about the parameter.

The denominator on the right hand side, $P(y)$ is the marginal distribution of the data which does not involve the parameter θ and the fact that the purpose of inference is about the θ, it is necessary to ignore the term $P(y)$ and write equation (1.4) as:

$$P\ (\theta|y) \propto P\ (y|\theta)P(\theta) \qquad (1.5)$$

$P(\theta|y)$ is called the Posterior distribution, $P\ (y|\theta)$ is the likelihood function while $P(\theta)$ is the prior density.

Thus, equation (1.5) can simply be interpreted as "Posterior distribution is proportionally related to the likelihood function times the prior density function".

Posterior distribution is an important concept in the study of Bayesian econometrics. It contains all the necessary and up-to-date information needed for Bayesian inference. It also provides a complete picture of current state of knowledge arising from both the data and prior information.

So many methods have been proposed to overcome the problem of multicollinearity in classical approach. These include the use of ridge estimator by Hoerl and Kennard (1970), Ordinary Ridge estimators (ORE) by Judge et al (1978), Judge et al (1989) and principal component, but all the aforementioned methods have shortcomings in that they do not make use of prior information on the parameters on interest.

In this study, Bayesian estimators were provided to handle the problem of multicollinearity in regression model. The Bayesian estimators are: Bayesian Non-informative Prior (BNIP) and Bayesian Informative Prior (BIP), while their estimates were then compared with the estimates of Classical method, namely, Likelihood Based (LB) method in order to investigate the sensitivity of these estimators under six degrees of collinearity; High Positive Collinearity (HPC), Moderate Positive Collinearity (MPC), Low Positive Collinearity (LPC), High Negative Collinearity (HNC), Moderate Negative Collinearity (MNC) and Low Negative Collinearity (LNC).

1.2 Statement of the Problem

Multicollinearity in a normal linear regression model is a serious problem in applied econometrics. A major drawback of the classical approach to the estimation of normal linear regression model is that it is indeterminate in the presence of extreme perfect multicollinearity. The use of out-of-sample information by the Bayesian approach to resolve this problem has not been fully explored in existing literature on the subject. It is therefore of interest to employ a Bayesian technique to derive estimators for normal linear regression model and investigate the sensitivity of the estimators to degree of collinearity among the regressors. It will enable future researchers to identify appropriate estimation method under different scenarios.

1.3 Justification

Regression models are the workhorse of econometrics that have a wide range of applications in various fields and are used in prediction of one variable from the other. One of the assumptions of classical linear regression model is, there is no multicollinearity among regressors. If this assumption is violated, the popular OLS estimator could become unstable due to large standard error and wider confidence interval. It can also lead to the difficulty in assessing individual effects of the correlated regressors on the dependent variable, and in turn, lead to wrong inferences on the model.

In literature, there are so many existing methods to solve the problem of multicollinearity using some classical approaches such as stepwise regression that entails adding and dropping of variables in a regression model but Leamer (1983) observed that dropping of variables in a regression model might lead to loss of vital information on the parameters of interest; Lee et al (2015) also observed that the use of principal component in multicollinearity may be inappropriate in the sense that, in principal component analysis, only major principal components in the regressors are retained while minor components are thrown out; the regression model that includes all major principal components might not have enough explanatory power on the dependent variable.

Classical methods can also be very sensitive to the slightest change in data and they do not make use of prior information on the parameters of interest while Bayesian method combines prior information with the likelihood function in providing its estimates.

1.4 Aim and Objectives of the Research

The aim of the work is to employ the Bayesian technique to derive estimators for Normal Linear Regression Model and investigate the sensitivity of the estimators to degree of collinearity among regressors.

The specific objectives of the research are outlined as follow to:

(1) Compare the derived Bayesian estimators with Likelihood Based (LB) method in the presence of multicollinearity.
(2) Compare the Bayesian posterior simulation method with analytical method in the presence of multicollinearity.
(3) Examine the sensitivity of Bayesian posterior simulation method on multicollinearity to replication.

1.5 Scope of the Study

This study focused on six degrees of collinearity; High Positive Collinearity (HPC), Moderate Positive Collinearity (MPC), Low Positive Collinearity (LPC), High Negative Collinearity (HNC), Moderate Negative Collinearity (MNC) and Low Negative Collinearity (LNC) in regression analysis using the Bayesian approach. Two out-of-sample priors namely, informative and non-informative priors were considered.

1.6 Organization of the Thesis

This thesis is organised as follows: Chapter one gives the introduction to the work; Chapter two reviews literature on Regression Model, concept of multicollinearity, multicollinearity with different classical approach and concept of Bayesian estimation methods; Chapter three discusses the theoretical framework for the research which involves the specification of the model, assumptions of the model, Bayesian estimator and statistical theories of various concepts.

Chapter four is the methodology which contains the step by step analysis of the experiments carried out and tools used for analysis. Chapter five presents the results of the analysis and discussion of the results while Chapter six concludes the work and appropriate recommendations based on the findings were made.

CHAPTER TWO

REVIEW OF LITERATURE

2.1 Introduction

This chapter reviews past and recent developments of central tool in applied econometrics, regression model; concept of multicollinearity; its causes, effect, and remedial measures; developments in the application of Classical and Bayesian estimators to handle the problem of multicollinearity in linear regression model and the concept of posterior simulation techniques in Bayesian approach.

2.2 Regression Model

The concept of regression which dates back to 1886 was introduced by Galton Francis and supported by his friend called Karl Pearson. It was used to explain the tendency for tall parents to have tall children, and short parents to have short children. The average height of children born of parents of a given height tends to move or "regress" toward the average height in the population as a whole.

Regression analysis is a statistical method used in investigating the relationship of variables, in order to estimate or predict the average value of one variable on the basisof the fixed values of other variables. One variable is called the dependent variable Y which is known to be statistical, random, or stochastic, that is, to have a probability distribution while the other variable is known as explanatory variable X that has fixed values. Regression can be either linear or non-linear. They are said to be linear; when the models are linear in the parameters while they are said to be non-linear when they are nonlinear in the parameters.

However, some regression models can look nonlinear in the parameters but are inherently or intrinsically linear because with suitable transformation, they can be made linear-in-parameters.

2.3 Collinearity

Collinearity often arises in many real-world applications and it refers to a situation in which there is an exact (or nearly exact) linear relationship between regressors, when there is more than one exact linear relationship, it is called multicollinearity (Hawking, 1983). When collinearity is exact, the regression coefficients using the Ordinary Least Squares (OLS) method will be indeterminate and their standard errors are infinite, when it is nearly exact, the regression coefficients are determinate but possess large standard errors. Thus, the parameters cannot be estimated with great accuracy Farrar and Glauber (1967) and Gujarati (1995).

Ballentine View of Collinearity

Collinearity can be visually represented with Venn diagrams called Ballentine diagram by Kennedy (1981). Figures 2.1, 2.2 and 2.3 represent different degrees of collinearity with the regression model given by:

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \mathrm{u} \quad (2.1)$$

Where y is the dependent variable, β_0, β_1, β_2 are the parameters to be estimated, X_1 and X_2 are the regressors and u is the error term.

The light blue colours represent regressors X_1 and X_2 while the deep blue colour represents the collinearity. The sizes of the circles and the manner in which they overlap illustrate various aspects of collinearity.

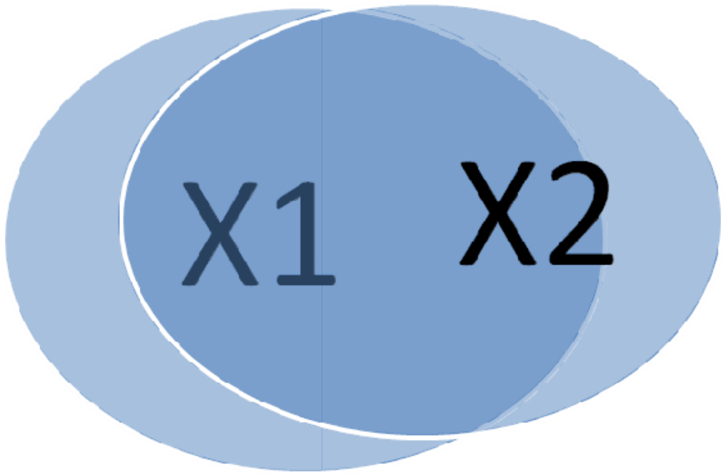

Figure 2.1: Diagram Showing High Collinearity

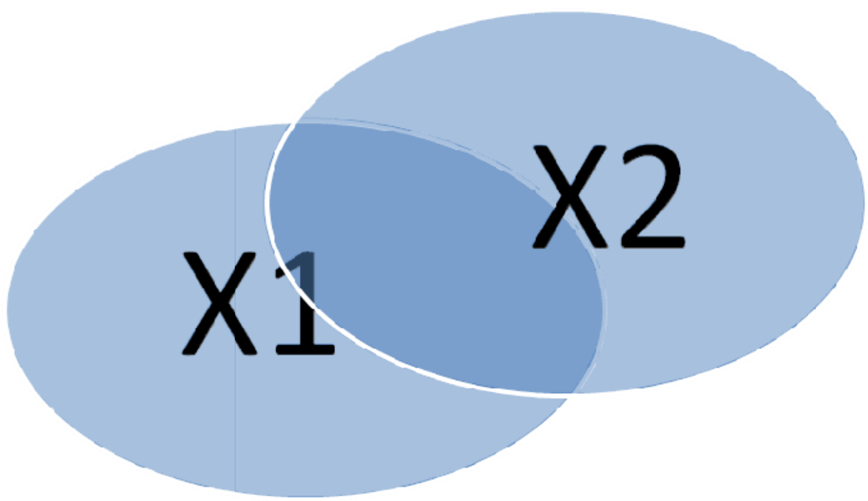

Figure 2.2: Diagram Showing Moderate Collinearity

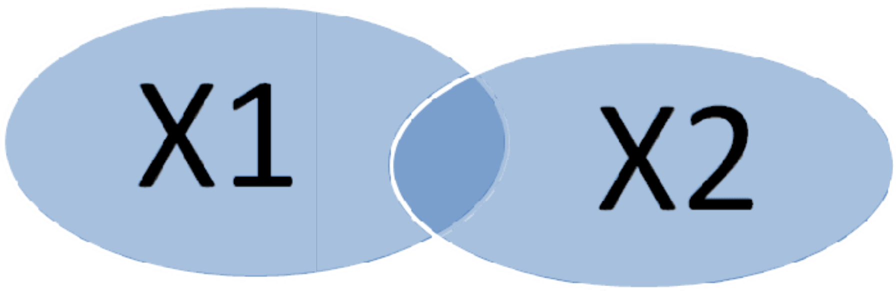

Figure 2.3: Diagram showing Low Collinearity

Causes of Collinearity

Several researchers like Mason et al (1975), Gunst and Mason (1977), Belsley (1980) and Montgomery and Peck (1982) carried out researches on the causes of collineairty; some of the causes are discussed below:

Belsley (1980) noted that method of data collection employed by a researcher can cause collinearity; it is expected that the data are collected over the whole cross-section of variables, but it may erroneously happen that such data might have been collected over a subspace of the explanatory variables where the variables are linearly dependent. Thus, the problem of collinearity arises.

An over determined model due to over zealousness of a researcher by including large number of regressors in the model in order to make it more realistic can also cause collinearity or an under-determined model, when a relevant explanatory variable is omitted in a model, for example, if we have a true model as:

$$y_i = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + u_i \qquad (2.2)$$

Where y_i is the dependent variable, β_1, β_2, β_3 are the parameters to be estimated, X_2 and X_3 are the regressors and u_i is the error term. But because of some reasons we fit the model in (2.2) by omitting X_3 and now have:

$$y_i = \alpha_1 + \alpha_2 X_2 + v_i \qquad (2.3)$$

where α_1 and α_2 are parameters and v_i is the error term.

If the omitted variable, X_3 is correlated with variable, X_2 , the correlation between the two variables is non-zero while the estimators $\widehat{\alpha_1}$ and $\widehat{\alpha_2}$ will be biased, the confidence interval and hypothesis testing procedures may give misleading conclusions.

Collinearity can also arise as a result of addition of polynomial terms to the regression model, especially when the range of the regressors is small or when some constraints on the model or on the population where the sample was drawn. The sample may be generated from the population having linear combinations which could not be so.

Consequences of Collinearity

One of the consequences of collinearity when there is exact linear relationship among the regressors, the least-squares estimator is not defined. This means that moment matrix $X'X$ will be singular and estimation of coefficients and standard errors will not be possible.

In the case of when regressors are highly related to one another, the OLS estimators tend to have large variances and standard errors, making precise estimation difficult, because of this consequence, confidence intervals also tend to be wider which can lead to the acceptance of the null hypothesis.

Collinearity can also results to large variances and standard errors of OLS estimator, while the t-ratio of one or more of the parameters of the regression model may likely also be insignificant and also the R^2value for the model may still be relatively high.

Detection for Collinearity

There are several methods to detect collinearity and they are highlighted as follows:
Variance Inflation factor (VIF) which measures the combined outcome of the dependences among the regressors on the variance can be used. The terminology, VIF, is due to Marquardt (1970) and can be computed as:

$$\text{VIF} = \frac{1}{1-R_j^2}, \qquad (2.4)$$

Where R_j^2 is the coefficient of determination in the regression of regressor X_j on the remaining regressors in the model. If VIF > 10, it is assumed that there exists a high collinearity.
Eigen values of $X'X$, say, $\lambda_1, \lambda_2, \ldots, \lambda_k$, can also be used to measure the extent of collinearity in the data. One or more small eigen values imply that there are near-linear dependences among the columns of X variable (Wetherill et al (1986), Greene (2000) and Draper and Smith (2003)). If the correlation exists between two regressors, it implies that there is presence of collinearity. If the correlation is high, say, 0.8, then, collinearity is a serious problem.

Farrar-Glauber Test

Another test for detecting collinearity apart from the ones mentioned above is called Farrar-Glauber test by Farrar and Glauber (1967). This involves the use of three test statistics (chi-square, t and F tests), but was criticised by Kumar (1975) and O'Hagan and Mcbabe (1975). The statistics are:

(i) **Chi-square test**: This is a test to determine the presence as well as the degree of collinearity in a model. A matrix of pair wise correlation coefficients (r_{ij}) is formed from the regressor variables, following the matrix obtained for k regressor.

$$\begin{bmatrix} 1 & r_{12} & r_{13} & \cdots & r_{1k} \\ r_{12} & 1 & r_{23} & \cdots & r_{2k} \\ \vdots & \vdots & & \cdots & \\ r_{k1} & r_{k2} & r_{k3} & \cdots & 1 \end{bmatrix}$$

Then the determinant D of the matrix is computed. It is evident that if all the regressors are perfectly correlated, many r will be unity and the determinant of the matrix will be zero. At the other extreme, if all regressors are orthogonal, the elements will be zero. The determinant will be unity. It is intuitively clear that, the determinant of this type of matrix will be between zero and unity ($0 < D < 1$); depending on the degree of collinearity. The closer the value of the determinant is to zero, the more is the degree of colinearity and vice-versa.

The chi-square involving the calculated determinant can be simply carried out and the test statistic is given by:

$$x^2 = [n\text{-}1\text{-}\tfrac{1}{6}(2k+5)] \text{ X } \ln D \qquad (2.5)$$

Where x^2 is the calculated chi-square statistic

n is the sample size

k is the number of parameters in the model.

The chi-square has $\frac{1}{2}k$(k-1) df. The null hypotheses to be tested are:

H_0: r = 0

H_1: r ≠ 0

Decision: If the chi-square calculated is greater than chi-square tabulated, H_0 will be rejected and there is significant collinearity but otherwise accept H_0.

(ii) **t-test:** The t-test can also be used to know the variables that are actually responsible for collinearity among the variables, in other words, it is to determine the pattern of collinearity. The t-statistic can be computed as:

$$t_{cal} = \frac{r^2\sqrt{n-k}}{\sqrt{1-r^2}} \tag{2.6}$$

$$\text{t-tab} = t_{n-k,\alpha} \tag{2.7}$$

Where n is the sample size and k is the number of parameters, α is the level of significance.

Decision: If the t-cal > t-tab at a chosen level of significance, it means that two regressors x_i and x_j are responsible for the collinearity, if otherwise the two regressors are not responsible for collinearity.

(iii) **F-test:** The F-test can be used to identify the variables that are significantly affected by collinearity. This is done by regressing each regressor on other variables. i.e

$$\begin{aligned}
X_1 &= g(X_2, X_3, \dots, X_k) \\
X_2 &= g(X_1, X_3, \dots, X_k) \\
X_3 &= g(X_1, X_2, \dots, X_k) \\
&\vdots \qquad\qquad (2.8) \\
&\vdots \\
X_k &= g(X_1, X_2, \dots, X_{k-1})
\end{aligned}$$

R^2 for each of the equation can be tested for statistical significance and the test statistic is given as:

$$\text{F} = \frac{R^2/k-1}{(1-R^2)/(n-k)} \tag{2.9}$$

While the

$$F_{tab} = F_{k-1,n-k} \tag{2.10}$$

Decision: If F $>F_{tab}$, it means that the variable X_i , i = 1, . . ., k, which was regressed on others is correlated with some other regressors, if otherwise $X_i$is not correlated with some other regressors.

Remedial Measures for Collinearity

When faced with problem of collinearity, one of the things to do is to drop one of the collinear variables, but dropping a variable could lead to specification error and specification error arises from incorrect specification of the model used in the analysis. Hence, this remedy may be worse than the problem of collinearity.

Another suggested remedy to the problem of collinearity is to do nothing; this was a school of thought of Kennedy (1998). Sometimes the problem of collinearity may not necessarily be bad or unavoidable. If the $R^2$of the regression is high, there should not be much worry. Also, if the t-statistics for all the parameters in a regression model are statistically insignificant, there should be no cause for alarm. If the estimation equation is used for prediction and the collinearity problem is expected to prevail in the situation to be predicted, we should not be concerned much about collinearity.

Collinearity is known to be a sample problem. It is possible that in another sample involving the same variables, collinearity may not be as serious as in the first sample, but increase in sample size would help reduce the severity of collinearity problem.

Combining both time-series and cross-sectional data known as pooling the data and transformation of variables by using suitable transformations like logarithms, forming ratios, etc were also found to be of great help in the reduction of collinearity problem, but pooling of data may create the problem of interpretation Stewart and Kenneth (1981).

2.4 Applications of Bayesian

Reverend Thomas Bayes in 1763 discovered the theorem called “Bayes theorem” while Pierre Simon Laplace gave the modern mathematical form and scientific application. This theorem is based on the method of statistical inference on how one could learn the probability of a future event occurring and how many times such event might have occurred or not occurred in the past.

After the work of Bayes, the applications of Bayesian now cut across different fields. In Biological sciences, we have the works of DeJong and Whiteman (1991), Albert and Chib (1993a), Barberis (2000), DeJong et al(2010) in finance, Crome (1996), Dennis (1996), Volinsky et al (1997), Anderson et al (2000), O'Hagan and Luce (2003), Wintle et al (2003), McCarthy and Parris (2004), Fidler (2004), Clark (2005), Martin et al (2005) and McCarthy and Masters (2005).

In social sciences, we have the works of Green (1962), Green and Frank (1966), Faigman and Baglioni (1988), Western and Jackman (1994),Fenton et al (2003), Rossi et al (2005) and Gill (2017) while in the field of Econometrics, we have Zellner (1976), Bauwens (1984), Geweke (1989), Chib et al (1998), Brown et al (1999), Chib and Hamilton (2000) among others.

2.5 Bayesian in Different Econometric Models

Recently, Bayesian methods are increasingly becoming attractive to researchers and can be applied in many models like regression, simultaneous equation model, seemingly unrelated model, vector autoregressive model, state space model, and some other time series models. The works using Bayesian approach for different models were accounted for in the works of Dreze (1962), Tsurumi (1980), Dreze and Richard (1983), Tsurumi (1985), Tsurumi (1990), Albert and Chib (1993), Chib and Greenberg (1994), Chib and Greenberg (1995), Bauwens and Lubrano (1996), Zellner (1998), Chao and Phillips (1998), Bauwens and Lubrano (1998), Kleibergen and Van Dijk (1998), Kleibergen and Zivot (1998), Tsurumi (2000), Gao and Lahiri (2001), Radchenko and Tsurumi (2004), Verzilli et al (2005), Ando and Zellner (2010), Mi et al (2012) and Choy and Charles (2017) etc.

In simultaneous equation model, Tsurumi (1985), Chao and Phillips (1998), Chao and Phillips (2000), Gao and Lahiri (2001), Zellner (1998), Radchenko and Tsurumi (2004) all focused on the development, derivation of the posterior distribution for the structural parameters, development of framework for construction of prior probability density functions, development of weak instrument in the limited information analysis, development of algorithms to estimate the parameters in order to solve the problem of weak exogeneity of endogenous variables, they concluded that Bayesian estimates are more highly concentrated about the true value of the coefficient being estimated.

Chib and Greenberg (1995), Verzilli et al (2005), Wang (2010), Ando and Zellner (2010), Choy and Charles (2017), and Billio et al (2017) all considered the use of Bayesian approach in SUR model. Chib and Greeberg (1995) in their studies developed efficient algorithms to estimate Markov time-varying parameters of SUR model using Bayesian approach, their algorithms was found to be useful for structural models with different identification restrictions. Verzilli et al (2005) investigated the use of Bayesian approach in order to model the statistical association between markers at multiple loci and multivariate quantitative traits using SUR model. They concluded that the use of Bayesian approach perform excellently well due to the use of prior distribution.

Wang (2010) considered a Sparse SUR model to generate relevant structures when there are high-dimensional distributions of SUR model parameters. He proposed a fully Bayesian analysis that can provide effective methods for computation using a specified graphical model to structure of covariance matrix. Their model was applied to a macroeconomic and finance which was shown to have practical significance.

Billio et al (2017) used a Bayesian approach in SUR models to study the interactions among different variables. They also proposed a hierarchical Dirichlet process prior for SUR models that allows the shrinkage of SUR parameters toward multiple locations and identification of group of parameters. It was observed that their approach can also be applied to other complex models.

In Vector Autoregressive (VAR) model, the works using Bayesian approach involve the works of Brandt and Freeman (2005), Adenomon et al (2015), Carriero et al (2015), Nicolalde (2016), Koop et al (2016), Kalli and Griffin (2018) among others.

Brandt and Freeman (2005) applied a Bayesian approach to time series model in the study of politics to analyze political data of Israeli-Palestinian conflict of the 1980s.
A reference prior was used to forecast over the short and medium terms. They concluded that their developed Bayesian procedure can also be used for economic data especially in volatility clustering.

Adenomon et al (2015) explored the short term forecast when there was problem of limited data in time series analysis. They evaluated the performances of both the classical VAR and

Bayesian VAR (BVAR) for short term series at different levels of collinearity. Their results showed that the BVAR models outperformed the classical VAR for time series length of 8 for all levels of collinearity, while the classical VAR was effective for time series length of 16 for all collinearity levels.

Carriero et al (2015) examined how the forecasting performance of Bayesian VAR (BVAR) was affected by a number of specification choices. They used a Normal-Inverted Wishart prior combined with a pseudo-iterated approach that made the analytical computation of multi-step forecasts feasible and simple. With the aid of empirical data, it was shown that very small losses from the adoption of specification choices made BVAR modeling quick and easy.

Nicolalde (2016) explored Bayesian Vector Autoregression (BVAR) priors which served as high-dimensional models. He carried out a forecasting exercise using the dummy observations prior and Conjugate Stochastic search variable selection while five models with different size and large number of lags were specified. Comparison based on his proposed method and OLS estimator showed that larger models outperformed the OLS using Mean Square Forecast Error as criterion.

Koop et al (2016) developed an alternative idea when large VAR model is involved by using a Bayesian approach, and this method involves randomly compressing the explanatory variables prior to analysis. Their developed Bayesian method was compared with the classical approach in the analysis of macroeconomic data which performed very well than the classical approach.

Kalli and Griffin (2018) proposed a nonparametric VAR model that allowed nonlinearity in the conditional mean, heteroscedasticity in the conditional variance, and non-Gaussian innovations while their proposed method was applied to US and UK macroeconomic time series, and compared to other Bayesian VAR models. Their approach showed that Bayesian nonparametric VAR was able to account for nonlinear relationships and heteroscedasticity in the data used while the short-run out-of-sample forecasts showed that the Bayesian nonparametric VAR predictably outperformed competing models.

2.6 Literature on Bayesian Regression Estimation

Bayesian method is universal and used by the researcher to learn about a phenomenon using data Koop (2003). Estimation of parameters is one of the things an econometrician would be interested in. A lot of works have been done on the estimation of parameters in Bayesian framework. These can be seen in the works of Poirier (1995), Zellner (1971), Zellner (1986), Zellner et al (1988), Gunn and Misbau (1993), Geweke, and Tanizaki, (2001), Coelho et al (2011), Handwin et al (2016), Ismail (2010), England and Gottschalk (2002), Dey (2012), Lopez (2013), Aliyu and Yahya (2016), Leon-Novelo and Savitsky (2018).

Yahya et al (2014) also examined the performance of Bayesian conjugate normal linear regression method with classical ordinary least squares when data satisfy all the necessary assumptions of OLS and the prior information on functional forms of regression parameters is available using a Monte Carlo study. Their results showed that Bayesian estimator is more efficient and consistent, and relatively more stable than the classical least squares method even when the sample data satisfy all the necessary assumptions of the OLS method.

Ahmad et al (2016) employed a Bayesian method of estimation to estimate the scale parameter of Nakagami distribution by using Jeffreys' extension and quasi priors, under three different loss functions with three different sample sizes while their results were compared with the classical Maximum likelihood method. They concluded that the Bayesian approach outperformed the classical Maximum likelihood method.

Dey (2012) obtained the Bayes estimators for the unknown parameters of an inverse Rayleigh distribution. The Bayes estimators were obtained under squared error (SE) loss and asymmetric linear exponential loss functions using a non-informative prior. They assessed the performance of the estimators based on the basis of their relative risk under the two loss functions. They concluded that estimation under the LINEX loss function was superior to the SE loss function with respect to the root mean squared error measure.

Lopez (2013) used a Bayesian technique to estimate the parameters of the simplex regression, and compared it with Beta regression using a simulation approach. He considered both the models with constant variance and models with variance heterogeneity while the Regressions were exemplified with heteroscedasticity. It was shown that when the true model was homogeneous simplex and also the estimates were closer to the true value parameters than the

Beta model, the same conclusion was also applied to the case when the models were heterogeneous.

Najafabadi and Najafabadi (2016) considered the problem of estimating Cronbach's alpha using a Bayesian approach, and employed a non-informative prior distribution under squared-error and LINEX loss functions. A simulation study was carried out and it suggested that the Bayes estimator under LINEX loss function reduced biasedness of the ordinary maximum likelihood estimator, and also, LINEX Bayes estimator was not sensitive with respect to choice of hyperparameters of prior distribution.

Neon-Lovelo and Savitsky (2018) considered a Bayesian estimation using a regression model based by incorporating the sampling weights into the estimation, to support policymaking using informative sampling designs where subject inclusion probabilities were designed to be correlated with the response variable of interest. It was observed that their Bayesian approach performs credibly well.

2.7 Literature on Other Area of Interest in Bayesian Modelling

In Bayesian modelling, apart from estimation of parameters of models, some of the things an econometrician would also wish to do is to compare different models or obtain predictions from a model. All these can also be done under Bayesian framework.

Bayesian in Area of Model Comparison in Regression

There are wide literatures on Bayesian in the area of model comparison in Regression. These include the works of Bos (2002), Kass and Waseerman (2010), Hagan (1995), Smith and Spiegelhalter (1980), Zellner and Tobias (2001), Chib (1993), Rodriguez et al (2004), Griffiths and Wan (1994), Koop and Poirier (2001), Aitkin (1991), Kass and Raftery (1995), Carlin and Chib (1995), Verdinelli and Wasserman (1995) and Wetzels et al (2010).

Hagan (1995) proposed a fractional Bayes factor for Bayesian comparison of models. His approach was found to be consistent, simple, robust and coherent while Chib (1993) developed a practical framework for Bayesian analysis of Gaussian and standard-t regression models with auto correlated errors. He made use of Gibbs sampling, an iterative Markovian sampling method, and showed that his proposed approach can deal with high-order autoregressive process without requiring an important sampling.

Rodriguez et al (2004) proposed an efficient Gibbs sampler for simulations of a multivariate normal random vector when subject to inequality linear constraints. It was observed that their proposed approach can allow for number of constraints and also can cope with equality linear constraints.

Koop and Poirier (2001) developed new Bayesian methods for semi-parametric inference in the partial linear Normal Regression Model, by considering a constrained and unconstrained methods in testing of parametric regression models against semi-parametric alternativeness and prediction. Their developed method was able to handle both the constrained and unconstrained methods in parametric regression models.

Verdinelli and Wasserman (1995) developed a generalized method called a Savage-Dickey Density Ratio (SDDR) for computing a Bayes factor in regression model in the area of model comparison. They concluded that their methods in terms of computational complexity can be extended to other models, while Wetzls et al (2010) in their work proposed an Encompassing Prior (EP) approach to facilitate Bayesian model selection for nested models with inequality constraints. Their EP approach was able to generalize the Savage Dickey Density ratio method by accommodating both inequality and exact equality constraints.

Bayesian on Area of Prediction in Regression

Bayesian reasoning says that we should summarize our uncertainty about what we do not know, that is, y^* through a conditional probability statement, which means, prediction should be based on predictive density P$(y^*|y)$ Koop (2003). The interest of a Bayesian econometrician can also be in the area of prediction, that is given the observed data, say y, the econometrician may be interested in predicting some future unobserved data y^*. Koop et al (2007).

Bayesian works in the area of prediction can be found in the works of McCormic et al (2012), Zhong et al (2013) and Gillberg et al (2013) amongst others.

Zhong et al (2013) carried out a comparative analysis between two modelling techniques, Bayesian network and Regression models, by employing them in the study of accident severity analysis. Their results showed that the goodness of fit of Bayesian network is higher than that of Regression models in accident severity modelling while the results obtained can also be applied to the prediction of accident severity, which is one of the essential steps in accident management process.

Gillberg et al (2013) considered Bayesian approach in the prediction of weak effects in a multiple-output regression set-up, when the covariates were expected to explain a small amount of less than 1% of the variance of the target variables. It was observed that their approach outperformed other alternatives in genomic prediction of rich phenotype data, especially the information sharing between the noise and regression models which led to significant improvement in prediction accuracy.

2.8 Literature on use of Priors in Bayesian Inference

Development of prior distributions is undoubtedly the most controversial aspect of any Bayesian analysis (Lindley, 1983; Walters and Ludwig, 1994). A proper Bayesian analysis will always incorporate genuine prior information, which will help to strengthen inferences about the true value of the parameter, and ensure that any relevant information about it is not wasted.

However, considerable care should be taken when selecting priors, and the process by which priors are selected must be documented carefully. This is because inappropriate choices for priors can lead to incorrect inferences.

There are two (2) types of priors:

(1) Non-informative prior
(2) Informative prior

Non-informative prior: It is a kind of prior where little is known about the parameters. Non-informative prior was first used by Laplace, Bayes, Jeffreys and Gauss. Jeffreys prior was widely accepted in univariate case, but it is often criticized in multivariate settings. The reason is, a priori parameter independence must be assumed for the prior to be found.

After the work of Jeffrey, so many researches were carried out on the use of non-informative prior. These include the works of Aliyu and Yahya (2016), Rodriguez et al (2004), Zhou et al (2009), Coles and Tawn (1994), Gelman et al (2008), Banerjee and Bhattarchaya (2012), Gelma (2006), Kass and Wasserman (1996) and Wan and Griffiths (1998) amongst others.

Aliyu and Yahya (2016) explored the use of non-informative prior in Bayesian estimation. The estimates were obtained under the squared error, entropy and precautionary loss functions while extensive Monte Carlo simulations were carried out to compare the performances of the Bayes estimates with that of MLEs. Their estimates under the Entropy loss function outperformed squared error loss function and MLEs.

Zhou et al (2009) proposed a diriclet process and a probit stick-breaking process using a non-informative prior in Non-parametric Bayesian techniques with applications in denoising, inpainting and Compressive Sensing (CS). Coles and Tawn (1994) developed the technique that can make optimal use of available data. In their work, a daily rainfall series was analysed within a Bayesian framework. They suggested that careful elicitation of prior expert information can supplement data and lead to improved estimates of external behaviour.

Gelman et al (2008) proposed a new prior distribution by placing independent Student-*t* prior distributions on the coefficients, which in the simplest setting is a longer-tailed version of the distribution attained by assuming one-half additional success and one-half additional failure in a logistic regression. It was observed that their prior outperformed existing Gaussian and Laplace priors.

Informative prior: Most orthodox works believe that the term prior information, does notexist at all. Jimmie Savage (1954) was believed to be the first author to incorporate prior opinions into scientific inference. Aftermaths of the work of Jimmie Savage brought about the use of informative prior.

Informative prior is referred to as a prior whereby information is available about the prior distribution. It summarises the evidence about the parameters concerned from many sources.

Sasaki and Kondo (2015) studied the use of paleodemography with Bayesian informative prior approach in order to provide an effective means by which mortality profiles of past populations can be adequately estimated. They also proposed an application of the Gompertz model to avoid the problems of "age-mimicry" inherent in conventional approaches.

2.9 Bayesian Posterior Simulation Techniques

Posterior distribution is an important concept in the study of Bayesian econometrics. It contains all the necessary and up-to-date information needed for Bayesian inference. It also provides a complete picture of current state of knowledge arising from both the data and prior information.

In Econometrics, there are various numeric summaries that are made e.g., mean, variance, median etc. These numeric summaries are obtained through the Posterior distribution by using integrations and most of these integrals have high-dimensional functions which cannot be solved analytically. If it cannot be solved analytically, it will make the computation very difficult. This is a major setback to the implementation of Bayesian approach.

Any Posterior features of interest in Bayesian inference meant for computation according to Geweke (1989), Tanner (1996) has this form:

$$E[u(\theta)|y] = \int u(\theta)p(\theta|y)\,\partial\theta \qquad (2.11)$$

Where $u(\theta)$ is the function of interest.

However, there is a recent development of powerful computing methods in Bayesian econometrics, thereby solving the difficult analytical calculations. The method is called a Posterior Simulation. There are many posterior simulation techniques that have been designed by many scholars for the implementation of Bayesian computation. Bayesian Posterior simulations are divided into two: *Direct simulation method* and *Iterative simulation method*:

Direct Simulation

It is a kind of simulation whereby samples are not obtained iteratively (depend on previous samples). Examples are: Monte Carlo Integration by Geweke (1989),importance sampling by Kloek and van Dijk (1978), Bauwens (1984) and Richard and Zhang (2000), rejection sampling and sequential Monte Carlo sampling.

Monte Carlo Integration

It is the simplest posterior simulation method and can be used when exact result cannot be obtained analytically. One of the advantages of Monte Carlo Integration is that large number of Posterior moments can be estimated at reasonable computational efforts while estimates of numerical accuracy of the results can also be obtained in a very simple way Kloek and Dijk (1978).

Considerable research have been carried out on the use of Monte Carlo Integration (MCI) in regression model and other models in literature and various issues have been the subject of discourse. Early works on MCI are the works of Kloek and Dijk (1978),Dijk and Kloek (1980),Bauwens (1984), Geweke (1988), Phillips and Marks (1996), Yool (1999) and recent ones are: Hakanson (2000), Richard and Zhang (2000) and van Horssen et al (2002) among others.

Dijk and Kloek (1980) employed a MCI for nine dimensional parameter space of Klein's model. They also showed how Monte Carlo can be used as a tool for the elicitation of *prior information,* and how the initial prior information on structural parameters can be modified by specifying prior information on multipliers and the period of oscillation.

The application of MCI as a posterior simulator in ecological modelling was also demonstrated in the works of Phillips and Marks (1996), Yool (1999), Hakanson (2000), van Horssen et al (2002) for uncertainty analysis.

Importance Sampling:

It is an approach for approximating the integral associated with E(h(θ) for some function h.

The classical approach using a Monte Carlo algorithm is known to begin by drawing N samples, $x^{(i)}$, i = 1, . . . , N which is uniformly over Θand approximates the integral by the sample mean of h $(\theta^{(i)})$f $(\theta^{(i)})$. Importance sampling extends this by drawing samples from a trial distribution g. More efficient algorithm was obtained when g is close to f. Importance sampling produces weighted samples with weights given by the ratio f/g. One can work either directly with the weighted samples, or resample with respect to the weights for a set of un-weighted samples.

The steps involved in Importance sampling are:

Step 1: Draw N samples $\theta^{(1)}, \ldots, \_\theta^{(N)}$ from g((θ)

Step 2: Evaluate weights $\frac{f(\theta^{(i)})}{g(\theta^{(i)}}$ for i = 1, . . . , N

Sequential Monte Carlo:

The sequential Monte Carlo (SMC) sampler can be viewed as an extension of importance sampling. It allows intermediary steps and propagates moves within each distribution. Crucially, SMC does not require an initial distribution which takes the same support as the target distribution. This can be considered a major advantage, particularly, for high dimensional problems. Furthermore, SMC is able to deal with far more complex problems by allowing corrections to the initial samples iteratively.

As in importance sampling, SMC produces weighted samples. The following shows the steps involved using SMC:

Step 1: Draw samples for $\theta_0^{(1)}, \ldots, \theta_0^{(N)}$ from initial distribution $f_0(\theta)$

Step 2: Initialise weights w(i)

for i = 1, . . . ,N, Move samples $\theta_{t-1}^{(s)}$ according to forward transition kernel

Iterative Simulation

It is a simulation technique that relies on the construction of a Markov chain unlike direct simulation. All posterior simulation that belongs to classes of this iterative simulation is also referred to as Markov chain Monte Carlo (MCMC) methods. According to Gelman and Rubin (1992), Raftery and Lewis (1992), Gilks et al. (1996), Gilks and Roberts (1996), Gilks (1996), Gelman (1996), Gelman et al (2004), before starting the Markov chain at any (arbitrarily) starting point, the standard MCMC theory guarantees that the chain will converge to the correct distribution.

One crucial difference between the iterative methods and the direct simulation methods is that iterative methods produce serially correlated samples. It is also very important that the initial portions of the MCMC sample be discarded (usually termed burn-in).

The determination of the length of burn-in and the total length of Markov chain is collectively known as convergence diagnostics. Cowles and Carlin (1996) gave a comparative review of the various methods available in literature for the assessment of convergence. Examples of iterative simulation are: Gibbs sampler introduced by Gelfand and

Smith (1990), followed by works of Carlin and Polson (1991), Casella and George (1992), Dellaportas and Smith (1993), Chib (1993),Chib (1995), Bauwens and Lubrano (1998), Damien et al (1999), Rodriguez et al (2004); Adaptive rejection sampling introduced by Gilks and Wild (1992); Slice sampling, Metropolis-Hastings sampling by Hastings (1970), followed by works of Chib and Greenberg (1995), Geweke and Tanizaki (2001), Chib and Jeliazkov (2001).

Gibbs Sampler

The Gibbs sampler is a Markov chain sampler that starts at any arbitrary initial state. The chain then gets iteratively updated for some specified N iterations. At every iteration, it cycles through each of the k components of the parameter $\theta = (\theta_1, \ldots, \theta_k)$ in turn. The parameters are updated through the new sample according to their distributions conditioned on current values of other parameters. Casella and George (1992) provided an easy to read explanation of how the Gibbs sampler works.

Here is a typical example of algorithm of Gibbs sampler:

Steps for Gibbs sampler

Step 1: Choose a Starting value $\theta^{(0)}$ for s = 1, . . . , S

Step 2: Take a random draw, $\theta_{(1)}^{(s)}$ from P $(\theta_{(1)} | \, y, \theta_{(2)}^{(s-1)}, \theta_{(3)}^{(s-1)} \ldots, \theta_{k}^{(s-1)})$, $\theta_{(2)}^{(s)}$ from P$(\theta_{(2)} | \, y, \theta_{(1)}^{(s-1)}, \theta_{(3)}^{(s-1)} \ldots, \theta_{k}^{(s-1)})$, . . . , $\theta_{(k)}^{(s)}$ from P$(\theta_{(k)} | \, y, \theta_{(1)}^{(s-1)}, \theta_{(2)}^{(s-1)} \ldots, \theta_{(k-1)}^{(s)})$

Step 3: Discard the burn-in- period and focus on the retained one S_1

Step 4: Then carry out analysis on the remaining retained.

Where, S = S_0 + S_1 , where S = replication, S_0 =Burn-in-period and S_1 =the retained one.

Adaptive Rejection Sampling (ARS)

The Adaptive Rejection Sampling (ARS) was first introduced by Gilks and Wild in 1992, strictly for log-concave densities. The algorithm proceeds as in the Gibbs sampler, cycling through each of the univariate parameters, in turn, sampling from the conditional densities. Whereas the Gibbs sampler requires these conditional densities to be a standard distribution such that sampling from it will be easy. The adaptive rejection sampling method will work for any logconcave conditional densities. Specifically, the difference between the Gibbs sampler and adaptive rejection sampling is the conditional distribution.

A typical adaptive rejection sampling algorithm is given below:

Step 1: Initialise the K abscissa TK = {x_j , j = 1, . . . ,K}

Step 2: Sample y from s(θ) and sample w from Unif(0,1).

Step 3: If w ≤ exp{l(y) − u(y)}, set $\theta^{(i+1)}$= y. Otherwise go to Step 2.

Step 4: If w ≤ exp{h(y) − u(y)}, set $\theta^{(i+1)}$= y. Otherwise go to Step 3.

Step 5: TK+1 = TK ∪[{y}, K = K + 1] and go to Step 2.

2.10 Review of Some Studies on Classical Method of Estimation Using Ridge Estimator for Collinearity

One of the suggested solutions to the problem of collinearity in regression models by different scholars is the use of ridge estimators. The use of ridge estimator first came into existence by Hoerl and Kennard (1970). The purpose is to handle collinearity in engineering data. Their findings state that there is a non-zero value of ridge parameter called k for which the Mean Squared Error (MSE) for the ridge estimator has a minimum variance than the Ordinary Least Squares (OLS) Kibra and Banik (2016).

The ridge solution suggested by Hoerl and Kennard (1970) is given as:

$$\beta\ (\hat{k}) = (X'X + kI)^{-1}X'Y\ , \quad k \geq 0 \qquad (2.12)$$

After the works of Hoerl and Kennard (1970a), and Hoerl and Kennard (1970b), more research had been carried out on the use of ridge estimator since then. This can be seen in the works of Lawless and Wang (1976), Dempster et al. (1977), Gibbons (1981), Gibbons and McDonald (1984), Nomura (1988), Kibria (2003), Khalaf and Shukur (2005), Zhang and Ibrahim (2005), Alkhamisi et al. (2006), and recent ones by Muniz and Kibria (2009), El-Dereny and Rashwan (2011), Khalaf (2011), Jensen and Ramirez (2012), MacDonald and Galameau (2012), Khalaf (2013), Duzan and Shariff (2015),Duzan and Shariff (2016),Khalaf and Iguernane (2016), Iguernane (2016) and Shariff and Ferdaos (2017).

The usage of ridge estimator in some works is highlighted:

Muniz and Kibria (2009)

They proposed three ridge estimators called K_{kM4} , K_{kM5} and K_{kM6} by applying arithmetic mean, geometric mean and square root based on the existing proposed methods by Kibria (2003), Khalaf and Shukur (2005) and Alkhamisi et al. (2006)for estimating the ridge parameter k and compared their performances of the estimators to Ordinary Least Squares (OLS) estimator by simulation study, using different degrees of correlation between the regressors, it was observed that the proposed estimators performed more than the OLS estimator.

El-Dereny and Rashwan (2011)

These authors considered the methods for reducing the influence of collinearity by using two classes of regression models while a great attention was paid to the use of ridge estimators. They proposed two alternative approaches to resolve the collinearity issue. The proposed two methods are: an application of the known Inequality Constrained Least Squares method and the Dual estimator method.

MacDonalad and Galameau (2012)

Their study proposed and also evaluated two analytic methods on how to specify k parameter in ridge regression with three explanatory variables, imposing four different kinds of correlations among the regressors. Their ridge estimators were evaluated by estimating the squared length of the unknown parameters, and then, choose the k parameter in the class of ridge estimators so that the corresponding ridge estimator has a squared length equal to the estimated quantity. They concluded by using a Monte Carlo simulation and observed that the proposed method performed well.

Duzan and Shariff (2015)

The authors reviewed the literature from years 1964 to 2014 on the proposed ridge estimators, to evaluate the ridge parameter k in order to provide guidance to users of regression models in handling the problems of collinearity. They concluded that the various estimations of ridge regression parameter k had improved and the estimation methods provided by a number of researchers were working well.

Duzan and Shariff (2016)

Robustness of ridge estimators were investigated in order to identify the most relevant k-value of ridge regression in four variable regression model, while the results of ridge estimators were compared with least squares method using a simulation study. They concluded that a ridge regression must be used when collinearity occurs in the estimation of parameters in regression model.

Khalaf and Iguernane (2016)

Their study focused on proposing a new estimator of ridge regression parameter when there is collinearity in a regression model. They modified the estimator of Khalaf and Shukur (2005) known as *KS* by finding the square root now called *KSM* estimator.

Their proposed estimator is given by:

$$\hat{k}_{KSM} = \sqrt{\hat{k}_{KS}} \tag{2.13}$$

And

$$\hat{k}_{KS} \text{ is } \frac{\lambda_{max}\widehat{\sigma^2}}{(n-p)\widehat{\sigma^2} + \lambda_{max}\widehat{\beta_{max}}} \tag{2.14}$$

Where λ_{max} is the largest eigenvalue of $X'X$

Results of their simulation study showed that the estimator *KSM* dominates Ordinary Least Squares (OLS) estimator, Khalaf and Shukur (2005) estimator, *KS* and Hoerl et al (1970) estimator with respect toMSE.

Iguername (2016)

He considered the problem of multicollinearity in the estimation of regression model when the degree is not high. He proposed two methods of finding the ridge regression parameter k called MI_1 and MI_2. The results of his simulation study using MSE criteria indicated that the proposed estimators performed better than the Ordinary Least Squares (OLS) estimator and HK estimator by Hoerl and Kennard (1970), *HKB* by Hoerl et al (1970), and *LW* estimator by Lawless and Wang (1976). OLS estimator got the worst in all cases using MSE criterion.

Shariff and Ferdaos (2017)

As an application, the authors combined the Generalized-M called GM estimation technique, the estimator proposed by Bagheri and Midi (2009), and ridge parameter in the presence of outliers and multicollinearity inorder to find the relationship between stock market price and some macroeconomic variables in Malaysia. They considered four macroeconomic factors namely; Consumer Price Index (CPI), Gross Domestic Product (GDP), Base Lending Rate and Money Supply. The GM estimator used is given as:

$$\hat{\beta}_{ROB} = (X'WX)^{-1}X'WY \qquad (2.15)$$

By introducing the technique of Bagheri and Midi (2009) and k-parameter, their estimator becomes:

$$\hat{\beta}_{ROBR} = (X'X + k\,I)^{-1}X'X\hat{\beta}_{ROB} \qquad (2.16)$$

Where $\hat{\beta}_{ROB}$ and $\hat{\beta}_{ROBR}$ are the Robust estimator and Robust Ridge estimator respectively. They concluded that their proposed estimator outperformed the earlier proposed method of Bagheri and Midi (2009).

2.11 Review of Some Studies on Classical Method of Estimation Using Other Methods for Collinearity

Reviews of some studies of other classical methods of estimation for collinearity are:

Oduntan (2004)

He carried out a research on the performance of six estimators in the presence of multicollinearity using a two-equation of just identified simultaneous equations model. Two levels of positive correlation among the predetermined variables known as the low and high multicollinearity were considered. His result shows that in the presence of multicollinearity, whether low or high, indirect least squares and OLS had better performance while other estimators performed poorly. It was also observed that, the estimators were not sensitive to sample sizes.

Agunbiade (2008)

A three-equation of just identified simultaneous equation model was considered when there is multicollineairty in order to compare the performance of six estimators using three levels of multicollinearity. The estimators considered are OLS, Two stage least squares (2SLS), Three stage Least Squares (3SLS), Limited Information Maximum Likelihood (LIML), Indirect Least Squares (ILS) and Full Information Maximum Likelihood (FIML) and the levels of multicollineairty are; the relatively highly negative correlation, relatively highly positive correlation and feebly negatively or positively correlation levels. He concluded that LIML, 2SLS and ILS are the best for estimating parameters of a model having the relatively highly negative correlation level of multicollinearity while OLS performed poorly under this scenario but performed best in the relatively highly positive correlation level of multicollinearity.

Other suggested methods to deal with the problem of collinearity in Regression analysis that had received a lot of attention in literature is the method of principal components. This method was proposed by Pearson (1901) and Hotelling (1933); and their concern was to find the best way to represent samples by using vectors with predictors, in such a manner so that the similar samples can be represented by points as close as possible. Some other authors that conducted research in this regard are; Jolliffe (1973), Mansfield et al. (1977), Mason and Gunst (1985), Boneh and Mendiet (1992), Tibshirani (1996), Angelo et al. (2012), Kim and Lee (2014) and Lee et al. (2015).

2.12 Review of Some Existing Studies on Bayesian Method of Estimation for Collinearity

There are limited literatures on the problem of collinearity using a Bayesian approach. These are reported in the works of Curtis and Ghosh (2011), Rajaratna et al. (2016), Ijarchelo et al. (2016), Hassan (2016), Efendi and Effrihan (2017). Most of these listed works made use of variable selection procedures.

Some of these existing literatures are highlighted as:

Curtis and Ghosh (2011)

They proposed a Bayesian model that accounted for correlation among the predictors by simultaneously performing selection and clustering of the predictors. They used a Dirichlet process prior and a variable selection prior for regression coefficient while redundant predictors were removed from the model. They concluded that Bayes method proposed did not outperform all other methods in all situations, but often the best in high collinearity.

Rajaratna et al. (2015)

They developed an algorithm called deterministic Bayesian LASSO. It was mainly designed to handle low to moderate multicollinearity settings. Their algorithm was based on exploiting the structure of the Bayesian LASSO, and the corresponding Gibbs sampler. The Bayesian LASSO is given by:

$$b^{(k+1)} = [X'X + \lambda(\beta^{(k)})^{-1}]^{-1}X'y \quad (2.32)$$

Hassan (2016)

He proposed a model selection procedure for the problem of high multicollinearity. His method led to the best m-models in terms of posterior model probability; a simulation study was carried out in order to compare the estimates of LASSO estimator obtained by Tibshirani (1996) with the estimates of Bayesian approach. His proposed Bayesian method performed better than the LASSO estimator.

Ijarchelo et al. (2016)

They developed a Bayesian regression procedure for variable selection under collinearity of parameters using a Zellner's g-prior given by:

$$\mathrm{P}\,(\beta_0, \psi |\, \Upsilon) \propto \frac{1}{\psi} \quad (2.33)$$

Their results showed that a strong collinearity may lead to a multimodal posterior distribution over models in which joint summaries are more appropriate than marginal summaries. They concluded that their posterior distribution were not available in closed form, and that can make the problem of collinearity become computationally challenging.

Londono (2016)

He proposed a model selection procedure when faced with the problem of high collinearity levels, and applied it to the inference over a treatment effect. He showed different frequentist and Bayesian approaches in the application to a model selection procedure based on a post double estimation procedure. His simulation results had evidence in favour of Bayesian procedures when the number of observations was not much higher than the number of possible controls, while a real life data of the impact of legalized abortion crimes rates were also used with a post double Markov Chain Monte Carlo Model Composition called MC^3.

Efendi and Effrihan (2017)

They conducted a simulation study in the implementation and evaluation of ridge regression model with Bayesian estimation method when the degree of collinearity is high using a Gibbs sampler; their posterior distribution obtained for ridge parameter is unknown. They concluded that their estimates of ridge regression models from both least squares and Bayesian methods have similar properties, while Bayesian method was better in small sample size setting.

CHAPTER THREE

THEORETICAL FRAMEWORK

3.1 Introduction

In this chapter, the model specification, classical methods of estimation of regression model and basic assumptions/principles guiding the application of Bayesian method are presented.

3.2 Linear Regression Model

Linear regression is probably the most widely used statistical technique for solving economic problems. Linear regression models are extremely powerful, and have the power to empirically simplify very complicated relationships between variables.
In general, the technique is useful among other applications to help in predicting observations of a dependent variable, usually denoted Y, with observed values of one or more independent variables, usually denoted by $X_1, X_2, \ldots$.

A key feature of all regression models is the inclusion of the error term, which captures sources of error that are not captured by other variables, the dependent variable Y, and an independent variable X. Hence, the simple linear regression model for Y on X is given by:

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i \qquad i = 1, \ldots, n. \tag{3.1}$$

Model Specification

The general idea of a simple linear regression model is that the dependent variable Y_i is a straight-line function of a single explanatory variable X_i. Here, we extend the simple linear regression model in (3.1) to multiple linear regression model by considering the dependent variable to be a function of k explanatory variables $X_{i1}, X_{i2}, \ldots, X_{ik}$. This relationship is a straight-line and can thus be written as:

$$Y_i = \theta_0 + \theta_1 X_{i1} + \theta_2 X_{i2} + \ldots + \theta_k X_{ik} + \varepsilon_i \tag{3.2}$$

Where the random errors ε_i, i = 1, …,n are independently and normally distributed random variables with zero mean and constant variance σ^2. The linear regression model in (3.2) means that the mean of the dependent variable can be expressed as:

$$E(Y_i) = \theta_0 + \theta_1 X_{i1} + \theta_2 X_{i2} + \ldots + \theta_k X_{ik} \quad (3.3)$$

The common assumption in linear regression model is the assumption of normality. In the case where the normality assumption was not satisfied, the use of generalized linear model becomes relevant.

Matrix Notation

Statistical results for multiple linear regression models such as parameter estimates, test statistic, etc., can become complex and tedious to write out, particularly, when the numbers of explanatory variables are more than two. A very useful approach is to simplify the complex expressions by introducing matrix notation.
The Linear Regression Model (LRG) in (3.2) can also be written as:

$$y = X\theta + \varepsilon \quad (3.4)$$

Where

y is N x 1 vector of the dependent variable

X is a N x k matrix of explanatory variables

θ is k x 1 of parameters vector

ε is a N x 1 vector of error terms

N and k are the number of observations and parameters, respectively.

Using matrix notation, equation (3.4) can be written as:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} 1 & X_{11} & \cdots & X_{1k} \\ 1 & X_{21} & \cdots & X_{2k} \\ \vdots & \vdots & \cdots & \vdots \\ 1 & X_{N1} & \cdots & X_{Nk} \end{bmatrix} \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_k \end{bmatrix} + \begin{bmatrix} \varepsilon_0 \\ \varepsilon_1 \\ \vdots \\ \varepsilon_N \end{bmatrix} \quad (3.5)$$

where

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}, \quad X = \begin{bmatrix} 1 & X_{11} & \cdots & X_{1k} \\ 1 & X_{21} & \cdots & X_{2k} \\ \vdots & \vdots & \cdots & \vdots \\ 1 & X_{N1} & \cdots & X_{Nk} \end{bmatrix}, \quad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_k \end{bmatrix}, \quad \varepsilon = \begin{bmatrix} \varepsilon_0 \\ \varepsilon_1 \\ \vdots \\ \varepsilon_N \end{bmatrix}$$

3.3 Assumptions Underlying Multiple Regression

Most statistical techniques have a set of underlying assumptions that guide their use in applications. The assumptions underlying regression model can be generally categorized into two: assumptions of the model and assumptions about the error term.

Assumptions of the Model

The assumptions about the model in equation (3.2) are as follows:

(i) It is a linear regression model i.e., linear in parameters.

(ii) The X's have fixed values which are independent of error term.

(iii) The number of observations n must be greater than the number of parameters to be estimated.

(iv) There should be no multicollinearity.

Assumptions on the Error Term

The following are the assumptions on the error term of the regression model in (3.4).

The mean of the probability distribution of the error term is zero ($E(\varepsilon_i) = 0$).

This is true by design of the estimator of OLS, but it also reflects the notion that it is not expected of the error terms to be mostly positive or negative (overestimation or underestimation of the regression line), but it should be centered on the regression line.

The probability distribution of error term has constant variance ($Var(\varepsilon_i)= \sigma^2$).

It implies that a constant variance for Y variable across all the levels of the independent variables is assumed. This is also called homoscedasticity, and it enables the pooling of information from all the data to make a single estimate of the variance. When data do not have constant error variance, we have heteroscedasticity.

The error terms are independent of each other and with the independent variables in the model ($Cov(\varepsilon_i, \varepsilon_j) =0$ and $Cov(X_i, \varepsilon_i) =0$).

It means that the error terms are uncorrelated with each other or with any of the independent variables in the model. Correlated error terms are common in time series data, and are known as auto-correlation. If there is correlation among the error terms and the independent variables, it usually implies that the model is mis-specified.

3.4 Estimation in Classical Regression Model

The econometrician is interested in estimating the parameters θ and σ^2, the Classical econometrician therefore obtains data y and X and simply write the likelihood function of the model in (3.0) as follows:

$$P(y|\theta, \sigma^2) = \frac{1}{(\sigma^2)^{N/2}(2\pi)^{N/2}} \exp\left[-\frac{1}{2\sigma^2}(y - X\theta)'(y - X\theta)\right] \quad (3.6)$$

There are two (2) generally used methods of estimation in Classical Regression model:

(1) Ordinary Least Squares (OLS)
(2) Maximum Likelihood method.

Ordinary Least Squares

This method is used extensively in regression analysis, primarily, because it is intuitively appealing and mathematically much simpler than the method of maximum likelihood, Cohen et al. (2003), Hung et al. (2012), Lavallee (2007), and Michalos and Kahlke (2010). However, the two methods OLS and maximum likelihood generally give similar results.

The principle of least squares is to find the ‘best fitting’ model. According to this principle, the best fitting model is the one that minimizes the sum of squared residuals, where the residuals are the difference between the observed variables and the values predicted by the fitted model. The smaller the residuals, the closer the fit.

The residuals $\hat{\varepsilon}_i$ can be obtained using the expression given by:

$$\hat{\varepsilon}_i = Y_i - [\hat{\theta}_0 + \hat{\theta}_1 X_{i1} + \hat{\theta}_2 X_{i2} + \ldots + \hat{\theta}_k X_{ik}] \quad (3.7)$$

We illustrate the derivation for the case of two regressors, X_{1i} and X_{2i}, with Y the dependant variable, α and β’s as parameters. The model in (3.2) now becomes:

$$Y_i = \alpha + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i \quad (3.8)$$

We look for estimators $\hat{\alpha}, \hat{\beta}_1, \hat{\beta}_2$ so as to minimise the sum of squared errors,

$$S = \sum_{i=1}^{n} (Y_i - \hat{\alpha} - \hat{\beta}_1 X_{1i} - \hat{\beta}_2 X_{2i})^2 \tag{3.9}$$

Differentiating and setting the partial differentials to zero we obtain the following normal equations:

$$\frac{\partial S}{\partial \hat{\alpha}} = \sum_{i=1}^{n} 2(Y_i - \hat{\alpha} - \hat{\beta}_1 X_{1i} - \hat{\beta}_2 X_{2i})(-1) = 0 \tag{3.10}$$

$$\frac{\partial S}{\partial \hat{\beta}_1} = \sum_{i=1}^{n} 2(Y_i - \hat{\alpha} - \hat{\beta}_1 X_{1i} - \hat{\beta}_2 X_{2i})(-X_{1i}) = 0 \tag{3.11}$$

$$\frac{\partial S}{\partial \hat{\beta}_2} = \sum_{i=1}^{n} 2(Y_i - \hat{\alpha} - \hat{\beta}_1 X_{1i} - \hat{\beta}_2 X_{2i})(-X_{2i}) = 0 \tag{3.12}$$

The three equations (3.10), (3.11) and (3.12) are called the "normal equations". Equation (3.10) can be written as

$$\sum_{i=1}^{n} Y_i = n\hat{\alpha} + \hat{\beta}_1 \sum_{i=1}^{n} X_{1i} + \hat{\beta}_2 \sum_{i=1}^{n} X_{2i} \tag{3.13}$$

or

$$\overline{Y} = \hat{\alpha} + \hat{\beta}_1 \overline{X}_1 + \hat{\beta}_2 \overline{X}_2 \tag{3.14}$$

Where the bar over Y, X_1 and X_2 indicates sample mean.

Equation (3.12) can also be written as

$$\sum_{i=1}^{n} X_{1i} Y_i = \hat{\alpha} \sum_{i=1}^{n} X_{1i} + \hat{\beta}_1 \sum_{i=1}^{n} X_{1i}{}^2 + \hat{\beta}_2 \sum_{i=1}^{n} X_{1i} X_{2i} \tag{3.15}$$

Substituting in the value of $\hat{\alpha}$ from (3.15), we get:

$$\sum X_{1i} Y_i = n\overline{X}_1 (\overline{Y} - \hat{\beta}_1 \overline{X}_1 - \hat{\beta}_2 \overline{X}_2) + \hat{\beta}_1 \sum X_{1i}{}^2 + \hat{\beta}_2 \sum X_{1i} X_{2i} \tag{3.16}$$

Similar equations result from (3.12) and (3.14). The equations can be simplified using the following notations:

$$S_{11} = \sum X_{1i}{}^2 - n\overline{X}_1{}^2 \qquad S_{1Y} = \sum X_{1i}Y_i - n\overline{X}_1\overline{Y}$$

$$S_{12} = \sum X_{1i}X_{2i} - n\overline{X}_1\overline{X}_2 \qquad S_{2Y} = \sum X_{2i}Y_i - n\overline{X}_2\overline{Y}$$

$$S_{22} = \sum X_{2i}{}^2 - n\overline{X}_2{}^2 \qquad S_{YY} = \sum Y_i^2 - n\overline{Y}^2$$

Equation (3.16) can then be written

$$S_{1Y} = \hat{\beta}_1 S_{11} + \hat{\beta}_2 S_{12} \qquad (3.17)$$

Similarly, equation (3.12) becomes

$$S_{2Y} = \hat{\beta}_1 S_{12} + \hat{\beta}_2 S_{22} \qquad (3.18)$$

The two equations (3.17) and (3.18) can then give:

$$\hat{\beta}_1 = \frac{S_{22}S_{1Y} - S_{12}S_{2Y}}{\Delta} \qquad (3.19)$$

and

$$\hat{\beta}_2 = \frac{S_{11}S_{2Y} - S_{12}S_{1Y}}{\Delta} \qquad (3.20)$$

Where $\Delta = S_{11}S_{22} - S_{12}^2$. $\hat{\alpha}$ can be obtained from equation (3.14).

RSS, ESS and TSS can also be calculated in the same way simple regression is calculated, that is:

$$RSS = \sum (Y_i - \hat{\alpha} - \hat{\beta}_1 X_{1i} - \hat{\beta}_2 X_{2i})^2 \qquad (3.21)$$

$$ESS = \sum (\hat{Y}_i - \overline{Y})^2 \qquad (3.22)$$

$$TSS = \sum (Y_i - \overline{Y})^2 \qquad (3.23)$$

Where RSS = Residual Sum of Squares, ESS = Explained Sum of Squares and TSS = Total Sum of Squares.

And, the coefficient of multiple determination is given by:

$$R^2 = ESS/TSS \tag{3.24}$$

R^2 is the proportion of the variation in Y explained by the regression.

The variances of the estimators are given by:

$$Var(\hat{\beta}_1) = \frac{\sigma^2}{S_{11}(1 - r_{12}{}^2)} \tag{3.25}$$

and

$$Var(\hat{\beta}_2) = \frac{\sigma^2}{S_{22}(1 - r_{12}{}^2)} \tag{3.26}$$

where $r_{12}{}^2$ is the squared correlation coefficient between X_1 and X_2. Thus, the greater the correlation between the two explanatory variables, the greater the variance of the estimators, i.e., the harder it is to get significant results.

In order to give explicit formulae for the least squares estimates of the regression parameters, it is convenient to switch to matrix notation. Without matrix notation, the formulae quickly become unmanageable when the number of explanatory variables increase. Recall that the multiple linear regression model (3.4) is given in matrix form, where the random errors, ε_i , i=1, . . . ,n are independently normally distributed random variables with zero mean and constant variance σ^2. It can be shown that the vector $\hat{\theta}$of least squares estimates of θ is given by :

$$\hat{\theta} = (X'\,X)^{-1}X'y \tag{3.27}$$

where y is the vector of observed response variables, and where the superscripts $'$and $-$denote transposed and inverse matrices, respectively.

Maximum Likelihood Based Estimation

When the maximum likelihood estimation procedure is applied to the classical linear regression model, the result is to get the maximum likelihood estimator. The maximum likelihood estimation procedure implies choosing estimates of the unknown parameters of those values that maximize the likelihood function for the sample of data. However, once the sample is obtained, the values of Y and the X's are known, but the values of the θ's are unknown. The likelihood function is a function of the unknown θ's, because one chooses the values of the θ's that maximize the likelihood function and the sample is more likely to come from a population with these parameter values than any other parameter values.

Recall the model in equation (3.4)

$$y = X\theta + \varepsilon$$

And

$$\varepsilon = y - \theta X$$

The multivariate normal distribution for ε in (3.4) is given by:

$$f(\varepsilon) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\{-\frac{\varepsilon'\varepsilon}{2\sigma^2}\} \quad (3.28)$$

$$= \frac{1}{(\sigma^2)^{N/2}(2\pi)^{N/2}} \exp\{-\frac{1}{2\sigma^2}(y-\theta X)'(y-\theta X)\}$$

The Likelihood function can be obtained as:

$$L(\theta;y) = f(y;\theta) \quad (3.29)$$

Where f (y; θ) is the joint density function of y.

$$L(\theta,\sigma^2;y) = (2\pi\sigma^2)^{-N/2}(\sigma^2)^{-N/2}\exp\{-\frac{1}{2\sigma^2}(y-\theta X)'(y-\theta X)\} \quad (3.30)$$

Taking the natural log of likelihood function in equation (3.30), we have:

$$l = \ln L(\theta,\sigma^2;y) \quad (3.31)$$

$$l = -\frac{N}{2}\ln(2\pi) - \frac{N}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\{-\frac{1}{2\sigma^2}(y-\theta X)'(y-\theta X)\} \quad (3.32)$$

We take the partial derivative of l with respect to θ which gives the score function as:

$$S(\theta;y) = \frac{\partial l}{\partial\theta} \quad (3.33)$$

The vector of unknown parameters has (k+1) elements, therefore the score function is written as:

$$S(\begin{bmatrix}\theta\\ \sigma^2\end{bmatrix};y) = \begin{bmatrix}\frac{\partial l}{\partial\theta}\\ \frac{\partial l}{\partial\sigma^2}\end{bmatrix} \quad (3.34)$$

$$\frac{\partial l}{\partial \theta} = -\frac{1}{\sigma^2}(-X'y + X'x\theta) \quad (3.35)$$

$$\frac{\partial l}{\partial \sigma^2} = -\frac{N}{2\sigma^2} + \frac{N}{2(\sigma^2)^2}(y-\theta X)'(y-\theta X) \quad (3.36)$$

The MLE can be obtained by setting equation (3.35) to zero.

From equation (3.35), we have;

$$X'y - X'X\theta = 0 \quad (3.37)$$

$$-X'X\theta = -X'y \quad (3.38)$$

$$\hat{\theta} = (X'X)^{-1}X'y \quad (3.39)$$

The MLE for θ was derived in (3.39).

Also, we set equation (3.36) to zero; the MLE can be obtained as:

$$\sigma^2 = \frac{(y-\theta X)'(y-\theta X)}{N} \quad (3.40)$$

3.5 Bayesian Estimation Method

In statistical modelling, one of the important interests of a researcher is to estimate the parameters such as what is obtained in classical approach. However, estimation of parameters in Bayesian approach is rigorous due to the use of prior information and likelihood function.

The derivation of Bayesian estimator or estimation of the linear regression model using Bayesian techniques can be performed through the following three steps (Simon, 2009);

1) Determine the likelihood function of the unknown parameters to be estimated given the data.
2) Specify the prior distribution for all the unknown parameters.
3) Obtain the posterior distribution of the parameters given the data and prior distribution.

The relationship between the three steps can be written as:

$$\mathrm{P}\,(\theta|y) \propto \mathrm{P}\,(\theta)\,\mathrm{P}\,(y|\theta) \quad (3.41)$$

P $(\theta|y)$ is referred to as posterior density function, P (θ) is the prior density function and P $(y|\theta)$ is the likelihood function.

Likelihood Function

Using the properties of a multivariate normal distribution, the likelihood function is given as:

$$\text{P}(\text{y}|\theta, h) = \frac{h^{N/2}}{(2\pi)^{N/2}} \exp\left[-\frac{h}{2}(y - X\theta)'(y - X\theta)\right] \qquad (3.42)$$

For convenience, it is better to write (3.42) in terms of Ordinary Least Squares (OLS) estimator:

$$(y - X\theta)'(y - X\theta) = (y - X\theta + x\hat{\theta} - X\hat{\theta})'(y - X\theta + X\hat{\theta} - X\hat{\theta}) \qquad (3.43)$$

$$= (y - X\hat{\theta})'(y - xX) + (\hat{\theta} - \theta)'X'X(\hat{\theta} - \theta) \qquad (3.44)$$

$$= \text{SSE} + (\hat{\theta} - \theta)'X'X(\hat{\theta} - \theta) \qquad (3.45)$$

Where SSE is the Sum of Squares of Error

And

$$\hat{\theta} = (X'X)^{-\prime}X'y \qquad (3.46)$$

While the variance of $\hat{\theta}$ is given as:

$$\text{v}(\hat{\theta}) = S^2(X'X)^{-\prime} \qquad (3.47)$$

The variance of the model is given as:

$$S^2 = \frac{(y - X\hat{\theta})'(y - X\hat{\theta})}{v} \qquad (3.48)$$

S^2 is the estimator of variance of the model (3.4)

And

$$v = N - k \quad \text{, the degree of freedom}$$

Equation (3.47) can be written as:

$$vS^2 = (y - X\hat{\theta})'(y - X\hat{\theta}) \qquad (3.49)$$

Hence, the likelihood is written as:

$$P(y|\theta, h) = \frac{h^{N/2}}{(2\pi)^{N/2}} \exp \left[-\frac{h}{2} \{SSE + (\hat{\theta} - \theta)' X' X(\hat{\theta} - \theta)\}\right] \qquad (3.50)$$

Prior Distribution

Prior distributions are divided into two as explained in chapter two;

(i) Informative
(ii) Non-Informative

Priors are meant to reflect any information the researcher has before seeing the data, which he wishes to incorporate in the analysis of the data. Hence, priors can take any form. Often time, particular classes of priors are chosen to make computation and interpretation easier, Koop (2003), Gelman (2006). Natural conjugate, an example of informative prior and non-informative prior using local uniform distributions, belongs to this class of priors.
Hence, informative prior (natural conjugate) and non-informative prior (local uniform distribution) will be used in this study. Therefore, two estimators will be derived as:

(i) Bayesian with Informative prior
(ii) Bayesian with Non-informative prior

3.51 Bayesian Estimator with Informative Prior (Natural Conjugate Prior)

To carry out Bayesian inference in the presence of multicollinearity with an informative prior, a natural conjugate will be utilized in developing the estimator.

Natural Conjugate Prior is a type of prior when combined with the likelihood function, gives a posterior distribution that falls in the same class of distribution, Raifa and Sclaifer (1961). Examples are Normal-Gamma and Normal priors. A natural conjugate prior was also found to have additional property of the same functional form with the likelihood function,Dreze and Richard (1983), Richard and Steel (1988), and Koop and Poirier (1993).

In the linear regression model given in (3.4), we must elicit prior distribution for parameter $\theta$and the precision h which is given by $P(\theta, h)$, in the sense that we are not conditioning on the data but on parameters, which implies that $P(\theta, h)$ is a prior distribution. Prior distribution can now be written as:

$$P(\theta, h) = P(\theta|h)P(h)$$

One then think about of a prior for $\theta|h$ and the other one for h.Tsionas (2000).

The likelihood function in (3.50) suggests a prior in form of Normal distribution for $\theta|h$ and a Gamma distribution for h. The name of such prior which is a product of Gamma and a conditional Normal is called a Normal-Gamma distribution.

Based on the above premise, it follows that:

$$\theta|\, h \sim N(\, \theta_0, h^{-1}Q_0) \qquad (3.51)$$

This implies that it follows a Normal distribution.

This can also be written as:

$$\mathrm{P}\,(\theta|\, h) \;=\; \frac{h^{k/2}}{2\pi^{k/2}\,|Q_o|^{1/2}}\,\{\exp[-\frac{h}{2}\,(\theta-\theta_o)'\,(Q_0)^{-1}\,(\theta-\theta_o)]\} \qquad (3.52)$$

And also,

$$h\sim G(\, S_0^{-2}, v_0) \qquad (3.53)$$

This also implies that (3.53) follows a Gamma distribution.

(3.53) can also be written as:

$$\mathrm{P}(h) \;=\; \frac{1}{\Gamma\left(\frac{v_0}{2}\right)\left(\frac{2S_0^{-2}}{v_0}\right)^{\frac{v_0}{2}}}\, h^{\frac{v_0-2}{2}} \exp\,(-\frac{hv_0}{2S_0^{-2}}) \qquad (3.54)$$

Where ,

$\Gamma\left(\frac{v_0}{2}\right)\left(\frac{2S_0^{-2}}{v_0}\right)^{\frac{v_0}{2}}$ is the integrating constant.

In the distribution of (3.52) and (3.54), θ_o denotes the prior mean for parameter θ, Q_0 is the un-scaled variance-covariance matrix for parameter θ, S_0^{-2} is the prior mean of gamma density function for the model precision h and v_0 is the prior degree of freedom of gamma distribution for the model precision h.

Recall the rule of probability,

$$P(B,A) = P(B|A)P(A) \qquad (3.55)$$

Therefore,

$$P(\theta, h) = P(\theta|h)P(h) \quad (3.56)$$

Hence, equations (3.52) and (3.54), the natural conjugate prior for θ and h can be simply written as:

$$P(\theta, h) = \frac{h^{k/2}}{2\pi^{k/2} |Q_o|^{1/2}} \{\exp[-\frac{h}{2} (\theta - \theta_o)' (Q_o)^{-1} (\theta - \theta_o)]\}$$

$$X \quad \frac{1}{\Gamma\left(\frac{v_o}{2}\right)\left(\frac{2S_o^{-2}}{v_0}\right)^{\frac{v_o}{2}}} h^{\frac{v_o-2}{2}} \exp\left(-\frac{hv_o}{2S_0^{-2}}\right) \quad (3.57)$$

$$P(\theta, h) = \frac{h^{\frac{v_o+k}{2}-1}}{2\pi^{k/2} |Q_o|^{1/2} \Gamma\left(\frac{v_o}{2}\right)\left(\frac{2S_o^{-2}}{v_o}\right)^{\frac{v_o}{2}}} \{\exp[-\frac{h}{2} (\theta - \theta_o)' (Q_o)^{-1}(\theta - \theta_o) + \frac{v_o}{S_o^{-2}}]\}$$

(3.58)

Equation (3.58) can also be written as:

$$\theta, h \sim NG\ (\theta_o, Q_o\ , S_o^{-2}, v_o) \quad (3.59)$$

Equation (3.59) above implies that the distribution of the prior, $P\ (\theta, h)$ for θ and h is a multivariate Normal-Gamma.

N.B: The symbol " o'' under the parameters is the prior, while symbol represented by * over the parameters indicate the posterior parameters.

Posterior Distribution for Informative Prior

The work of posterior distribution is to summarize the information from both the data, and prior about the unknown parameters $\theta$and h.

For the linear regression model in (3.4), it can be shown that the posterior distribution is also a Normal-Gamma distribution form, which also confirmed that the prior obtained earlier is a natural conjugate prior for θ and h, Koop (2003) and Koop et al (2007).

The posterior distribution is then obtained from the relation as follows:

$$\mathrm{P}\ (\theta, h|y) \propto \mathrm{P}\ (\theta, h\ )\ \mathrm{P}\ (y|\theta, h) \quad (3.60)$$

This expression means that we should multiply (3.58) and (3.41), which gives the joint posterior distribution as:

$$\theta, h|y\ \sim NG\ (\theta^*, Q^*\ , S_o^{-2}, v^*) \quad (3.61)$$

Equation (3.61) follows a Normal-Gamma posterior distribution.

Hence, the hyper-parameters given in (3.61) are:

$$Q^{*} = ({Q_0}^{-1} + X' X)^{-1} \quad (3.62)$$

$$\theta^{*} = Q^{*} ({Q_0}^{-1}\theta_o + X'X \hat{\theta}) \quad (3.63)$$

$$v^{*} = \mathrm{N} + v_o \quad (3.64)$$

Equations (3.62), (3.63) and (3.64) are the estimators for un-scaled variance-covariance matrix (which is a $k \times k$ matrix), posterior mean and degree of freedom of posterior, respectively.

While the Sum of Squares of Error (SSE) and Variance of the error of the model in (3.4) can also be given respectively as:

$$\mathrm{SSE} = (vS^2)_0 + vS^2 + (\hat{\theta} - \theta_0)' [Q_0 + (X'X]^{-1} (\hat{\theta} - \theta_0) \quad (3.65)$$

$$S^{2\,*} = \frac{(vS^2)_0 + vS^2 + (\hat{\theta} - \theta_0)' [Q_0 + (X'X)^{-1}]^{-1} (\hat{\theta} - \theta_0)}{v^{*}} \quad (3.66)$$

In regression modelling, the coefficient on the regressors, θ is usually a primary focus, and a measure of marginal effect of the regressors on the dependent variable. The posterior mean, E $(\theta|y)$ is the point estimate, and $v(\theta)$ is a metric for measuring the uncertainty associated with the point estimate.

Since the interest is on θ, we integrate out$h$ in (3.61) to obtain the marginal posterior for θ. Applying the rule of probability we have:

$$\mathrm{E}\,(\theta|y) = \iint \theta \, \mathrm{P}(\theta, h|y)\partial h \, \partial\theta = \int \theta \, \mathrm{P}(\theta|y)\partial\theta \quad (3.67)$$

Where,

$$\mathrm{P}(\theta|y) = \int \mathrm{P}(\theta, h|y)\partial h \quad (3.68)$$

Hence, equation (3.68) becomes:

$$\mathrm{P}(\theta|y) = \frac{v^{\frac{v^*}{2}}\Gamma\left(\frac{v^*+k}{2}\right)}{\pi^{k/2}\Gamma\left(\frac{v^*}{2}\right)} |S^{2\,*}Q^{*}|^{-1/2} \left[v^{*} + (\theta - \theta^{*})' (S^{2\,*} Q^{*})^{-1}(\theta - \theta^{*})\right]^{\frac{v^*+k}{2}} \quad (3.69)$$

Equation (3.68) follows a t-distribution which can also be written as:

$$\theta|y \sim t\ (\theta^*\ , S^{2\ *}\ Q^*, v^*) \qquad (3.70)$$

And from the definition of t-distribution, the mean and variance can be obtained as:

$$\mathrm{E}\ (\theta|y) = \ \theta^* \qquad (3.71)$$

$$v(\theta) \quad = \frac{\mathrm{SSE}}{v^*-2} Q^* \qquad (3.72)$$

Equations (3.71) and (3.72) are mean and variance estimators used to obtain the values for parameter, θ for different degree of collinearity analytically.

SE (θ^*) is the standard error of Bayesian estimator of θ^* which can also be obtained as:

$$\mathrm{SE}\ (\theta^*) \ = \sqrt{v(\theta^*)} \qquad (3.73)$$

The credible interval for estimators of Bayesian in the same way we have confidence interval in the classical is given by:

$$\theta^* \pm t_{1-\alpha/2}, v^* \ \ \mathrm{SE}\ (\theta^*) \qquad (3.74)$$

Hence, equations (3.71) to (3.74) provide an insight on how Bayesian methods combine the prior (informative) and data information, using model (3.4). The results Bayesian econometrician will report can then be written analytically.

3.52 Bayesian Estimator with Non-Informative Prior

In deriving the Bayesian estimator with non-informative prior, non-informative prior will be multiplied with the likelihood function in the manner as obtained for informative prior.

Recall the relationship between the posterior distribution, likelihood function and prior distribution is given by:

$$P\ (\theta, h|y) \propto \mathrm{P}(\theta, h)\ \times \mathrm{P}(y|\theta, h)$$

Non-informative prior: Prior elicitation often lead to wide disagreement about the choice of prior which in turn gave rise to the use of non-informative prior, in some cases, it is desirable for data information to be predominant over prior information. Non-informative priors are used to make inferences which are not greatly affected by external information or when external information is not provided. As noted by Jeffreys (1961), non-informative prior tend to be proper in most models, and two rules must be adhered to when choosing a non-informative prior distribution:

(1) If a parameter have any value in a finite range from -∞ to +∞ , the prior probability should be taken as uniformly distributed.
(2) If the parameter by nature can take any value from 0 to ∞, the prior probability of the logarithm should be taken as uniformly distributed.

It is assumed that $\theta$and h are independently distributed, and then prior distributions can be written as:

$$\mathrm{P}(\theta, h) = \mathrm{P}(\theta)\ \mathrm{P}(h) \tag{3.75}$$

Jeffreys' non-informative prior is based on invariant principle, which states that transformation, θ = h (θ) of a non-informative prior should not yield additional information.

Using Jeffreys' invariant theory proposed by Zellner (1971), we then write the prior as:

$$\mathrm{P}(\theta) = \text{constant} = 1 \qquad -\infty \text{ to } +\infty\ , \tag{3.76}$$

Equation (3.76) is called uniform distribution

$$\mathrm{P}(h) \propto h^{-1} = {}^{1}/_{h} \tag{3.77}$$

the non-informative prior combining (3.76) and (3.77) is then given by:

$$\mathrm{P}(\theta, h) \propto 1 \times {}^{1}/_{h} \tag{3.78}$$

$$\mathrm{P}(\theta, h) \propto {}^{1}/_{h} \tag{3.79}$$

Likelihood Function

The likelihood function is given by:

$$\mathrm{P}(\mathrm{y}|\theta, h) = \frac{h^{N/2}}{(2\pi)^{N/2}} \exp\left[-\frac{h}{2}\ (y - X\theta)'\ (y - X\theta)\right] \tag{3.80}$$

It is convenient to re-write the likelihood function in (3.80) in a slightly different way by focussing on the exponent part as:

$$(y - X\theta)'\,(y - X\theta) \quad = \,(y - X\theta + X\hat{\theta} - x\hat{\theta}\,)'\,(y - X\theta \,+\, X\hat{\theta} - X\hat{\theta}\,) \qquad (3.81)$$

Thus,

$$(y - X\theta)'\,(y - X\theta) \quad = \,(y - X\hat{\theta})'(y - X\hat{\theta}\,) + (\theta - \hat{\theta}\,)'X'\,X\,(\theta - \hat{\theta}\,) \qquad (3.82)$$

Recall from (3.48), that:

$$S^2 = \frac{(y - X\hat{\theta})'\,(y - X\hat{\theta})}{v} \qquad (3.83)$$

$$\text{where } \; v = \text{N-k} \qquad (3.84)$$

Equation (3.82) becomes:

$$S^2 = \frac{(y - X\hat{\theta})'\,(y - X\hat{\theta})}{\text{N}-\text{k}} \qquad (3.85)$$

Then, we have:

$$(\text{N} - \text{k})S^2 \; = (y - X\hat{\theta})'(y - X\hat{\theta}\,) \qquad (3.86)$$

Substitute $(\text{N} - \text{k})S^2$ for $(y - X\hat{\theta})'(y - X\hat{\theta}\,)$ in (3.82), we have:

$$(y - X\theta)'\,(y - X\theta) \; = \; (\text{N} - \text{k})S^2 + (\theta - \hat{\theta}\,)'X'\,X\,(\theta - \hat{\theta}\,) \qquad (3.87)$$

Substitute (3.87) into (3.80); the likelihood function then becomes:

$$\text{P}(\text{y}|\theta, h) = \quad \frac{h^{N/2}}{(2\pi)^{N/2}} \exp\,[-\frac{h}{2}\,\{(\text{N} - \text{k})S^2 \,+\, (\theta - \hat{\theta}\,)'X'\,X\big(\theta - \hat{\theta}\,\big)\}] \qquad (3.88)$$

If we combine equation (3.80) with (3.88), it will yield posterior distribution as:

$$\text{P}(\theta, h|y) = \frac{h^{N/2-1}}{(2\pi)^{N/2}} \; \exp\,[-\frac{h}{2}\,\{(\text{N} - \text{k})S^2 \,+\, (\theta - \hat{\theta}\,)'X'\,X\big(\theta - \hat{\theta}\,\big)\}] (3.89)$$

$$\text{P}(\theta, h|y) = \frac{h^{N/2-1}}{(2\pi)^{N/2}} \; \exp\,[-\frac{h}{2}(\text{N} - \text{k})S^2] \; \exp\,[-\frac{h}{2}(\theta - \hat{\theta}\,)'X'\,X\big(\theta - \hat{\theta}\,\big)] \qquad (3.90)$$

Since the interest is on parameter θ, by examination of (3.90), treating parameter h as fixed and ignoring the terms that do not involve parameter θ, we have:

$$\text{P}\,(\theta|h, y) = \; \exp\,[-\frac{h}{2}(\theta - \theta^*\,)'X'\,X(\theta - \theta^*)] \qquad (3.91)$$

Equation (3.91) is the kernel of multivariate Normal distribution

NB: * over the parameter indicates a parameter of posterior distribution

3.6 Bayesian Monte Carlo Integration

As stated in chapter two, the posterior distribution is an important aspect of Bayesian whereby numeric summaries are made from. e.g. mean, standard deviation etc. If the numeric summaries cannot be obtained analytically, the best way is to obtain the results by using a method called posterior simulation methods. An example of such is Monte Carlo Integration.

The posterior simulation using Monte Carlo Integration (MCI) method will be compared with analytical method obtained in (3.71) to (3.74) under informative prior for all the degrees of collinearity in order to compare their performance in the presence of multicollinearity.

MCI is a widely used technique in many branches of mathematics and engineering. Suppose the random variable X has arbitrary probability distribution p(x), and we have an algorithm for generating a large number of independent realisations $x^{(1)}, x^{(2)}, \ldots, x^{(T)}$ from this distribution, then;

$$E\,(X) = \int x\, p(X)\partial x \approx \frac{1}{T}\sum_{i=1}^{T} X^{(t)} \qquad (3.92)$$

In other words, the theoretical expectation on X can be approximated by the sample mean of a set of independent realisations drawn from $p(X)$. By the strong law of large numbers, the approximation becomes arbitrarily exact as $T \to \infty$. For example, the expectation of any function of X, g(X), can be obtained as:

$$E\,(g(X)) = \int g(x)\, p(x)\partial x \approx \frac{1}{T}\sum_{i=1}^{T} g(x^{(t)}) \qquad (3.93)$$

That is, the sample mean of the function of the simulated values. In particular, since the variance of X is simply a function of the expectations of X and X^2, this too may be approximated in a natural way using MCI. Not surprisingly, the estimate turns out to be the sample variance of the realisations: $x^{(1)}, x^{(2)}, \ldots, x^{(T)}$ from $p(x)$.

Another important function of X is the indicator function, I(L < X < U), which takes value 1 if X lies in the interval (L, U) and 0 otherwise. The expectation of l(L< X < U) with respect to $p(x)$ gives the probability that X lies within the specified interval, Pr (L < X < U), and may be approximated using MCI by taking the sample average of the value of indicator function for each realisations $x^{(i)}$.

Straightforwardly, it gives:

$$\Pr(L < X < U) \approx \frac{number\ of\ realisations\ \ x^{(t)} \in (L,U)}{T} \tag{3.94}$$

Hence, any desired summary of $p(x)$ may be approximated by calculating the corresponding summary of the sampled values generated from $p(x)$, with the approximation becoming increasingly exact as the sample size increases.

The algorithm for MCI is provided in chapter four of this work.

CHAPTER FOUR

METHODOLOGY

4.1 Introduction

This chapter discusses the design of the experiment and procedure used in generation of data for the research, estimation of the parameters using the derived estimators provided in chapter three, different criteria used for evaluation and summarising the results.

4.2 Design

1. Data generation of regressors involving six cases of collinearity among regressors.

   a. High Positive Collinearity (HPC): when there is high positive level of collinearity among regressors in the model. ρ = 0.95, 0.90 and 0.80.
   b. Moderate Positive Collinearity (MPC): when there is moderate positive level of collinearity among regressors in the model. ρ = 0.49, 0.46 and 0.36.
   c. Low Positive Collinearity (LPC): when there is low positive level of collinearity among regressors in the model. ρ = 0.20, 0.17 and 0.15.
   d. High Negative Collinearity (HNC): when there is high negative level of collinearity among regressors in the model. ρ = -0.95, -0.90 and -0.80.
   e. Moderate Negative Collinearity (MNC): when there is moderate negative level of collinearity among regressors in the model. ρ = -0.49, -0.46 and -0.36
   f. Low Negative Collinearity (LNC): when there is low negative level of collinearity among regressors in the model. ρ = -0.20, -0.17 and -0.15.

2. Generation of the error term.

3. Specification of the true parameter values.

4. Use the regressors, the error term and true parameter values to generate the dependent variable.

5. Specify the prior values for the hyper-parameters for the derived Bayesian estimators (Informative and Non-informative) as contained in chapter three.

6. Use the data generated for both the regressors and dependent variable to obtain the estimates of the parameters from the Bayesian estimators (informative and non-informative).

7. For the posterior simulation using Monte Carlo Integration, the specified hyper-parameters will be used with the aid of computer.

8. Collate and summarize the results for clear conclusions and interpretations.

4.3 Data Generation Procedure for the Study

The model for this study is given by:

$$y = \theta_0 + \theta_1 X_1 + \theta_2 X_2 + \theta_3 X_3 + \varepsilon_i \qquad (4.0)$$

Where, y is the dependent variable θ_0, θ_1, θ_2 and $\theta_3$are the parameters to be estimated, X_i, $i = 1, 2, 3$ are the regressors and ε_i is the error term.

In order to estimate the parameters of the model in (4.0), we simulate the data as illustrated below:

(i) The error term, ε_i were generated from a normal distribution with mean zero and unit variance, i.e $\varepsilon_i \sim$ N(0,1).

(ii) The explanatory variables, X_1, X_2, X_3 were generated using the procedure by Wichem and Churchilll (1978), Alkhamisi et al (2006) and Muniz et al (2012). It follows as:

$$X_{ij} = (1 - \rho^2)^{1/2} X_{ij}^* + \rho X_{ij}^* \qquad (4.1)$$

Where ρ is the correlation between regressors and X_{ij}^* is independent variables obtained from uniform distribution i.e. $X_{ij}^* \sim$ U(0,1).

(iii) The true parameter values were set as: θ_0 = 17, θ_1 = 8.5 , θ_2= 5.0 , θ_3 = 2.0

(iv) The dependent variable y can then be obtained given the values of θ_0, θ_1, θ_2, θ_3 ,X_1, X_2, X_3 and ε_i

(v) Sample sizes are set as: 10, 30, 70, 100, 200 and 300.

4.4 Prior Specification

The following prior specifications are used:

(1) Informative prior:

$$v_0 = 4, \qquad Q_0 = \begin{pmatrix} 2.4 & 0 & 0 & 0 \\ 0 & 6\,X\,10^{-7} & 0 & 0 \\ 0 & 0 & 0.15 & 0 \\ 0 & 0 & 0 & 0.6 \end{pmatrix},$$

$$h = S_o^{-2} = 1.5 \;, \quad \theta_o = \begin{pmatrix} 15 \\ 10 \\ 5.5 \\ 2.5 \end{pmatrix}$$

(2) Non-informative prior:

$$Q_0 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad v_0 = 0, \;\; \theta_o = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad h = S_o^{-2} = 1.5$$

4.5 Algorithms of Bayesian Monte Carlo Integration (MCI)

The following are the algorithms to evaluate the model in equation (4.0):

(1) Select a random draw $\theta^{(s)}$ from the joint Posterior given in equation (3.60) for θ_0 , θ_1 , θ_2 and θ_3 using a random number generator.

(2) Obtain $g(\theta^{(S)})$ for θ_0 , θ_1 ,θ_2 and θ_3 and keep the results.

(3) Repeat (1) and (2) S-times

(4) Obtain the average of S draws for:

$g(\theta_{(0)}^{(1)}), . . . \;, g(\theta_{(0)}^{(S)})$ for θ_0 ,

$g(\theta_{(1)}^{(1)}), . . . \;, g(\theta_{(1)}^{(S)})$ for θ_1 ,

$g(\theta_{(2)}^{(1)}), . . . \;, g(\theta_{(2)}^{(S)})$ for θ_2

$g(\theta_{(3)}^{(1)}), . . . \;, g(\theta_{(3)}^{(S)})$ for θ_3

(5) Carry out analysis of interest.

$\theta_{(0)}^{(1)}, . . . \;, \theta_{(0)}^{(S)}$ are the draws of replication for θ_0 for analysis using MCI

$\theta_{(1)}^{(1)}, . . . , \theta_{(1)}^{(S)}$ are the draws of replication for θ_1 for analysis using MCI

$\theta_{(2)}^{(1)}, . . . , \theta_{(2)}^{(S)}$ are the draws of replication forθ_2 for analysis using MCI

$\theta_{(3)}^{(1)}, . . . , \theta_{(3)}^{(S)}$ are the draws of replication forθ_3 for analysis using MCI

The algorithms illustrated above will yield an estimate of E[g(θ)|y] for any function of interest like mean, variance etc., with the aid of computer by taking a random sample from the posterior. MCI yields only approximation for E[g(θ)|y] since the replication S, cannot be set to infinity. However, when selecting S, the researcher can control the degrees of approximation error.

For example, if the interest is centred on the mean, it can be calculated as:

$$\widehat{g_s} = \frac{1}{S}\sum_{i=1}^{S} g(\theta^{(s)})$$

The replications to perform the MCI were set as:

1. S= 1000 $\Rightarrow$ MCI (1000)
2. S= 10000 $\Rightarrow$ MCI (10000)
3. S=100000 $\Rightarrow$ MCI (100000)

4.6 Criteria for Assessing the Performances of the Estimators

Some of the criteria used in literature will also be used to judge the performances of the estimators for the six cases of collinearity:

1. Standard Error (SE)
2. Credible Interval and Confidence Interval (CI) for the Bayesian estimators and Likelihood based, respectively
3. Mean

CHAPTER FIVE

DISCUSSION OF RESULTS

5.1 Introduction

This chapter presents the discussion of the results of the analysis carried out. The performances of the estimators using model in (4.0) of chapter four are done across sample sizes N= 10, 30, 70, 100, 200 and 300 for the degrees of collinearity while the performances of Bayesian posterior simulation, and analytical methods in the presence of collineairty for varying level of collineairty, ρ = 0.95, 0.90 etc using means, standard errors and Confidence/Credible Intervals of estimators are also carried out.

The degrees of collinearity considered were:

- High Positive Collinearity (HPC); ρ = 0.95, 0.90 and 0.80
- Moderate Positive Collinearity (MPC); ρ = 0.49, 0.46 and 0.36
- Low Positive Collinearity (LPC); ρ = 0.20, 0.17 and 0.15
- High Negative Collinearity (HNC); ρ = -0.95, -0.90 and -0.80
- Moderate Negative Collinearity (MNC); ρ = -0.49, -0.46 and -0.36
- Low Negative Collinearity (LNC); ρ = -0.20, -0.17 and -0.15

The following notations are used in the presentation of the results.

- Likelihood Based method- **LB**
- Bayesian with Non-informative Prior- **BNIP**
- Bayesian with Informative Prior- **BIP**

Tables 5.1-5.130 present the means, standard errors and 95% and 99% confidence/credible intervals for sample sizes of 10, 30, 70, 100, 200 and 300 in the presence of collinearity while Tables 5.133-5.168 present the means, standard errors, and 95% and 99% confidence/credible intervals for Bayesian Analytical method and Bayesian posterior simulation (MCI) methods, using informative prior. The posterior simulation methods were replicated 1000, 10000 and 100000 times to examine the sensitivity of Bayesian posterior simulation methods on multicollinearity to increasing number of replication.

Figures 5.1-5.28 present the plots of the estimators for different sample sizes for the six degrees of collinearity.

5.2 Performances of the Estimators In The Presence of Multicollinearity.

Table 5.1: High Positive Collinearity, ρ = 0.95 and sample size, N=10.

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.8594	1.2872	(13.7099, 20.0091)	(12.0874, 21.6315)
(17)	**BNIP**	16.8594	0.9970	(14.6379, 19.0810)	(13.6996, 20.0193)
	BIP	15.4915	0.4130	(14.6217, 16.3614)	(14.2842, 16.6989)
θ_1	**LB**	13.2207	7.5473	(-5.2469, 31.6883)	(-14.7604, 41.2018)
(8.5)	**BNIP**	13.2207	5.8461	(0.1947, 26.2467)	(-5.3073, 31.7487)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9978, 10.0022)
θ_2	**LB**	1.7932	9.1344	(-20.5578, 24.1442)	(-32.0718, 35.6582)
(5.0)	**BNIP**	1.7932	7.0755	(-13.9719, 17.5583)	(-20.6308, 24.2172)
	BIP	5.5092	0.3874	(4.7113, 6.3070)	(4.4018, 6.6165)
θ_3	**LB**	-3.8846	9.6393	(-27.4711, 19.7019)	(-39.6216, 31.8524)
(2.0)	**BNIP**	-3.8846	7.4666	(-20.5211, 12.7520)	(-27.5481, 19.7790)
	BIP	2.5116	0.7362	(0.9658, 4.0575)	(0.3661, 4.6571)

Table 5.2: High Positive Collinearity , ρ = 0.90 and sample size, N=10.

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.3852	0.5068	(16.1451, 18.6252)	(15.5063, 19.2640)
(17)	**BNIP**	17.3852	0.3926	(16.5105, 18.2598)	(16.1410, 18.6293)
	BIP	16.5191	0.3657	(15.7347, 17.3035)	(15.4303, 17.6078)
θ_1	**LB**	6.9682	1.8906	(2.3420, 11.5943)	(-0.0411, 13.9774)
(8.5)	**BNIP**	6.9682	1.4645	(3.7052, 10.2312)	(2.3269, 11.6094)
	BIP	10.0000	0.0007	(9.9984, 10.0016)	(9.9978, 10.0022)
θ_2	**LB**	8.2848	3.1037	(0.6403, 15.8793)	(-3.2270, 19.7916)
(5.0)	**BNIP**	8.2848	2.4041	(2.9281, 13.6416)	(0.6655, 15.9042)
	BIP	5.3799	0.3677	(4.5912, 6.1687)	(4.2852, 6.4747)
θ_3	**LB**	0.4720	2.7504	(-6.258, 7.2021)	(-9.7250, 10.6690)
(2.0)	**BNIP**	0.4720	2.1305	(-4.2749, 5.2190)	(-6.2800, 7.2241)
	BIP	1.8349	0.6856	(0.3644, 3.3054)	(-0.2061, 3.8759)

Table 5.3: High Positive Collinearity, ρ = 0.80 and sample size, N=10.

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.0313	1.5473	(12.2451, 19.817)	(10.2947, 21.7679)
(17)	**BNIP**	16.0313	1.1986	(13.3607, 18.7018)	(12.2327, 19.8298)
	BIP	15.4915	0.4056	(14.6217, 16.3614)	(14.2842, 16.6989)
θ_1	**LB**	12.8662	3.2158	(4.9975, 20.7348)	(0.9440, 24.7884)
(8.5)	**BNIP**	12.8662	2.4909	(7.3161, 18.4163)	(4.9718, 20.7606)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9978, 10.0022)
θ_2	**LB**	3.9309	8.1683	(-16.0563, 23.91808)	(-26.3526, 34.2144)
(5.0)	**BNIP**	3.9309	6.3272	(-10.1669, 18.0287)	(-16.1216, 23.9834)
	BIP	5.5092	0.3720	(4.7113, 6.3070)	(4.4018, 6.6165)
θ_3	**LB**	-1.5202	8.1652	(-21.4998, 18.4593)	(-31.7922, 28.7517)
(2.0)	**BNIP**	-1.5202	6.3247	(-15.6126, 12.5722)	(-21.5651, 18.5246)
	BIP	2.5116	0.7207	(0.9658, 4.0575)	(0.3661, 4.6571)

Table 5.4: Moderate Positive Collinearity, ρ = 0.49 and sample size, N=10.

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	18.6167	1.3629	(15.2817, 21.9517)	(13.5637, 23.6698)
(17)	**BNIP**	18.6167	1.0557	(16.2644, 20.9691)	(15.2708, 21.9626)
	BIP	15.6370	0.5752	(14.4033, 16.8706)	(13.9247, 17.3492)
θ_1	**LB**	9.0641	2.0404	(4.0715, 14.0568)	(1.4995, 16.6288)
(8.5)	**BNIP**	9.0641	1.5805	(5.5426, 12.5857)	(4.0551, 14.0731)
	BIP	10.0000	0.0011	(9.9976, 10.0023)	(9.9967, 10.0033)
θ_2	**LB**	3.3798	4.7171	(-8.1626, 14.9221)	(-14.1086, 20.8081)
(5.0)	**BNIP**	3.3798	3.6539	(-4.7615, 11.5211)	(-8.2003, 14.9598)
	BIP	5.3373	0.5394	(4.1804, 6.4942)	(3.7316, 6.9430)
θ_3	**LB**	-3.2938	4.8666	(-15.2019, 8.6142)	(-21.3363, 14.7486)
(2.0)	**BNIP**	-3.2938	3.7696	(-11.6931, 5.1054)	(-15.2408, 8.6531)
	BIP	1.6390	1.0248	(-0.5590, 3.8370)	(-1.4117, 4.6896)

.

Table 5.5: Moderate Positive Collinearity, ρ = 0.46 and sample size, N=10.

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	18.6286	1.0971	(15.9441, 21.3130)	(14.5612, 22.6959)
(17)	**BNIP**	18.6286	0.8498	(16.7351, 20.5220)	(15.9354, 21.3218)
	BIP	15.7337	0.5356	(14.5850, 16.8823)	(14.1394, 17.3279)
θ_1	**LB**	6.5615	1.2958	(3.3908, 9.7323)	(1.7574, 11.3657)
(8.5)	**BNIP**	6.5615	1.0037	(4.3251, 8.7980)	(3.3804, 9.7427)
	BIP	10.0000	0.0010	(9.9978, 10.0022)	(9.9970, 10.0030)
θ_2	**LB**	2.3067	3.3697	(-5.9388, 10.5522)	(-10.1864, 14.7998)
(5.0)	**BNIP**	2.3067	2.6102	(-3.5092, 8.1225)	(-5.9657, 10.5791)
	BIP	5.3927	0.5008	(4.3186, 6.4668)	(3.9019, 6.8835)
θ_3	**LB**	1.8186	2.5253	(-4.3607, 7.9979)	(-7.5439, 11.1811)
(2.0)	**BNIP**	1.8186	1.9561	(-2.5399, 6.1771)	(-4.3809, 8.0181)
	BIP	1.9958	0.9389	(-0.0181, 4.0096)	(-0.7993, 4.7908)

Table 5.6: Moderate Positive Collinearity, ρ = 0.36 and sample size, N=10.

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.6114	1.3042	(13.4200, 19.8028)	(11.7759, 21.4469)
(17)	**BNIP**	16.6114	1.0103	(14.3603, 18.8625)	(13.4095, 19.8133)
	BIP	15.7337	0.3945	(14.5850, 16.8823)	(15.0005, 17.3493)
θ_1	**LB**	10.3367	1.4125	(6.8803, 13.7930)	(5.0998, 15.5736)
(8.5)	**BNIP**	10.3367	1.0941	(7.8988, 12.7746)	(6.8690, 13.8043)
	BIP	10.0000	0.0008	(9.9978, 10.0022)	(9.9977, 10.0023)
θ_2	**LB**	6.9233	3.4316	(-1.4735, 15.3200)	(-5.7991, 19.6457)
(5.0)	**BNIP**	6.9233	2.6581	(1.0006, 12.8459)	(-1.5010, 15.3475)
	BIP	5.3927	0.3775	(4.3186, 6.4668)	(4.4097, 6.6574)
θ_3	**LB**	-1.1555	3.1432	(-8.8467, 6.53571)	(-12.8088, 10.4978)
(2.0)	**BNIP**	-1.1555	2.4347	(-6.5804, 4.2694)	(-8.8718, 6.5608)
	BIP	1.9958	0.7176	(-0.0181, 4.0096)	(0.1417, 4.4139)

Table 5.7: Low Positive Collinearity, ρ = 0.20 and sample size, N=10.

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.8763	0.9143	(15.6390, 20.1136)	(14.4865, 21.2662)
(17)	**BNIP**	17.8763	0.7082	(16.2983, 19.4544)	(15.6317, 20.1209)
	BIP	16.0203	0.3488	(15.2722, 16.7684)	(14.9820, 17.0587)
θ_1	**LB**	8.2282	1.1826	(5.3345, 11.1219)	(3.8438, 12.6125)
(8.5)	**BNIP**	8.2282	0.9160	(6.1871, 10.2692)	(5.3250, 11.1313)
	BIP	10.0000	0.0007	(9.9985, 10.0015)	(9.9979, 10.0021)
θ_2	**LB**	3.7700	2.3684	(-2.0254, 9.5654)	(-5.0104, 12.5508)
(5.0)	**BNIP**	3.7700	1.8346	(-0.3177, 7.8577)	(-2.0443, 9.5843)
	BIP	5.4410	0.3467	(4.6974, 6.1847)	(4.4089, 6.4732)
θ_3	**LB**	0.3099	1.8120	(-4.1238, 4.7437)	(-6.4079, 7.0277)
(2.0)	**BNIP**	0.3099	1.4036	(-2.8174, 3.4372)	(-4.1383, 4.7582)
	BIP	2.2905	0.6426	(0.9122, 3.6688)	(0.3775, 4.2035)

Table 5.8: Low Positive Collinearity regressors, ρ = 0.17 and sample size, N=10.

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	14.2507	1.0575	(13.4200, 19.8028)	(11.7759, 21.4469)
(17)	**BNIP**	14.2507	0.8192	(12.4255, 16.0758)	(11.6545, 16.8468)
	BIP	16.0892	0.3134	(15.4171, 16.7613)	(15.1564, 17.0221)
θ_1	**LB**	10.8832	0.9618	(6.8803, 13.7930)	(5.0998, 15.5736)
(8.5)	**BNIP**	10.8832	0.7450	(9.2233, 12.5431)	(8.5222, 13.2443)
	BIP	10.0000	0.0006	(9.9986, 10.0014)	(9.9981, 10.0019)
θ_2	**LB**	9.2429	2.8139	(-1.4735, 15.3200)	(-5.7991, 19.6457)
(5.0)	**BNIP**	9.2429	2.1797	(4.3863, 14.0995)	(2.3349, 16.1508)
	BIP	5.5819	0.3157	(4.9048, 6.2591)	(4.6421, 6.5217)
θ_3	**LB**	3.4000	2.2126	(-8.8467, 6.53571)	(-12.8088, 10.4978)
(2.0)	**BNIP**	3.4000	1.7139	(-0.4187, 7.2188)	(-2.0317, 8.8317)
	BIP	2.7162	0.6079	(1.4124, 4.0200)	(0.9066, 4.5258)

Table 5.9: Low Positive Collinearity regressors, ρ = 0.15 and sample size, N=10.

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.6192	0.9534	(15.2865, 19.9520)	(14.0847, 21.1538)
(17)	**BNIP**	17.6192	0.7385	(15.9738, 19.2647)	(15.2788, 19.9596)
	BIP	16.3954	0.3794	(15.5818, 17.2090)	(15.2661, 17.5247)
θ_1	**LB**	8.4999	1.2881	(5.3481, 11.6517)	(3.7244, 13.2753)
(8.5)	**BNIP**	8.4999	0.9977	(6.2768, 10.7230)	(5.3378, 11.6620)
	BIP	10.0000	0.0008	(9.9983, 10.0017)	(9.9977, 10.0023)
θ_2	**LB**	2.7172	2.1093	(-2.444, 7.8785)	(-5.10278, 10.5373)
(5.0)	**BNIP**	2.7172	1.6338	(-0.9232, 6.3577)	(-2.4609, 7.8953)
	BIP	5.4141	0.3835	(4.5916, 6.2366)	(4.2725, 6.5557)
θ_3	**LB**	4.0064	2.2311	(-1.4529, 9.4658)	(-4.2653, 12.2782)
(2.0)	**BNIP**	4.0064	1.7282	(0.1557, 7.8571)	(-1.4708, 9.4836)
	BIP	2.6546	0.7285	(1.0921, 4.2171)	(0.4859, 4.8233)

From Tables 5.1-5.9, the following are observed:

In terms of CI, the CI of Bayesian estimators at 95% and 99% are more compact than the LB estimator most especially the BIP for all the parameters considered across the three levels of collineairty (HPC, MPC and LPC).

The standard errors of Bayesian estimators (BNIP and BIP) are smaller than the LB method for HPC, MPC and LPC for sample size 10. In table 5.1, the SE for parameters, θ_0, θ_1, θ_2 and θ_3 are (1.2872,0.9970 and 0.4130), (7.5473, 5.8461 and 0.0008), (9.1344, 7.0755 and 0.3874) and (9.6393, 7.4666 and 0.7362) respectively for HPC, MPC and LPC, when the sample size, is 10.
The means of the estimators especially the BIP are not too far from the initial values of the simulated data, the means of LB and BNIP are the same for all the parameters across the levels of collinearity.

The CI of LPC is more compact than the HPC, for instance; parameterθ_2 when LPC (ρ = 0.15), the CI for LB, BNIP and BIP are (-2.444≤CI≤7.8785), (-0.9232≤CI≤6.3577) and (4.5916≤CI≤ 6.2366), respectively but when HPC (ρ = 0.95), the CI for LB, BNIP and BIP are (-20.5578≤CI≤24.1442), (-13.9719≤CI≤17.5583) and (4.7113≤CI≤ 6.3070), respectively.

In Low Positive Collinearity, it shows that the performance of Likelihood Based (LB) method becomes better than HPC and MPC. Hence, the collinearity does not have much effect on the LB for LPC.

Table 5.10: Summary of Tables 5.1 -5.9 for Standard Error when the sample size, N= 10.

Parameters	Estimators	0.95	0.90	0.80	0.49	0.46	0.36	0.20	0.17	0.15
θ_0	**LB**	1.2872	0.5060	1.5473	1.3629	1.0971	1.3042	0.9143	1.0575	0.9534
	BNIP	0.9970	0.3956	1.1986	1.0557	0.8498	1.0103	0.7082	0.8192	0.7385
	BIP	0.4130	0.3657	0.4045	0.5752	0.5356	0.3945	0.3488	0.3134	0.3794
θ_1	**LB**	7.5473	1.8906	3.2158	2.0404	1.2958	1.4125	1.1826	0.9618	1.2881
	BNIP	5.8461	1.4645	2.4909	1.5805	1.0037	1.0941	0.9160	0.7450	0.9977
	BIP	0.0008	0.0007	0.0008	0.0071	0.0010	0.0008	0.0007	0.0006	0.0008
θ_2	**LB**	9.1344	3.1037	8.1683	4.7171	3.3697	3.4316	2.3684	2.8139	2.1093
	BNIP	7.0755	2.4041	6.3272	3.6539	2.6102	2.6581	1.8346	2.1797	1.6338
	BIP	0.3874	0.3677	0.3720	0.5394	0.5008	0.3775	0.3467	0.3157	0.3835
θ_3	**LB**	9.6393	2.7504	8.1652	4.8666	2.5253	3.1432	1.8120	2.2126	2.2311
	BNIP	7.4666	2.1305	6.3247	3.7696	1.9561	2.4347	1.4036	1.7139	1.7282
	BIP	0.7362	0.6856	0.7207	1.0248	0.9389	0.7176	0.6426	0.6079	0.7285

Table 5.10 shows the summary of SE for multicollinearity (HPC, MPC and LPC) of the estimators across the parameters (θ_0, θ_1, θ_2 and θ_3) when the sample size is 10. There seems to be no fixed pattern in the performance of the estimators for the levels of multicollinearity ($\rho =$ 0.15-0.95). It is also observed that as ρ decreases, the SE of estimators also decreases for all the parameters. When $\rho = 0.95$, all the estimators have the highest value of SE for the parameters except for the intercept parameter θ_0.

The Bayesian estimators (BIP and BNIP) have the smallest SE for all the levels of multicollinearity considered (ρ =0.15-0.95) when the sample size, N=10. It is also observed that LB has the highest value of SE, when ρ = 0.95, for parameter θ_3 being 9.6293. The SE of BIP for parameter θ_1 for ρ's are almost the same. It was also observed in table 5.10, that HPC is characterized with large SE especially when the ρ = 0.95 and as the level of collinearity move from high positive to low positive, the SE also reduces consistently.

Hence, BIP outperformed other estimators (BNIP and LP).

Table 5.11: Summary of Tables 5.1-5.9 for Mean for sample size, N= 10.

Parameters	Estimators	0.95	0.90	0.80	0.49	0.46	0.36	0.20	0.17	0.15
θ_0	**LB**	16.8594	17.3852	16.0313	18.6167	18.6286	16.6114	17.8763	14.2507	17.6192
(17.00)	**BNIP**	16.8594	17.3852	16.0313	18.6167	18.6286	16.6114	17.8763	14.2507	17.6192
	BIP	15.4915	16.5191	15.4915	15.6370	15.7337	15.7337	16.0203	16.0892	16.3954
θ_1	**LB**	13.2207	6.9682	12.8662	9.0641	6.5615	10.3367	8.2282	10.8832	8.4999
(8.5)	**BNIP**	13.2207	6.9682	12.8662	9.0641	6.5615	10.3367	8.2282	10.8832	8.4999
	BIP	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000
θ_2	**LB**	1.7932	8.2848	3.9309	3.3798	2.3067	6.9233	3.7700	9.2429	2.7172
(5.00)	**BNIP**	1.7932	8.2848	3.9309	3.3798	2.3067	6.9233	3.7700	9.2429	2.7172
	BIP	5.5092	5.3799	5.5092	5.3373	5.3927	5.3927	5.4410	5.5819	5.4141
θ_3	**LB**	-3.8846	0.4720	-1.5202	-3.2938	1.8186	-1.1555	0.3099	3.4000	4.0064
(2.00)	**BNIP**	-3.8846	0.4720	-1.5202	-3.2938	1.8186	-1.1555	0.3099	3.4000	4.0064
	BIP	2.5116	1.8349	2.5116	1.6390	1.9958	1.9958	2.2905	2.7162	2.6546

Table 5.11 summarizes the mean estimates of all the estimators for parameters (θ_0, θ_1, θ_2 and θ_3), when the sample size is 10.

The mean of LB and BNIP are the same for all levels of multicollinearity across the parameters. However, there is evidence to suggest that BIP is the best for estimating parameters of the regression model because the means are closer to the true parameter value for all the levels of multicollinearity.

The mean estimates of the estimators get closer to the true parameter values for LPC (0.20, 0.17 and 0.15). Also none of the estimators generated negative average estimates and none generated large positive estimates except for parameter $\theta_3$under LB and BNIP estimators. The average estimates have shown no consistent pattern for all levels of multicollinearity across the parameters when the sample size is 10.

Table 5.12: High Positive Collinearity, ρ = 0.95 and sample size, N=30

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.9670	0.6965	(16.5354,19.3985)	(16.03172, 19.9022)
(17)	**BNIP**	17.9670	0.6484	(16.6428, 19.2911)	(16.1840, 19.7499)
	BIP	16.5256	0.3642	(15.7856, 17.2657)	(15.5321, 17.5192)
θ_1	**LB**	9.2698	2.8858	(3.3379,15.2017)	(1.25093, 17.2887)
(8.5)	**BNIP**	9.2698	2.6866	(3.7831, 14.7565)	(1.8818, 16.6579)
	BIP	10.0000	0.0009	(9.9981, 10.0019)	(9.9974, 10.0026)
θ_2	**LB**	2.2399	4.9174	(-7.8681, 12.3478)	(-11.4243, 15.9039)
(5.0)	**BNIP**	2.2399	4.5779	(-7.1094, 11.5891)	(-10.3493, 14.8290)
	BIP	5.1549	0.4546	(4.2310, 6.0787)	(3.9146, 6.3952)
θ_3	**LB**	0.9541	5.2168	(-9.7692, 11.6775)	(-13.542, 15.4502)
(2.0)	**BNIP**	0.9541	4.8566	(-8.9644, 10.8726)	(-12.4015, 14.3098)
	BIP	1.1338	0.8081	(-0.5084, 2.7760)	(-1.0709, 3.3385)

Table 5.13: High Positive Collinearity, ρ = 0.90 and sample size, N=30.

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.9105	0.5528	(15.7743, 18.0468)	(15.3745,18.4466)
(17)	**BNIP**	16.9105	0.5146	(15.8595, 17.9615)	(15.4953, 18.3257)
	BIP	16.5785	0.3091	(15.9503, 17.2068)	(15.7351, 17.4220)
θ_1	**LB**	7.8741	2.4975	(2.7404, 13.0079)	(0.9342, 14.8141)
(8.5)	**BNIP**	7.8741	2.3251	(3.1257, 12.6226)	(1.4802, 14.2681)
	BIP	10.0000	0.0009	(9.9982, 10.0018)	(9.9975, 10.0025)
θ_2	**LB**	9.7744	3.4381	(2.7072, 16.8415)	(0.2208, 19.3279)
(5.0)	**BNIP**	9.7744	3.2007	(3.2377, 16.3111)	(0.9724, 18.5763)
	BIP	5.3574	0.4338	(4.4759, 6.2389)	(4.1740, 6.5409)
θ_3	**LB**	-1.2346	3.3749	(-8.1718, 5.7026)	(-10.6125, 8.1432)
(2.0)	**BNIP**	-1.2346	3.1419	(-7.6512, 5.1819)	(-9.8747, 7.4054)
	BIP	1.3367	0.7483	(-0.1841, 2.8575)	(-0.7051,3.3785)

Table 5.14: High Positive Collinearity, ρ = 0.80 and sample size, N=30.

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.1931	0.6793	(15.7968, 18.5894)	(15.3055, 19.0807)
(17)	**BNIP**	17.1931	0.6324	(15.9015, 18.4846)	(15.4540, 18.9322)
	BIP	16.3754	0.3371	(15.6903, 17.0605)	(15.4556, 17.2952)
θ_1	**LB**	8.6261	1.5259	(5.4896, 11.7626)	(4.3861, 12.8660)
(8.5)	**BNIP**	8.6261	1.4205	(5.7250, 11.5272)	(4.7196, 12.5325)
	BIP	10.0000	0.0008	(9.9983, 10.0017)	(9.9977, 10.0023)
θ_2	**LB**	4.4173	2.4797	(-0.6799, 9.5144)	(-2.4732, 11.3078)
(5.0)	**BNIP**	4.4173	2.3085	(-0.2973, 9.1319)	(-1.9311, 10.7657)
	BIP	5.3091	0.4078	(4.4804, 6.1378)	(4.1965, 6.4217)
θ_3	**LB**	2.5640	2.6355	(-2.8534, 7.9813)	(-4.7593, 9.8873)
(2.0)	**BNIP**	2.5640	2.4535	(-2.4468, 7.5748)	(-4.1832, 9.3112)
	BIP	1.8231	0.7238	(0.3521, 3.2941)	(-0.1518, 3.7980)

Table 5.15 Moderate Positive Collinearity, ρ = 0.49 and sample size, N=30.

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.8159	0.4069	(16.9796, 18.6523)	(16.6853, 18.9465)
(17)	**BNIP**	17.8159	0.3788	(17.0424, 18.5895)	(16.7743, 18.8576)
	BIP	16.1627	0.2686	(15.6168, 16.7086)	(15.4298, 16.8956)
θ_1	**LB**	9.6426	0.6478	(8.3110, 10.9742)	(7.8425, 11.4427)
(8.5)	**BNIP**	9.6426	0.6031	(8.4110, 10.8743)	(7.9842, 11.3011)
	BIP	10.0000	0.0007	(9.9987, 10.0013)	(9.9982, 10.0018)
θ_2	**LB**	3.1630	1.0671	(0.9695, 5.3565)	(0.1978, 6.1282)
(5.0)	**BNIP**	3.1630	0.9934	(1.1341, 5.1918)	(0.4311, 5.8949)
	BIP	5.2558	0.3145	(4.6167, 5.8949)	(4.3977, 6.1138)
θ_3	**LB**	-0.3051	1.0492	(-2.4618, 1.8515)	(-3.2206, 2.6103)
(2.0)	**BNIP**	-0.3051	0.9767	(-2.2999, 1.6896)	(-2.9912, 2.3809)
	BIP	1.5739	0.5596	(0.4366, 2.7112)	(0.0470, 3.1008)

Table 5.16 Moderate Positive Collinearity, ρ = 0.46 and sample size, N=30.

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	18.0459	0.5174	(16.3655, 19.1095)	(16.6081, 19.4837)
(17)	**BNIP**	18.0459	0.4817	(17.0621, 19.0297)	(16.7212, 19.3706)
	BIP	16.4598	0.3476	(15.7534, 17.1662)	(15.5114, 17.4082)
θ_1	**LB**	7.6960	0.5931	(6.4770, 8.9150)	(6.0481, 9.3440)
(8.5)	**BNIP**	7.6960	0.5521	(6.5685, 8.8236)	(6.1777, 9.2143)
	BIP	10.0000	0.0010	(9.9981, 10.0019)	(9.9974, 10.0026)
θ_2	**LB**	4.2493	1.5303	(1.1037, 7.3950)	(-0.0031, 8.5016)
(5.0)	**BNIP**	4.2493	1.4247	(1.3398, 7.1589)	(0.3315, 8.1671)
	BIP	5.2562	0.4569	(4.3277, 6.1847)	(4.0096, 6.5028)
θ_3	**LB**	1.7192	1.4710	(-1.3045, 4.7428)	(-2.3683, 5.8067)
(2.0)	**BNIP**	1.7192	1.3694	(-1.0776, 4.5159)	(-2.0468, 5.4851)
	BIP	1.7676	0.8176	(0.1061, 3.4291)	(-0.4630, 3.9982)

Table 5.17: Moderate Positive Collinearity, ρ = 0.36 and sample size, N=30.

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.7265	0.4843	(16.3655, 18.5295)	(15.9848, 18.9102)
(17)	**BNIP**	17.7265	0.4509	(16.8057, 18.6474)	(16.4866, 18.9665)
	BIP	16.2950	0.2979	(15.6896, 16.9003)	(15.4823, 17.1077)
θ_1	**LB**	8.6495	0.6465	(6.7665, 9.4106)	(6.3002, 9.8759)
(8.5)	**BNIP**	8.6495	0.6019	(7.4202, 9.8787)	(6.9942, 10.3047)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
θ_2	**LB**	4.0811	1.2169	(1.2469, 7.3829)	(0.1675, 8.4623)
(5.0)	**BNIP**	4.0811	1.1329	(1.7674, 6.3948)	(0.9656, 7.1966)
	BIP	5.2628	0.3685	(4.5139, 6.0117)	(4.2574, 6.2682)
θ_3	**LB**	0.5145	1.3533	(1.4159, 5.9183)	(0.6239, 6.7103)
(2.0)	**BNIP**	0.5145	1.2599	(-2.0585, 3.0875)	(-2.9502, 3.9791)
	BIP	1.7875	0.6697	(0.4265, 3.1486)	(-0.0398, 3.6149)

Table 5.18: Low Positive Collinearity, ρ = 0.20 and sample size, N=30.

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.4475	0.5264	(16.3655, 18.5295)	(15.9848, 18.9102)
(17)	**BNIP**	17.4475	0.4900	(16.4467, 18.4483)	(16.0999, 18.7951)
	BIP	16.3871	0.2725	(15.8334, 16.9408)	(15.6437, 17.1305)
θ_1	**LB**	8.0880	0.6434	(6.7665, 9.4106)	(6.3002, 9.8759)
(8.5)	**BNIP**	8.0880	0.5990	(6.8647, 9.3113)	(6.4408, 9.7353)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
θ_2	**LB**	4.3149	1.4926	(1.2469, 7.3829)	(0.1675, 8.4623)
(5.0)	**BNIP**	4.3149	1.3895	(1.4772, 7.1526)	(0.4938, 8.1360)
	BIP	5.4291	0.3711	(4.6750, 6.1832)	(4.4167, 6.4415)
θ_3	**LB**	3.6671	1.0952	(1.4159, 5.9183)	(0.6239, 6.7103)
(2.0)	**BNIP**	3.6671	1.0196	(1.5849, 5.7493)	(0.8633, 6.4709)
	BIP	2.4619	0.6192	(1.2035, 3.7203)	(0.7724, 4.1514)

Table 5.19: Low Positive Collinearity, ρ = 0.17 and sample size, N=30.

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.9460	0.6337	(16.3746, 18.9796)	(15.9163, 19.4379)
(17)	**BNIP**	17.6771	0.5899	(16.4723, 18.8819)	(16.0548, 19.2994)
	BIP	15.8294	0.3537	(15.1107, 16.5481)	(14.8644, 16.7944)
θ_1	**LB**	7.3974	0.6856	(5.9880, 8.8067)	(5.4922, 9.3026)
(8.5)	**BNIP**	7.3974	0.6383	(6.0938, 8.7010)	(5.6421, 9.1527)
	BIP	10.0000	0.0010	(9.9980, 10.0020)	(9.9973, 10.0027)
θ_2	**LB**	2.7093	1.5297	(-0.4351, 5.8536)	(-1.5414, 6.9599)
(5.0)	**BNIP**	2.7093	1.4241	(-0.1991, 5.6176)	(-1.2069, 6.6255)
	BIP	5.2537	0.4741	(4.2903, 6.2172)	(3.9603, 6.5472)
θ_3	**LB**	2.8356	1.3558	(0.0486, 5.6226)	(-0.9319, 6.6031)
(2.0)	**BNIP**	2.8356	1.2622	(0.2578, 5.4134)	(-0.6355, 6.3067)
	BIP	2.5796	0.8355	(0.8817, 4.2775)	(0.3001, 4.8591)

Table 5.20: Low Positive Collinearity, ρ = 0.15 and sample size, N=30.

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.6501	0.4818	(15.6597,17.6405)	(15.3112,17.9890)
(17)	**BNIP**	16.6501	0.4486	(15.7340, 17.5662)	(15.4166,17.8837)
	BIP	16.0062	0.2340	(15.5307, 16.4817)	(15.3678,16.6446)
θ_1	**LB**	8.7457	0.5508	(7.6135, 9.8780)	(7.2152, 10.2763)
(8.5)	**BNIP**	8.7457	0.5128	(7.6985, 9.7930)	(7.3356, 10.1559)
	BIP	10.0000	0.0007	(9.9987, 10.0013)	(9.9982, 10.0018)
θ_2	**LB**	4.0759	1.2089	(1.5910, 6.5608)	(0.7168, 7.4351)
(5.0)	**BNIP**	4.0759	1.1254	(1.7775, 6.3743)	(0.9811, 7.1708)
	BIP	5.4219	0.3155	(4.7807, 6.0630)	(4.5611, 6.2827)
θ_3	**LB**	4.2218	0.9666	(2.2350, 6.2086)	(1.5360, 6.9076)
(2.0)	**BNIP**	4.2218	0.8998	(2.3841, 6.0595)	(1.7473, 6.6963)
	BIP	2.9690	0.5488	(1.8537, 4.0844)	(1.4716, 4.4665)

From Tables 5.12-5.20, the following observations were made when the sample size is 30:

The mean estimates of all the estimators (LB, BNIP and BIP) are close to the true parameter values, but BIP outperformed all other estimators except few cases; for HPC when the sample size is 30, the means of the estimators are 17.9670, 17.9670, 16.5256 for LB, BNIP and BIP respectively for parameter θ_0 with true value of the parameter of 17.00.

BNIP and BIP estimator have smaller SE than LB estimator, the SE of all the estimators also decreases as sample size increases. HPC has the highest values of SE among the levels of collinearity, the values also reduced consistently compared to sample size of 10, but for LPC, the SE increases when ρ = 0.17.

The confidence interval of LB becomes more compact compared to when the sample size is 10 for HPC, MPC and LPC. The CI of intercept parameter is compact for all the estimators across the levels of collinearity. BIP has the smallest SE values for all the parameters considered while the 95% and 99% CI of BIP are also more compact than LB and BNIP for HPC, MPC and LPC.

Table 5.21: Summary of Tables 5.12 -5.20 for Standard Error when the sample size, N= 30.

Parameters	Estimators	0.95	0.90	0.80	0.49	0.46	0.36	0.20	0.17	0.15
θ_0	**LB**	0.6965	0.5528	0.6793	0.4069	0.5174	0.4843	0.5264	0.6337	0.4818
	BNIP	0.6484	0.5146	0.6324	0.3788	0.4817	0.4509	0.4900	0.5899	0.4486
	BIP	0.3642	0.3091	0.3371	0.2686	0.3476	0.2979	0.2725	0.3537	0.2340
θ_1	**LB**	2.8858	2.4975	1.5259	0.6478	0.5931	0.6465	0.6434	0.6856	0.5508
	BNIP	2.6866	2.3251	1.4205	0.6031	0.5521	0.6019	0.5990	0.6383	0.5128
	BIP	0.0009	0.0009	0.0008	0.0007	0.0010	0.0008	0.0008	0.0010	0.0008
θ_2	**LB**	4.9174	3.4381	2.4797	1.0671	1.5303	1.2169	1.4926	1.5297	1.2089
	BNIP	4.5779	3.2007	2.3085	0.9934	1.4247	1.1329	1.3895	1.4241	1.1254
	BIP	0.4546	0.4338	0.4078	0.3145	0.4569	0.3685	0.3467	1.2622	0.3155
θ_3	**LB**	5.2168	3.3749	2.6355	1.0492	1.4710	1.3533	1.0952	1.3558	0.9666
	BNIP	4.8566	3.1419	2.4535	0.9767	1.3694	1.2599	1.0196	1.2622	0.8998
	BIP	0.8081	0.7483	0.7238	0.5596	0.8176	0.6697	0.6192	0.8355	0.5488

Table 5.21 gives the summary of SE in tables 5.12-5.20 when the sample size is 30. $\rho = 0.95$ gives the highest SE among the levels of multicollinearity for all the estimators across the parmaters.

Bayesian estimators (BNIP and BIP) have the smallest SE for all the ρ's (0.15-0.95) across the parameters. It was also observed that when $\rho = 0.15$, all the estimators have minimum SE values for all the parameters.

Among the three estimators, BIP outperformed all the estimators at all the levels of multicollinearity having the smallest SE in most cases. Hence, SE has not shown any consistent pattern within the three levels of multicollinearity (high, moderate and low positive collinearities).

Table 5.22: Summary of Tables 5.12-5.20 for Mean for sample size, N= 30.

Parameters	Estimators	0.95	0.90	0.80	0.49	0.46	0.36	0.20	0.17	0.15
θ_0	**LB**	17.9670	16.9105	17.1931	17.8159	18.0459	17.7265	17.4475	16.9460	16.6501
(17.00)	**BNIP**	17.9670	16.9105	17.1931	17.8159	18.0459	17.7265	17.4475	16.9460	16.6501
	BIP	16.5256	16.5785	16.3754	16.1627	16.4598	16.2950	16.3871	15.8294	16.0062
θ_1	**LB**	9.2698	7.8741	8.6261	9.6426	7.6960	8.6485	8.0880	7.3974	8.7457
(8.5)	**BNIP**	9.2698	7.8741	8.6261	9.6426	7.6960	8.6495	8.0880	7.3974	8.7457
	BIP	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000
θ_2	**LB**	2.2399	9.7744	4.4173	3.1630	4.2493	4.0811	4.3149	2.7093	4.0759
(5.00)	**BNIP**	2.2399	9.7744	4.4173	3.1630	4.2493	4.0811	4.3149	2.7093	4.0759
	BIP	5.1549	5.3574	5.3091	5.2558	5.2562	5.2628	5.4291	5.2537	5.4219
θ_3	**LB**	0.9541	-1.2346	2.5640	-0.3051	1.7192	0.5145	3.6671	2.8356	4.2218
(2.00)	**BNIP**	0.9541	-1.2346	2.5640	-0.3051	1.7192	0.5145	3.6671	2.8356	4.2218
	BIP	1.1338	1.3367	1.8231	1.5739	1.7676	1.7875	2.4619	2.5796	2.9690

Table 5.22 shows the summary of mean for tables 5-12-5.20 for all the levels of multicollinearity when the sample size is 30. The means of LB and BNIP are the same for all levels of multicollinearity across the parameters. The means of BIP are not too far from true parameter values for all the levels of multicollinearity for all the parameters considered in most cases.

Negative means was observed when ρ = 0.49 and 0.90 for LB and BNIP estimators for parameter θ_3.

Table 5.23: High Positive Collinearity, ρ = 0.95 and sample size, N=70

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.4391	0.3760	(16.6885, 18.1898)	(16.4419, 18.4364)
(17)	**BNIP**	17.4391	0.3651	(16.7110, 18.1673)	(16.4724, 18.4058)
	BIP	16.7363	0.2351	(16.2679, 17.2046)	(16.1148, 17.3577)
θ_1	**LB**	9.7138	1.8816	(5.9572, 13.4704)	(4.7232, 14.70447)
(8.5)	**BNIP**	9.7138	1.8270	(6.0700, 13.3577)	(4.8761, 14.5516)
	BIP	10.0000	0.0008	(9.9983, 10.0017)	(9.9978, 10.0022)
θ_2	**LB**	2.8381	2.5519	(-2.2569, 7.9331)	(-3.9304, 9.6067)
(5.0)	**BNIP**	2.8381	2.4779	(-2.1039, 7.7801)	(-3.7231, 9.3993)
	BIP	4.9526	0.3948	(4.1658, 5.7393)	(3.9086, 5.9965)
θ_3	**LB**	0.8514	2.9699	(-5.0783, 6.7811)	(-7.0261, 8.7288)
(2.0)	**BNIP**	0.8514	2.8838	(-4.9003, 6.6030)	(-6.7848, 8.4875)
	BIP	0.5777	0.6369	(-0.6912, 1.8467)	(-1.1060, 2.2615)

Table 5.24: High Positive Collinearity, ρ = 0.90 and sample size, N=70

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.9460	0.3787	(16.1899, 17.7021)	(15.9415, 17.9505)
(17)	**BNIP**	16.9460	0.3677	(16.2126, 17.6794)	(15.9723, 17.9197)
	BIP	16.2691	0.2396	(15.7917, 16.7465)	(15.6356, 16.9025)
θ_1	**LB**	10.8772	1.3907	(8.7007, 13.6538)	(7.1887,14.5658)
(8.5)	**BNIP**	10.8772	1.3503	(8.1841, 13.5704)	(7.3017, 14.4528)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9978, 10.0021)
θ_2	**LB**	3.7170	2.1729	(-0.6213, 8.0553)	(-2.0463, 9.4803)
(5.0)	**BNIP**	3.7170	2.1099	(-0.4910, 7.9250)	(-1.8697, 9.3038)
	BIP	5.2293	0.3843	(4.4636, 5.9949)	(4.2133, 6.2452)
θ_3	**LB**	-0.7270	2.0584	(-4.8367, 3.3827)	(-6.1866, 4.7327)
(2.0)	**BNIP**	-0.7270	1.9987	(-4.7133, 3.2593)	(-6.0194, 4.5654)
	BIP	1.1752	0.5952	(-0.0107, 2.3611)	(-0.3984, 2.7488)

Table 5.25: High Positive Collinearity, ρ = 0.80 and sample size, N=70

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.2351	0.4167	(16.4032, 18.0670)	(16.1300, 18.3402)
(17)	**BNIP**	17.2351	0.4046	(16.4282, 18.0420)	(16.1638, 18.3064)
	BIP	16.7268	0.2525	(16.2236, 17.2299)	(16.0591, 17.3944)
θ_1	**LB**	8.8310	1.0675	(6.6997, 10.9624)	(5.9995, 11.6626)
(8.5)	**BNIP**	8.8310	1.0366	(6.7636, 10.8984)	(6.0863, 11.5758)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
θ_2	**LB**	5.0839	1.4572	(2.1745, 7.9934)	(1.2188, 8.9491)
(5.0)	**BNIP**	5.0839	1.4150	(2.2618, 7.9060)	(1.3372, 8.8307)
	BIP	5.1653	0.3615	(4.4450, 5.8857	(4.2095, 6.1212)
θ_3	**LB**	1.6487	1.4436	(-1.2335, 4.5310)	(-2.1803, 5.4778)
(2.0)	**BNIP**	1.6487	1.4018	(-1.1470, 4.4445)	(-2.0630, 5.3605)
	BIP	1.2926	0.5747	(0.1474, 2.4377)	(-0.2270, 2.8121)

Table 5.26: Moderate Positive Collinearity, ρ = 0.49 and sample size, N=70

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.0015	0.4092	(16.1845, 17.8184)	(15.9162, 18.0865)
(17)	**BNIP**	17.0015	0.4031	(15.7654, 16.8137)	(15.9494, 18.0535)
	BIP	16.2896	0.2667	(15.7917, 16.7465)	(15.5941, 16.9851)
θ_1	**LB**	9.0363	0.6659	(7.7068,10.3659)	(7.2701,10.8026)
(8.5)	**BNIP**	9.0363	0.6560	(7.7467, 10.3260)	(7.3242, 10.7485)
	BIP	10.0000	0.0008	(9.9983, 10.0017)	(9.9978, 10.0022)
θ_2	**LB**	4.2182	1.1278	(1.9665, 6.4700)	(1.2268, 7.2096)
(5.0)	**BNIP**	4.2182	1.1111	(2.0341, 6.4023)	(1.3185, 7.1179)
	BIP	5.1215	0.3870	(4.3608, 5.8822)	(4.1122, 6.1308)
θ_3	**LB**	1.8422	1.0265	(-0.2073, 3.8918)	(-0.8806, 4.5650)
(2.0)	**BNIP**	1.8422	1.0113	(-0.1458, 3.8303)	(-0.7971, 4.4816)
	BIP	1.1752	0.6194	(0.5038, 2.9385)	(0.1058, 3.3364)

Table 5.27: Moderate Positive Collinearity, ρ = 0.46 and sample size, N=70

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.4871	0.3629	(16.7625, 18.2117)	(16.5244, 18.4498)
(17)	**BNIP**	17.4871	0.3524	(16.7842, 18.1900)	(16.5539, 18.4203)
	BIP	16.5841	0.2641	(16.0578, 17.1104)	(15.8857, 17.2825)
θ_1	**LB**	8.6788	0.4957	(7.6892, 9.6686)	(7.3642, 9.9935)
(8.5)	**BNIP**	8.6788	0.4813	(7.7189, 9.6387)	(7.4044, 9.9532)
	BIP	10.0000	0.0009	(9.9982, 10.0018)	(9.9977, 10.0023)
θ_2	**LB**	4.0078	0.9211	(2.1688, 5.8470)	(1.5648, 6.4508)
(5.0)	**BNIP**	4.0078	0.8943	(2.2240, 5.7915)	(1.6396, 6.3759)
	BIP	5.0442	0.3980	(4.2513, 5.8371)	(3.9920, 6.0964)
θ_3	**LB**	1.6666	0.8560	(-0.0424, 3.3755)	(-0.6038, 3.9369)
(2.0)	**BNIP**	1.6666	0.8311	(0.0089, 3.3242)	(-0.5342, 3.8673)
	BIP	1.4772	0.6059	(0.2698, 2.6845)	(-0.1249, 3.0792)

Table 5.28: Moderate Positive Collinearity, ρ = 0.36 and sample size, N=70

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.0015	0.4092	(16.1845, 17.8184)	(15.9162, 18.0865)
(17)	**BNIP**	17.0015	0.4031	(15.7654, 16.8137)	(15.9494, 18.0535)
	BIP	16.2896	0.2667	(15.7917, 16.7465)	(15.5941, 16.9851)
θ_1	**LB**	9.0363	0.6659	(7.7068,10.3659)	(7.2701,10.8026)
(8.5)	**BNIP**	9.0363	0.6560	(7.7467, 10.3260)	(7.3242, 10.7485)
	BIP	10.0000	0.0008	(9.9983, 10.0017)	(9.9978, 10.0022)
θ_2	**LB**	4.2182	1.1278	(1.9665, 6.4700)	(1.2268, 7.2096)
(5.0)	**BNIP**	4.2182	1.1111	(2.0341, 6.4023)	(1.3185, 7.1179)
	BIP	5.1215	0.3870	(4.3608, 5.8822)	(4.1122, 6.1308)
θ_3	**LB**	1.8422	1.0265	(-0.2073, 3.8918)	(-0.8806, 4.5650)
(2.0)	**BNIP**	1.8422	1.0113	(-0.1458, 3.8303)	(-0.7971, 4.4816)
	BIP	1.1752	0.6194	(0.5038, 2.9385)	(0.1058, 3.3364)

Table 5.29: Low Positive Collinearity, ρ = 0.20 and sample size, N=70

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.3447	0.4080	(16.5301, 18.1593)	(16.2625, 18.4269)
(17)	**BNIP**	17.3447	0.3962	(16.5546, 18.1349)	(16.2957, 18.3937)
	BIP	16.3027	0.2495	(15.8056, 16.7998)	(15.6431, 16.9623)
θ_1	**LB**	8.6242	0.4610	(7.7038, 9.5445)	(7.4015, 9.8469)
(8.5)	**BNIP**	8.6242	0.4476	(7.7314, 9.5169)	(7.4389, 9.8094)
	BIP	10.0000	0.0009	(9.9983, 10.0017)	(9.9977, 10.0023)
θ_2	**LB**	4.6462	0.9301	(2.7892, 6.5033)	(2.1792, 7.1133)
(5.0)	**BNIP**	4.6462	0.9032	(2.8449, 6.4475)	(2.2548, 7.0377)
	BIP	5.2790	0.3963	(4.4893, 6.0686)	(4.2312, 6.3267)
θ_3	**LB**	1.2628	0.8934	(-0.5210, 3.0466)	(-1.1070, 3.6325)
(2.0)	**BNIP**	1.2628	0.8675	(-0.4675, 2.9930)	(-1.0344, 3.5599)
	BIP	1.6263	0.6275	(0.3760, 2.8766)	(-0.0327, 3.2853)

Table 5.30: Low Positive Collinearity, ρ = 0.17 and sample size, N=70

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.9087	0.3868	(15.9175, 17.2918)	(15.6949, 17.5144)
(17)	**BNIP**	16.9087	0.3756	(16.1596, 17.6579)	(15.9141, 17.9033)
	BIP	16.0190	0.2353	(15.5502, 16.4878)	(15.3969, 16.6410)
θ_1	**LB**	8.5162	0.4059	(5.2364, 8.9481)	(4.6357, 9.5494)
(8.5)	**BNIP**	8.5162	0.3942	(7.7300, 9.3024)	(7.4725, 9.5599)
	BIP	10.0000	0.0009	(9.9983, 10.0017)	(9.9977, 10.0023)
θ_2	**LB**	4.9773	0.9050	(3.9008, 10.5841)	(2.8182, 11.6666)
(5.0)	**BNIP**	4.9773	0.8788	(3.2246, 6.7299)	(2.6504, 7.3042)
	BIP	5.3295	0.3908	(4.5508, 6.1082)	(4.2962, 6.3628)
θ_3	**LB**	2.2787	0.7948	(1.1373, 5.3914)	(0.4483, 6.0804)
(2.0)	**BNIP**	2.2787	0.7717	(0.7396, 3.8179)	(0.2353, 4.3222)
	BIP	2.3808	0.5991	(1.1871, 3.5745)	(0.7968, 3.9648)

Table 5.31: Low Positive Collinearity, ρ = 0.15 and sample size, N=70

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.2549	0.3766	(16.5029, 18.0069)	(16.2559, 18.2539)
(17)	**BNIP**	17.2549	0.3657	(16.5255, 17.9843)	(16.2865,18.2233)
	BIP	16.3744	0.2453	(15.8857, 16.8632)	(15.7259, 17.0229)
θ_1	**LB**	8.0682	0.4260	(7.2177, 8.9187)	(6.9382, 9.1981)
(8.5)	**BNIP**	8.0682	0.4137	(7.2431, 8.8932)	(6.9728, 9.1635)
	BIP	10.0000	0.0008	(9.9983, 10.0016)	(9.9978, 10.0022)
θ_2	**LB**	4.4982	0.8206	(2.8598, 6.1366)	(2.3217, 6.6748)
(5.0)	**BNIP**	4.4982	0.7968	(2.9090, 6.0874)	(2.3883, 6.6081)
	BIP	5.2396	0.3766	(4.4893, 5.9899)	(4.2440, 6.2352)
θ_3	**LB**	3.1267	0.9184	(1.2931, 4.9604)	(0.6908, 5.5627)
(2.0)	**BNIP**	3.1267	0.8918	(1.3482, 4.9053)	(0.7655, 5.4880)
	BIP	2.3696	0.6324	(1.1095, 3.6298)	0.6975, 4.0417)

Table 5.32: Summary of Tables 5.23 -5.31 for Standard Error when the sample size, N= 70.

Parameters	Estimators	0.95	0.90	0.80	0.49	0.46	0.36	0.20	0.17	0.15
θ_0	**LB**	0.3760	0.3787	0.4167	0.4092	0.3629	0.4092	0.4080	0.3868	0.3766
	BNIP	0.3651	0.3677	0.4046	0.4031	0.3524	0.4031	0.3962	0.3756	0.3657
	BIP	0.2351	0.2396	0.2525	0.2667	0.2641	0.2667	0.2495	0.2353	0.2453
θ_1	**LB**	1.8816	1.3907	1.0675	0.6659	0.4957	0.6659	0.4610	0.4059	0.4260
	BNIP	1.8270	1.3503	1.0366	0.6560	0.4813	0.6560	0.4476	0.3942	0.4137
	BIP	0.0008	0.0008	0.0008	0.0008	0.0009	0.0008	0.0009	0.0009	0.0008
θ_2	**LB**	2.5519	2.1729	1.4572	1.1278	0.9211	1.1278	0.9301	0.9050	0.8206
	BNIP	2.4779	2.1099	1.4150	1.1111	0.8943	1.1111	0.9032	0.8788	0.7968
	BIP	0.3948	0.3843	0.3615	0.3870	0.3980	0.3870	0.3963	0.3908	0.3766
θ_3	**LB**	2.9699	2.0584	1.4436	1.0265	0.8560	1.0265	0.8934	0.7948	0.9184
	BNIP	2.8838	1.9987	1.4018	1.0113	0.8311	1.0113	0.8675	0.7717	0.8918
	BIP	0.6369	0.5952	0.5747	0.6194	0.6059	0.6194	0.6275	0.5991	0.6324

Table 5.32 gives the summary of SE in tables 5.23-5.31. When ρ = 0.95, for all the estimators have the highest SE for all the parameters except for intercept parameter θ_0. The SE estimators decreases as the ρ decreases across the parameters but increases when ρ =0.36 and also decreases again when ρ =0.20. For all levels of multicollinearity, BIP has the smallest SE across the parameters followed by BNIP estimator. There is no fixed pattern in the performance of the estimators for the levels of multicollinearity (HPC, MPC and LPC).

Table 5.33: Summary of Tables 5.23-5.31 for Mean for sample size, N= 70.

Parameters	Estimators	0.95	0.90	0.80	0.49	0.46	0.36	0.20	0.17	0.15
θ_0	**LB**	17.4391	16.9460	17.2351	17.0015	17.4871	17.0015	17.3447	16.9087	17.2549
(17.00)	**BNIP**	17.4391	16.9460	17.2351	17.0015	17.4871	17.0015	17.3447	16.9087	17.2549
	BIP	16.7363	16.2691	16.7268	16.2896	16.5841	16.2896	16.3027	16.0190	16.3744
θ_1	**LB**	9.7138	10.8772	8.8310	9.0363	8.6788	9.0363	8.6242	8.5162	8.0682
(8.5)	**BNIP**	9.7138	10.8772	8.8310	9.0363	8.6788	9.0363	8.6242	8.5162	8.0682
	BIP	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000
θ_2	**LB**	2.8381	3.7170	5.0839	4.2182	4.0078	4.2182	4.6462	4.9773	4.4982
(5.00)	**BNIP**	2.8381	3.7170	5.0839	4.2182	4.0078	4.2182	4.6462	4.9773	4.4982
	BIP	4.9526	5.2293	5.1653	5.1215	5.0442	5.1215	5.2790	5.3295	5.2396
θ_3	**LB**	0.8514	-0.7270	1.6487	1.8422	1.6666	1.8422	1.2628	2.2787	3.1267
(2.00)	**BNIP**	0.8514	-0.7270	1.6487	1.8422	1.666	1.8422	1.2628	2.2787	3.1267
	BIP	0.5777	1.1752	1.2926	1.1752	1.4772	1.1752	1.6263	2.3808	2.3696

Table 5.33 shows the summary of mean for tables 5-23-5.31 when the sample size is 70, BIP has positive means values for all the levels of collinearity across the parameters. As the ρ reduces, the mean estimates tend toward the true parameter values. The mean values of LB and BNIP are the same for all the levels of collinearity for all the parameters considered while the negative mean values were observed for LB and BNIP when ρ= 0.90 for parameter θ_3.

Table 5.34: High Positive Collinearity, ρ = 0.95 and sample size, N=100

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.2229	0.2946	(16.6382, 17.8076)	(16.4488, 17.9970)
(17)	**BNIP**	17.2229	0.2886	(16.6503, 17.7955)	(16.4651, 17.9808)
	BIP	16.6918	0.2177	(16.2600, 17.1236)	(16.1205, 17.2632)
θ_1	**LB**	9.0684	1.5291	(6.0332, 12.1037)	(5.0499, 13.0870)
(8.5)	**BNIP**	9.0684	1.4982	(6.0960, 12.0408)	(5.1343, 13.0026)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9978, 10.0022)
θ_2	**LB**	5.2582	2.1747	(0.9413, 9.5750)	(-0.4571, 10.9735)
(5.0)	**BNIP**	5.2582	2.1308	(1.0307, 9.4856)	(-0.3371, 10.8534)
	BIP	4.9383	0.3778	(4.1890, 5.6875)	(3.9469, 5.9296)
θ_3	**LB**	-0.0828	2.1315	(-4.3138, 4.1481)	(-5.6844, 5.5187)
(2.0)	**BNIP**	-0.0828	2.0884	(-4.2262, 4.0605)	(-5.5667, 5.4011)
	BIP	0.2707	0.5756	(-0.8709, 1.4122)	(-1.2398, 1.7811)

Table 5.35: High Positive Collinearity, ρ = 0.90 and sample size, N=100

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.6046	0.3462	(15.9175, 17.2918)	(15.6949, 17.5144)
(17)	**BNIP**	16.6046	0.3392	(15.9317, 17.2775)	(15.7140, 17.4953)
	BIP	16.6678	0.2320	(16.2078, 17.1279)	(16.0591, 17.2766)
θ_1	**LB**	7.0922	0.9350	(5.2364, 8.9481)	(4.6357, 9.5494)
(8.5)	**BNIP**	7.0922	0.9161	(5.2748, 8.9097)	(4.6867, 9.4978)
	BIP	10.0000	0.0008	(9.9983, 10.0017)	(9.9978, 10.0022)
θ_2	**LB**	7.2424	1.6835	(3.9008, 10.5841)	(2.8182, 11.6666)
(5.0)	**BNIP**	7.2424	1.6495	(3.9699, 10.5149)	(2.9111, 11.5737)
	BIP	4.9277	0.3825	(4.1740, 5.6813)	(3.9304, 5.9249)
θ_3	**LB**	3.2644	1.0716	(1.1373, 5.3914)	(0.4483, 6.0804)
(2.0)	**BNIP**	3.2644	1.0499	(1.1814, 5.3473)	(0.5074, 6.0213)
	BIP	1.2330	0.5727	(0.0974, 2.3687)	(-0.2697, 2.7357)

Table 5.36: High Positive Collinearity, ρ = 0.80 and sample size, N=100

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.4488	0.3102	(15.833, 17.0644)	(15.6336,17.263)
(17)	**BNIP**	16.4488	0.3039	(15.8458, 17.0517)	(15.6507,17.246)
	BIP	16.3866	0.2239	(15.9427, 16.8306)	(15.7993,16.974)
θ_1	**LB**	7.5078	0.7894	(5.9408, 9.0748)	(5.4334, 9.5824)
(8.5)	**BNIP**	7.5078	0.7735	(5.9732, 9.0423)	(5.4767, 9.5389)
	BIP	10.0000	0.0008	(9.9985, 10.0015)	(9.9980,10.0020)
θ_2	**LB**	7.6124	1.0594	(5.5094, 9.7153)	(4.8282,10.3966)
(5.0)	**BNIP**	7.6124	1.0380	(5.5529, 9.6718)	(4.8866,10.3381)
	BIP	5.2772	0.3366	(4.6097, 5.9447)	(4.3940, 6.1605)
θ_3	**LB**	2.2977	1.1462	(0.0225, 4.5730)	(-0.7146,5.3100)
(2.0)	**BNIP**	2.2977	1.1231	(0.0696, 4.5259)	(-0.6513,5.2468)
	BIP	1.0252	0.5268	(-0.0195, 2.0699)	(-0.3572,2.4076)

Table 5.37: Moderate Positive Collinearity, ρ = 0.49 and sample size, N=100

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.3881	0.7826	(14.8415, 17.9415)	(14.3314, 18.4447)
(17)	**BNIP**	16.3881	0.7668	(14.8668, 17.9093)	(14.3746, 18.4015)
	BIP	16.2830	0.5786	(15.1356, 17.4304)	(14.7648, 17.8012)
θ_1	**LB**	4.4289	1.3624	(1.7246, 7.1331)	(0.8485, 8.0092)
(8.5)	**BNIP**	4.4289	1.3348	(1.7806, 7.0772)	(0.9237, 7.9340)
	BIP	10.0000	0.0022	(9.9957, 10.0043)	(9.9943, 10.0057)
θ_2	**LB**	8.9793	2.3137	(4.3868, 13.5719)	(2.8990, 15.0597)
(5.0)	**BNIP**	8.9793	2.2669	(4.4819, 13.4768)	(3.0267, 14.9320)
	BIP	4.9845	0.9454	(3.1098, 6.8592)	(2.5039, 7.4651)
θ_3	**LB**	5.6519	2.2472	(1.1912, 10.1126)	(-0.2538, 11.5576)
(2.0)	**BNIP**	5.6519	2.2018	(1.2836, 10.0202)	(-0.1298, 11.4336)
	BIP	1.3869	1.4472	(-1.4829, 4.2567)	(-2.4104, 5.1842)

Table 5.38: Moderate Positive Collinearity, ρ = 0.46 and sample size, N=100

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.4596	0.2905	(15.8829, 17.0362)	(15.6961, 17.2230)
(17)	**BNIP**	16.4596	0.2846	(15.8948, 17.0243)	(15.7121, 17.2070)
	BIP	16.2111	0.2286	(15.7578, 16.6645)	(15.6113, 16.8110)
θ_1	**LB**	8.1608	0.3873	(7.3919, 8.9296)	(7.1428, 9.1787)
(8.5)	**BNIP**	8.1608	0.3795	(7.4078, 8.9137)	(7.1642, 9.1573)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
θ_2	**LB**	6.6151	0.6907	(5.2441, 7.9861)	(4.8000, 8.4302)
(5.0)	**BNIP**	6.6151	0.6767	(5.2725, 7.9577)	(4.8381, 8.3921)
	BIP	5.4260	0.3463	(4.7392, 6.1127)	(4.5173, 6.3347)
θ_3	**LB**	2.2563	0.7361	(0.7953, 3.7174)	(0.3220, 4.1907)
(2.0)	**BNIP**	2.2563	0.7212	(0.8255, 3.6871)	(0.3626, 4.1501)
	BIP	1.6359	0.5357	(0.5736, 2.6981)	(0.2303, 3.0415)

Table 5.39: Moderate Positive Collinearity, ρ = 0.36 and sample size, N=100

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.7849	0.4135	(15.9642, 17.6057)	(15.6984, 17.8715)
(17)	**BNIP**	16.7849	0.4051	(15.9812, 17.5887)	(15.7212, 17.8487)
	BIP	16.1767	0.2699	(15.6415, 16.7120)	(15.4685, 16.8850)
θ_1	**LB**	8.6829	0.4816	(7.7269, 9.6390)	(7.4172, 9.9487)
(8.5)	**BNIP**	8.6829	0.4719	(7.7467, 9.6192)	(7.4437, 9.9221)
	BIP	10.0000	0.0009	(9.9982, 10.0018)	(9.9976, 10.0024)
θ_2	**LB**	4.6911	0.9750	2.7558, 6.6264)	(2.1289, 7.2534)
(5.0)	**BNIP**	4.6911	0.9553	(2.7959, 6.5864)	(2.1827, 7.1996)
	BIP	5.1728	0.4138	(4.3521, 5.9934)	(4.0869, 6.2586)
θ_3	**LB**	3.0622	0.8887	(1.2982, 4.8263)	(0.72679, 5.3977)
(2.0)	**BNIP**	3.0622	0.8707	(1.3348, 4.7897)	(0.7758, 5.3487)
	BIP	2.2752	0.6187	(1.0483, 3.5021)	(0.6518, 3.8986)

Table 5.40: Low Positive Collinearity, ρ = 0.20 and sample size, N=100

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.0939	0.3124	(16.4737, 17.7140)	(16.2728, 17.9149)
(17)	**BNIP**	17.0939	0.3061	(16.4866, 17.7012)	(16.290, 17.8977)
	BIP	16.2146	0.2199	(15.7786, 16.6506)	(15.6377, 16.7915)
θ_1	**LB**	8.2836	0.3602	(7.5687, 8.9986)	(7.3371, 9.2302)
(8.5)	**BNIP**	8.2836	0.3529	(7.5835, 8.9837)	(7.3569, 9.2103)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9978, 10.0022)
θ_2	**LB**	5.2181	0.6888	(3.8509, 6.5853)	(3.4080, 7.0282)
(5.0)	**BNIP**	5.2181	0.6749	(3.8792, 6.5570)	(3.4460, 6.9902)
	BIP	5.2852	0.3598	(4.5717, 5.9986)	(4.3411, 6.2292)
θ_3	**LB**	2.0146	0.6895	(0.6459, 3.3834)	(0.2025, 3.8268)
(2.0)	**BNIP**	2.0146	0.6756	(0.6742, 3.3550)	(0.2406, 3.7887)
	BIP	2.1230	0.5526	(1.0271, 3.2189)	(0.6729, 3.5731)

Table 5.41: Low Positive Collinearity, ρ = 0.17 and sample size, N=100

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.1693	0.2713	(16.6307, 17.7080)	(16.4563, 17.8824)
(17)	**BNIP**	17.1693	0.2659	(16.6419, 17.6968)	(16.4712, 17.8675)
	BIP	16.2104	0.2019	(15.8100, 16.6107)	(15.6806, 16.7401)
θ_1	**LB**	8.4619	0.3093	(7.8480, 9.0758)	(7.6491, 9.2746)
(8.5)	**BNIP**	8.4619	0.3030	(7.8607, 9.0630)	(7.6662, 9.2576)
	BIP	10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0020)
θ_2	**LB**	4.4443	0.6257	(3.2023, 5.6862)	(2.8000, 6.0885)
(5.0)	**BNIP**	4.4443	0.6130	(3.2281, 5.6605)	(2.8346, 6.0540)
	BIP	5.1451	0.3322	(4.4863, 5.8039)	(4.2734, 6.0168)
θ_3	**LB**	1.6344	0.6242	(0.3953, 2.8735)	(-0.0061, 3.2749)
(2.0)	**BNIP**	1.6344	0.6116	(0.4209, 2.8478)	(0.0283, 3.2404)
	BIP	1.7343	0.5089	(0.7252, 2.7435)	(0.3991, 3.0696)

Table 5.42: Low Positive Collinearity, ρ = 0.15 and sample size, N=100

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.8517	0.3297	(16.1972, 17.5061)	(15.9852, 17.7181)
(17)	**BNIP**	16.8517	0.3230	(16.2107, 17.4926)	(16.0034, 17.6999)
	BIP	16.1034	0.2148	(15.6775, 16.5293)	(15.5398, 16.6669)
θ_1	**LB**	8.7026	0.3474	(8.0130, 9.3922)	(7.7896, 9.6156)
(8.5)	**BNIP**	8.7026	0.3404	(8.0273, 9.3780)	(7.8088, 9.5965)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9978, 10.0022)
θ_2	**LB**	5.6415	0.6710	(4.3096, 6.9733)	(3.8782, 7.4048)
(5.0)	**BNIP**	5.6415	0.6574	(4.3372, 6.9458)	(3.9152, 7.3678)
	BIP	5.4518	0.3528	(4.7521, 6.1515)	(4.5259, 6.3776)
θ_3	**LB**	1.5498	0.7346	(0.0917, 3.0079)	(-0.3807, 3.4802)
(2.0)	**BNIP**	1.5498	0.7197	(0.1218, 2.9777)	(-0.3402, 3.4397)
	BIP	2.0187	0.5585	(0.9113, 3.1262)	(0.5533, 3.4841)

From Tables 5.23-5.42, the following observations were made when the sample sizes are 70 and 100:

The means of LB and BNIP are the same for the sample sizes considered across the parameters. The means of LB and BNIP for parameters θ_0, θ_1, θ_2, θ_3 are 17.4871, 8.6758, 4.0078 and 1.6666, respectively when the sample size is 70 for MPC.

The mean estimates of all the estimators (LB, BNIP and BIP) are in line with the true parameter values. Bayesian estimators (BNIP and BIP) have smaller SE than LB, for instance in table 5.36, the SE of LB, BNIP and BIP are 1.0594, 1.0380, 0.3366, respectively for parameter θ_2. HPC and MPC have the highest values of SE for sample sizes 70 and 100. The Confidence Intervals of LB are wider than the Bayesian estimators (BIP and BNIP) for HPC, MPC and LPC.

The CI for sample sizes of 70 and 100 for all the estimators are better compared to sample sizes of 10 and 30; for instance, when the sample size is 70 for parameter θ_2, CI for LB, BNIP and BIP are (2.8598, 6.1366), (2.9090, 6.0874) and (4.4893, 5.9899), respectively while sample size of 30 for parameter θ_2, the CI for LB, BNIP and BIP are (1.5910, 6.5608), (1.7775, 6.3743) and (4.7807, 6.0630) respectively under LPC. This means as the sample size increases the CI also becomes more compact.

Hence, BIP outperformed other estimators, having the smallest SE values for all the parameters considered and compact CI. It also appears that sample size 70 is a turning point that shows the asymptotic effect of the estimators.

Table 5.43: Summary of Tables 5.34 -5.42 for Standard Error when the sample size, N= 100.

Parameters	Estimators	0.95	0.90	0.80	0.49	0.46	0.36	0.20	0.17	0.15
θ_0	**LB**	0.2946	0.3462	0.3102	0.7826	0.2905	0.4135	0.3124	0.2713	0.3297
	BNIP	0.2886	0.3392	0.3039	0.7668	0.2846	0.4051	0.3061	0.2659	0.3230
	BIP	0.2177	0.2320	0.2239	0.5786	0.2286	0.2699	0.2199	0.2019	0.2148
θ_1	**LB**	1.5291	0.9350	0.7894	1.3624	0.3873	0.4816	0.3602	0.3093	0.3474
	BNIP	1.4982	0.9161	0.7735	1.3348	0.3795	0.4719	0.3529	0.3030	0.3404
	BIP	0.0008	0.0008	0.0008	0.0022	0.0008	0.0009	0.0008	0.0008	0.0008
θ_2	**LB**	2.1747	1.6835	1.0594	2.3137	0.6907	0.9750	0.6888	0.6257	0.6710
	BNIP	2.1308	1.6495	1.0380	2.2669	0.3463	0.9553	0.6749	0.6130	0.6574
	BIP	0.3778	0.3825	0.3366	0.9454	0.3463	0.4138	0.3598	0.3322	0.3528
θ_3	**LB**	2.1315	1.0716	1.1462	2.2472	0.7361	0.8887	0.6895	0.6242	0.7346
	BNIP	2.0884	1.0499	1.1231	2.2018	0.7212	0.8707	0.6756	0.6116	0.7197
	BIP	0.5756	0.5727	0.5268	1.4472	0.5357	0.6187	0.5526	0.5089	0.5585

Table 5.43 gives a summary of SE of Tables 5.34-5.42 for all the levels of collinearity.

Highest SE of estimate was observed mostly when ρ =0.49 for all the estimators while lowest SE of estimate was observed when ρ =0.17 for all the estimators across the parameters. BIP has the smallest SE compared to other estimators (LB and BNIP) for all the level of collinearity across the parameters. LB estimator has the highest SE for all the levels of collinearity across the parameters considered.

Hence, BIP outperformed other estimators in terms of SE.

Table 5.44: Summary of Tables 5.34-5.42 for Mean for sample size, N= 100.

Parameters	Estimators	0.95	0.90	0.80	0.49	0.46	0.36	0.20	0.17	0.15
θ_0 (17.00)	**LB**	17.229	16.6046	16.4488	16.3881	16.4596	16.7849	17.0939	17.1693	16.8517
	BNIP	17.229	16.6046	16.4488	16.3881	16.4596	16.7849	17.0939	17.1693	16.8517
	BIP	16.6918	16.6678	16.3866	16.2830	16.2111	16.1767	16.2146	16.2104	16.1034
θ_1 (8.5)	**LB**	9.0684	7.8772	7.5078	4.4289	8.1608	8.6829	8.2836	8.4619	8.7026
	BNIP	9.0684	7.8772	7.5078	4.4289	8.1608	8.6829	8.2836	8.4619	8.7026
	BIP	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000
θ_2 (5.00)	**LB**	5.2582	7.2424	7.6124	4.9845	6.6151	4.6911	5.2181	4.4443	5.6415
	BNIP	5.2582	7.2424	7.6124	4.9845	6.6151	4.6911	5.2181	4.4443	5.4515
	BIP	4.9383	4.9277	5.2772	4.9845	5.4260	5.1728	5.2852	5.1451	5.4518
θ_3 (2.00)	**LB**	-0.0828	3.2644	2.2977	5.6519	2.2563	3.0622	2.0146	1.6344	1.5498
	BNIP	-0.0828	3.2644	2.2977	5.6519	2.2563	3.0622	2.0146	1.6344	1.5498
	BIP	0.2707	1.2330	1.0252	1.3869	1.6359	2.2752	2.1230	1.7343	2.0187

Table 5.44 gives a summary of means of Tables 5.34-5.42 for all the levels of collinearity. Negative means of estimate was not observed at all the levels of collinearity using the estimators except for ρ =0.95 for LB and BNIP. Most of the means are not too far from the true parameter values except for parameter θ_3 when ρ =0.49 and 0.95.

Table 5.45: High Positive Collinearity, ρ = 0.95 and sample size, N=200

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.0056	0.2174	(16.5865, 17.4247)	(16.4528, 17.5583)
(17)	**BNIP**	17.0056	0.2104	(16.5907, 17.4204)	(16.4585, 17.5527)
	BIP	16.8949	0.2125	(16.5879, 17.2018)	(16.4900, 17.2997)
θ_1	**LB**	8.0263	1.0457	(5.9640, 10.0887)	(5.3062,10.7465)
(8.5)	**BNIP**	8.0263	1.0352	(5.9849, 10.0677)	(5.3341, 10.7186)
	BIP	10.0000	0.0008	(9.9983, 10.0016)	(9.9978, 10.0022)
θ_2	**LB**	2.4794	1.5373	(-0.5523, 5.5111)	(-1.5192, 6.4780)
(5.0)	**BNIP**	2.4794	1.5218	(-0.5214, 5.4803)	(-1.4782, 6.4371)
	BIP	4.5251	0.3689	(3.7979, 5.2524)	(3.5660, 5.4842)
θ_3	**LB**	5.2057	1.5661	(2.1171, 8.2944)	(1.1320, 9.2795)
(2.0)	**BNIP**	5.2057	1.5504	(2.1485, 8.2630)	(1.1737, 9.2377)
	BIP	0.3455	0.5015	(-0.6434, 1.3343)	(-0.9586, 1.6495)

Table 5.46: High Positive Collinearity, ρ = 0.90 and sample size, N=200

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.7492	0.2174	(16.3204, 17.1780)	(16.1836, 17.3148)
(17)	**BNIP**	16.7492	0.2152	(16.3247, 17.1736)	(16.1894, 17.3090)
	BIP	16.8698	0.1615	(16.5513, 17.1883)	(16.4498, 17.2898)
θ_1	**LB**	7.4293	0.7690	(5.9127, 8.9458)	(5.4290, 9.4295)
(8.5)	**BNIP**	7.4293	0.7613	(5.9281, 8.9304)	(5.4495, 9.4090)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
θ_2	**LB**	6.7790	1.0858	(4.6376, 8.9203)	(3.9546, 9.6033)
(5.0)	**BNIP**	6.7790	1.0749	(4.6594, 8.8986)	(3.9836, 9.5744)
	BIP	4.9483	0.3457	(4.2667, 5.6299)	(4.0494, 5.8471)
θ_3	**LB**	2.8935	1.1222	(0.6804, 5.1066)	(-0.0255, 5.8125)
(2.0)	**BNIP**	2.8935	1.1109	(0.7029, 5.0841)	(0.0044, 5.7826)
	BIP	0.4838	0.4612	(-0.4256, 1.3932)	(-0.7155, 1.6831)

Table 5.47: High Positive Collinearity, ρ = 0.80 and sample size, N=200

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.6954	0.2050	(16.2912, 17.0996)	(16.1623, 17.2285)
(17)	**BNIP**	16.6954	0.2029	(16.2953, 17.0955)	(16.1678,17.2231)
	BIP	16.6262	0.1660	(16.2988, 16.9536)	(16.1945, 17.0579)
θ_1	**LB**	8.2638	0.4811	(7.3150, 9.2127)	(7.0123, 9.5153)
(8.5)	**BNIP**	8.2638	0.4763	(7.3246, 9.2030)	(7.0251, 9.5025)
	BIP	10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0020)
θ_2	**LB**	5.7751	0.7675	(4.2616, 7.2887)	(3.7789, 7.7714)
(5.0)	**BNIP**	5.7751	0.7597	(4.2770, 7.2733)	(3.7993, 7.7509)
	BIP	4.9022	0.3240	(4.2634, 5.5410)	(4.0598, 5.7446)
θ_3	**LB**	1.8511	0.7885	(0.2960, 3.4062)	(-0.2000, 3.9022)
(2.0)	**BNIP**	1.8511	0.7806	(0.3118, 3.3904)	(-0.1790, 3.8812)
	BIP	0.3914	0.4334	(-0.4631, 1.2459)	(-0.7354, 1.5183)

Table 5.48: Moderate Positive Collinearity, ρ = 0.49 and sample size, N=200

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.8946	0.2141	(16.4724, 17.3168)	(16.3378, 17.4514)
(17)	**BNIP**	16.8946	0.2119	(16.4767, 17.3125)	(16.3435, 17.4457)
	BIP	16.6711	0.1870	(16.3023, 17.0398)	(16.1847, 17.1574)
θ_1	**LB**	7.9956	0.3617	(7.2823, 8.7089)	(7.0548, 8.9364)
(8.5)	**BNIP**	7.9956	0.3581	(7.2896, 8.7017)	(7.0644, 8.9268)
	BIP	10.0000	0.0008	9.9983, 10.0016)	(9.9978, 10.0021)
θ_2	**LB**	5.3705	0.5955	(4.1961, 6.5450)	(3.8215, 6.9195)
(5.0)	**BNIP**	5.3705	0.5895	(4.2080, 6.5330)	(3.8373, 6.9037)
	BIP	4.7703	0.3299	(4.1198, 5.4208)	(3.9124, 5.6281)
θ_3	**LB**	2.6325	0.6206	(1.4085, 3.8565)	(1.0182, 4.2468)
(2.0)	**BNIP**	2.6325	0.6144	(1.4210, 3.8440)	(1.0347, 4.2303)
	BIP	1.0956	0.4612	(0.1863, 2.0050)	(-0.1036, 2.2948)

Table 5.49: Moderate Positive Collinearity, ρ = 0.46 and sample size, N=200

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.8689	0.2328	(16.4097, 17.3282)	(16.2633, 17.4746)
(17)	**BNIP**	16.8689	0.2305	(16.4144, 17.3235)	(16.2695, 17.4684)
	BIP	16.3679	0.2064	(15.9610, 16.7748)	(15.8313, 16.9045)
θ_1	**LB**	8.0520	0.2998	(7.4608, 8.6433)	(7.2722, 8.8318)
(8.5)	**BNIP**	8.0520	0.2968	(7.4668, 8.6372)	(7.2802, 8.8238)
	BIP	10.0000	0.0009	(9.9982, 10.0017)	(9.9977, 10.0023)
θ_2	**LB**	5.3563	0.5659	(4.2402, 6.4723)	(3.8842, 6.8283)
(5.0)	**BNIP**	5.3563	0.5602	(4.2515, 6.4610)	(3.8993, 6.8132)
	BIP	4.8690	0.3489	(4.1811, 5.5569)	(3.9619, 5.7761)
θ_3	**LB**	2.7603	0.5858	(1.6050, 3.9157)	(1.2365, 4.2842)
(2.0)	**BNIP**	2.7603	0.5800	(1.6167, 3.9040)	(1.2521, 4.2686)
	BIP	1.6816	0.4955	(0.7045, 2.6586)	(0.3931, 2.9700)

Table 5.50: Moderate Positive Collinearity, ρ = 0.36 and sample size, N=200

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.7701	0.2331	(16.3105, 17.2298)	(16.1639, 17.3764)
(17)	**BNIP**	16.7701	0.2307	(16.3151, 17.2251)	(16.1701, 17.3702)
	BIP	16.4683	0.1856	(16.1024, 16.8342)	(15.9858, 16.9509)
θ_1	**LB**	8.6002	0.2678	(8.0721, 9.1284)	(7.9036,9.2969)
(8.5)	**BNIP**	8.6002	0.2651	(8.0774, 9.1231)	(7.9107, 9.2898)
	BIP	10.0000	0.0008	(9.9984, 10.0015)	(9.9979, 10.0020)
θ_2	**LB**	6.0823	0.5189	(5.0588, 7.1057)	(4.7324, 7.4321)
(5.0)	**BNIP**	6.0823	0.5137	(5.0692, 7.0953)	(4.7462, 7.4183)
	BIP	5.3394	0.3094	(4.7294, 5.9494)	(4.5349, 6.1439)
θ_3	**LB**	1.7572	0.5056	(0.7601, 2.7543)	(0.4420,3.0723)
(2.0)	**BNIP**	1.7572	0.5005	(0.7702, 2.7442)	(0.4555, 3.0588)
	BIP	1.3304	0.4202	(0.5018, 2.1590)	(0.2377, 2.4231)

Table 5.51: Low Positive Collinearity, ρ = 0.20 and sample size, N=200

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.0776	0.2158	(16.6521, 17.5032)	(16.5164, 17.6389)
(17)	**BNIP**	17.0776	0.2136	(16.6564, 17.4989)	(16.5221, 17.6332)
	BIP	16.4234	0.1780	(16.0724, 16.7743)	(15.9606, 16.8862)
θ_1	**LB**	8.3348	0.2491	(7.8437, 8.8260)	(7.6870, 8.9827)
(8.5)	**BNIP**	8.3348	0.2465	(7.8487, 8.8210)	(7.6937, 8.9760)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
θ_2	**LB**	5.5911	0.4769	(4.6506, 6.5315)	(4.3507, 6.8314)
(5.0)	**BNIP**	5.5911	0.4721	(4.6602, 6.5219)	(4.3634, 6.8187)
	BIP	5.2829	0.3188	(4.6542, 5.9115)	(4.4539, 6.1119)
θ_3	**LB**	1.9444	0.4908	(0.9764, 2.9124)	(0.6676, 3.2211)
(2.0)	**BNIP**	1.9444	0.4859	(0.9862, 2.9025)	(0.6807, 3.2080)
	BIP	1.4727	0.4367	(0.6117, 2.3338)	(0.3372, 2.6083)

Table 5.52: Low Positive Collinearity, ρ = 0.17 and sample size, N=200

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.0260	0.2248	(16.5827, 17.4693)	(16.4413, 17.6107)
(17)	**BNIP**	17.0260	0.2225	(16.5872, 17.4648)	(16.4473, 17.6047)
	BIP	16.3895	0.1752	(16.0440, 16.7349)	(15.9339, 16.8450)
θ_1	**LB**	8.6144	0.2773	(8.0675, 9.1612)	(7.8931, 9.3357)
(8.5)	**BNIP**	8.6144	0.2745	(8.0730, 9.1557)	(7.9005, 9.3283)
	BIP	10.0000	0.0008	(9.9983, 10.0016)	(9.9978, 10.0022)
θ_2	**LB**	5.3104	0.5273	(4.27036, 6.3504)	(3.9387, 6.6821)
(5.0)	**BNIP**	5.3104	0.5220	(4.2809, 6.3398)	(3.9527, 6.6680)
	BIP	5.2743	0.3327	(4.6184, 5.9302)	(4.4094, 6.1393)
θ_3	**LB**	1.1954	0.4943	(0.2207, 2.1702)	(-0.0902, 2.4811)
(2.0)	**BNIP**	1.1954	0.4893	(0.2306, 2.1603)	(-0.0771, 2.4679)
	BIP	1.0690	0.4307	(0.2199, 1.9181)	(-0.0508, 2.1888)

Table 5.53: Low Positive Collinearity, ρ = 0.15 and sample size, N=200

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.0242	0.2260	(16.5786, 17.470)	(16.4365, 17.6120)
(17)	**BNIP**	17.0242	0.2237	(16.5831, 17.4653)	(16.4425, 17.6060)
	BIP	16.2998	0.1745	(15.9557, 16.6439)	(15.8461, 16.7535)
θ_1	**LB**	8.6313	0.2533	(8.1318, 9.1309)	(7.9725, 9.2902)
(8.5)	**BNIP**	8.6313	0.2508	(8.1369, 9.1258)	(7.9792, 9.2835)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9978, 10.0021)
θ_2	**LB**	4.7879	0.4853	(3.8308, 5.7450)	(3.5255, 6.0503)
(5.0)	**BNIP**	4.7879	0.4804	(3.8405, 5.7353)	(3.5385, 6.0374)
	BIP	5.0798	0.3220	(4.4450, 5.7146)	(4.2426, 5.9170)
θ_3	**LB**	2.2433	0.4850	(1.2868, 3.1997)	(0.9818, 3.5048)
(2.0)	**BNIP**	2.2433	0.4801	(1.2965, 3.1900)	(0.9947, 3.4919)
	BIP	2.0880	0.4351	(1.2302, 2.9458)	(0.9567, 3.2193)

Table 5.54: Summary of Table 5.45 -5.53 for Standard Error when the sample size, N=200.

Parameters	Estimators	0.95	0.90	0.80	0.49	0.46	0.36	0.20	0.17	0.15
θ_0	**LB**	0.2174	0.2174	0.2050	0.2141	0.2328	0.2331	0.2158	0.2248	0.2260
	BNIP	0.2104	0.2152	0.2029	0.2119	0.2305	0.2307	0.2136	0.2225	0.2237
	BIP	0.2125	0.1615	0.1660	0.1870	0.2064	0.1856	0.1780	0.1752	0.1745
θ_1	**LB**	1.0457	0.7690	0.4811	0.3617	0.2998	0.2678	0.2491	0.2773	0.2533
	BNIP	1.0352	0.7613	0.4763	0.3581	0.2968	0.2651	0.2465	0.2745	0.2508
	BIP	0.0008	0.0008	0.0008	0.0008	0.0009	0.0008	0.0008	0.0008	0.0008
θ_2	**LB**	1.5373	1.0858	0.7675	0.5955	0.5659	0.5189	0.4769	0.5273	0.4853
	BNIP	1.5218	1.0749	0.7597	0.5895	0.5602	0.5137	0.4721	0.5220	0.4804
	BIP	0.3689	0.3457	0.3240	0.3299	0.3489	0.3094	0.3188	0.3327	0.3220
θ_3	**LB**	1.5661	1.1222	0.7885	0.6206	0.5858	0.5056	0.4908	0.4943	0.4850
	BNIP	1.5504	1.1109	0.7806	0.6144	0.5800	0.5005	0.4859	0.4893	0.4801
	BIP	0.5015	0.4612	0.4334	0.4612	0.4955	0.4202	0.4367	0.4307	0.4351

Table 5.54 gives the summary of SE in tables 5.45-5.53 when the sample size is 200. As the ρ decreases, the SE of the estimators also decreases for all the parameters. It is also observed that the SE are smaller compared to sample sizes of 10, 30, 70 an 100 across the parameters.

Among all the estimators considered, BIP outperformed all other estimators having the smallest SE in all the cases of collinearity considered (0.15-0.95). However, as the ρ decreases, the SE of both LB and BNIP tends toward the SE of BIP.

Table 5.55: Summary of Tables 5.45-5.53 for Mean for sample size, N= 200.

Parameters	Estimators	0.95	0.90	0.80	0.49	0.46	0.36	0.20	0.17	0.15
θ_0	**LB**	17.0056	16.7492	16.6954	16.8946	16.8689	16.7701	17.0776	17.0260	17.0242
(17.00)	**BNIP**	17.0056	16.7492	16.6954	16.6711	16.8689	16.7701	17.0776	17.0260	17.0242
	BIP	16.8949	16.8698	16.6262	16.6711	16.3679	16.4683	16.4234	16.3895	16.2998
θ_1	**LB**	8.0263	7.4293	8.2638	7.9956	8.0520	8.6002	8.3348	8.6144	8.6313
(8.5)	**BNIP**	8.0263	7.4293	8.2638	7.9956	8.0520	8.6002	8.3348	8.6144	8.6313
	BIP	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000
θ_2	**LB**	2.4794	6.7790	5.7751	5.3705	5.3563	6.0823	5.5911	5.3104	4.7879
(5.00)	**BNIP**	2.4794	6.7790	5.7751	5.3705	5.3563	6.0823	5.5911	5.3104	4.7879
	BIP	4.5251	4.9483	4.9022	4.7703	4.8690	5.3394	5.2829	5.2743	5.0798
θ_3	**LB**	5.2057	2.8935	1.8511	2.6325	2.7603	1.7572	1.9444	1.1954	2.2433
(2.00)	**BNIP**	5.2057	2.8935	1.8511	2.6325	2.7603	1.7572	1.9444	1.1954	2.2433
	BIP	0.3455	0.4838	0.3914	1.0956	1.6816	1.3304	1.4727	1.0690	2.0880

Table 5.55 summarizes the mean estimates of all the estimators for parameters (θ_0, θ_1, θ_2 and θ_3), when the sample size is 200.

The means of LB and BNIP are the same for all the levels of collinearity considered across the parameters. All the means are positive and not too far from the true parameter values. For parameter θ_3, the means of BIP are far from the true parameter value except for ρ=0.46 and 0.15.

Table 5.56: High Positive Collinearity, ρ = 0.95 and sample size, N=300

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.6654	0.1720	(16.3269, 17.0038)	(16.2195, 17.1113)
(17)	**BNIP**	16.6654	0.1708	(16.3292, 17.0016)	(16.2225, 17.1083)
	BIP	16.8260	0.1297	(16.5707, 17.0812)	(16.4898, 17.1621)
θ_1	**LB**	6.9703	0.7995	(5.3969, 8.5436)	(4.8976, 9.0429)
(8.5)	**BNIP**	6.9703	0.7941	(5.4075, 8.5330)	(4.9117, 9.0289)
	BIP	10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0020)
θ_2	**LB**	5.4153	1.1662	(3.1202, 7.7103)	(2.3920, 8.4386)
(5.0)	**BNIP**	5.4153	1.1584	(3.1357, 7.6948)	(2.4124, 8.4181)
	BIP	4.6317	0.3327	(3.9769, 5.2864)	(3.7692, 5.4941)
θ_3	**LB**	4.9680	1.1761	(2.6534, 7.2825)	(1.9188, 8.0171)
(2.0)	**BNIP**	4.9680	1.1682	(2.6690, 7.2669)	(1.9395, 7.9964)
	BIP	0.4720	0.4245	(-0.3632, 1.3072)	(-0.6282, 1.5722)

Table 5.57: High Positive Collinearity, ρ = 0.90 and sample size, N=300

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.1138	0.1572	(16.8045, 17.4232)	(16.7063, 17.5213)
(17)	**BNIP**	17.1138	0.1561	(16.8066, 17.4211)	(16.7091, 17.5186)
	BIP	17.0194	0.1247	(16.7741, 17.2647)	(16.6962, 17.3425)
θ_1	**LB**	8.8058	0.6309	(7.5643, 10.0474)	(7.1703, 10.4414)
(8.5)	**BNIP**	8.8058	0.6266	(7.5727, 10.0390)	(7.1814, 10.4303)
	BIP	10.0000	0.0007	(9.9985, 10.0015)	(9.9981, 10.0019)
θ_2	**LB**	5.0298	0.8404	(3.3759, 6.6837)	(2.8577, 7.2085)
(5.0)	**BNIP**	5.0298	0.8348	(3.3871, 6.6725)	(4.3634, 6.8187)
	BIP	4.6994	0.3043	(4.1006, 5.2981)	(3.9106, 5.4881)
θ_3	**LB**	1.1155	0.8998	(-0.6553, 2.8863)	(-7.2173, 3.4482)
(2.0)	**BNIP**	1.1155	0.8938	(-0.6434, 2.8743)	(-1.2015, 3.4324)
	BIP	-0.0930	0.3860	(-0.8525, 0.6665)	(-1.0935, 0.9074)

Table 5.58: High Positive Collinearity, ρ = 0.80 and sample size, N=300

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.5598	0.1744	(16.2166, 16.9030)	(16.1077, 17.0120)
(17)	**BNIP**	16.5598	0.1732	(16.2189, 16.9007)	(16.1107, 17.0089)
	BIP	16.6580	0.1482	(16.3663, 16.9497)	(16.2737, 17.0422)
θ_1	**LB**	7.8726	0.4130	(7.0599, 8.6854)	(6.8020, 8.9433)
(8.5)	**BNIP**	7.8726	0.4102	(7.0654, 8.6799)	(6.8092, 8.9360)
	BIP	10.0000	0.0008	(9.9984, 10.0015)	(9.9980, 10.0020)
θ_2	**LB**	5.5473	0.6380	(4.2917, 6.8028)	(3.8933, 7.2013)
(5.0)	**BNIP**	5.5473	0.6337	(4.3002, 6.7944)	(3.9045, 7.1901)
	BIP	4.6711	0.3068	(4.0674, 5.2748)	(3.8758, 5.4663)
θ_3	**LB**	3.5240	0.6512	(2.2425, 4.8056)	(1.8358, 5.2123)
(2.0)	**BNIP**	3.5240	0.6468	(2.2511, 4.7969)	(1.8472, 5.2008)
	BIP	1.1061	0.3952	(0.3285, 1.8838)	(0.0818, 2.1305)

Table 5.59: Moderate Positive Collinearity, ρ = 0.49 and sample size, N=300.

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.1785	0.1854	(16.8136, 17.6978)	(6.6978, 17.6591)
(17)	**BNIP**	17.1785	0.1841	(16.8161, 17.5408)	(16.7011, 17.6558)
	BIP	16.9271	0.1613	(16.6098, 17.2445)	(16.5091, 17.3452)
θ_1	**LB**	8.6202	0.3067	(8.0165, 9.2237)	(7.8250, 9.4159)
(8.5)	**BNIP**	8.6202	0.3046	(8.0206, 9.2196)	(7.8304, 9.4098)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
θ_2	**LB**	4.4730	0.5210	(3.478, 5.4982)	(3.1224, 5.8236)
(5.0)	**BNIP**	4.4730	0.5175	(3.4547, 5.4913)	(3.1316, 5.8144)
	BIP	4.4304	0.3065	(3.8273, 5.0335)	(3.6360, 5.2248)
θ_3	**LB**	1.7868	0.5025	(0.7979, 2.7757)	(0.4841, 3.0895)
(2.0)	**BNIP**	1.7868	0.4991	(0.8046, 2.7690)	(0.4929, 3.0807)
	BIP	0.6372	0.3784	(-0.1074, 1.3818)	(-0.3436, 1.6180)

Table 5.60: Moderate Positive Collinearity, ρ = 0.46 and sample size, N=300

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.9032	0.1732	(16.5624, 17.2439)	(16.4542, 17.3521)
(17)	**BNIP**	16.9032	0.1720	(16.5647, 17.2416)	(16.4573, 17.3490)
	BIP	16.6155	0.1610	(16.2987, 16.9322)	(16.1982, 17.0327)
θ_1	**LB**	8.3735	0.2451	(7.8912, 8.8559)	(7.7381, 9.0090)
(8.5)	**BNIP**	8.3735	0.2435	(7.8944, 8.8526)	(7.7424, 9.0046)
	BIP	10.0000	0.0008	(9.9983, 10.0016)	(9.9978, 10.0022)
θ_2	**LB**	5.3975	0.4473	(4.5173, 6.2778)	(4.2380, 6.5571)
(5.0)	**BNIP**	5.3975	0.4443	(4.5232, 6.2718)	(4.2458, 6.5492)
	BIP	4.8655	0.3048	(4.2657, 5.4652)	(4.0755, 5.6555)
θ_3	**LB**	1.9901	0.4347	(1.1347, 2.8456)	(0.8633, 3.1170)
(2.0)	**BNIP**	1.9901	0.4318	(1.1405, 2.8398)	(0.8709, 3.1094)
	BIP	1.0248	0.3743	(0.2883, 1.7613)	(0.0546, 1.9950)

Table 5.61: Moderate Positive Collinearity, ρ = 0.36 and sample size, N=300

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.3307	0.1859	(16.9648, 17.6966)	(16.8487, 17.8127)
(17)	**BNIP**	17.3307	0.1847	(16.9672, 17.6941)	(16.8519, 17.8094)
	BIP	16.7763	0.1651	(16.4515, 17.1012)	(16.3484, 17.2042)
θ_1	**LB**	8.5606	0.2340	(8.1001, 9.0211)	(7.9539, 9.1673)
(8.5)	**BNIP**	8.5606	0.2324	(8.1032, 9.0180)	(7.9580, 9.1631)
	BIP	10.0000	0.0008	(9.9983, 10.0016)	(9.9978, 10.0021)
θ_2	**LB**	5.0745	0.4316	(4.2251, 5.9239)	(3.9556, 6.1935)
(5.0)	**BNIP**	5.0745	0.4287	(4.2309, 5.9182)	(3.9632, 6.1859)
	BIP	4.8544	0.3017	(4.2606, 5.4482)	(4.0722, 5.6365)
θ_3	**LB**	0.7574	0.4382	(-0.1050, 1.6197)	(-0.3786, 1.8933)
(2.0)	**BNIP**	0.7574	0.4352	(-0.0991, 1.6139)	(-0.3709, 1.8856)
	BIP	0.4090	0.3911	(-0.3605, 1.1785)	(-0.6047, 1.4226)

Table 5.62: Low Positive Collinearity, ρ = 0.20 and sample size, N=300

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.1668	0.1845	(16.8037, 17.5300)	(16.6885, 17.6452)
(17)	**BNIP**	17.1668	0.1833	(16.8061, 17.5275)	(16.6917, 17.6420)
	BIP	16.4520	0.1560	(16.1449, 16.7591)	(16.0475, 16.8565)
θ_1	**LB**	8.4631	0.2092	(8.0514, 8.8749)	(7.9207, 9.0056)
(8.5)	**BNIP**	8.4631	0.2078	(8.0541, 8.8721)	(7.9244, 9.0019)
	BIP	10.0000	0.0008	(9.9983, 10.0016)	(9.9978, 10.0022)
θ_2	**LB**	5.0539	0.4143	(4.2386, 5.8692)	(3.9800, 6.1279)
(5.0)	**BNIP**	5.0539	0.4115	(4.2441, 5.8637)	(3.9872, 6.1206)
	BIP	4.9868	0.3050	(4.3866, 5.5870)	(4.1962, 5.7774)
θ_3	**LB**	1.2353	0.4210	(0.4067, 2.0638)	(0.1438, 2.3267)
(2.0)	**BNIP**	1.2353	0.4182	(0.4123, 2.0582)	(0.1512, 2.3193)
	BIP	1.1284	0.3981	(0.3450, 1.9117)	(0.0965, 2.1603)

Table 5.63: Low Positive Collinearity, ρ = 0.17 and sample size, N=300

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.9019	0.1812	(16.5452, 17.2585)	(16.4320, 17.3717)
(17)	**BNIP**	16.9019	0.1800	(16.5476, 17.2561)	(16.4352, 17.3685)
	BIP	16.2381	0.1491	(15.9447, 16.5314)	(15.8517, 16.6245)
θ_1	**LB**	8.5005	0.2115	(8.0842, 8.9168)	(7.95211, 9.0489)
(8.5)	**BNIP**	8.5005	0.2101	(8.0870, 8.9140)	(7.9558, 9.0452)
	BIP	10.0000	0.0008	(9.9983, 10.0016)	(9.9978, 10.0021)
θ_2	**LB**	5.3246	0.4126	(4.5126, 6.1365)	(4.2549, 6.3942)
(5.0)	**BNIP**	5.3246	0.4098	(4.5181, 6.1311)	(4.2622, 6.3870)
	BIP	5.2013	0.3008	(4.6094, 5.7932)	(4.4216, 5.9810)
θ_3	**LB**	1.8227	0.4079	(1.0199, 2.6254)	(0.7652, 2.8802)
(2.0)	**BNIP**	1.8227	0.4052	(1.0253, 2.6200)	(0.7723, 2.8730)
	BIP	1.6928	0.3844	(0.9363, 2.4492)	(0.6963, 2.6892)

Table 5.64: Low Positive Collinearity, ρ = 0.15 and sample size, N=300

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.2024	0.1776	(16.8530, 17.5519)	(16.7421, 17.6628)
(17)	**BNIP**	17.2024	0.1764	(16.8553, 17.5496)	(16.7452, 17.6597)
	BIP	16.4611	0.1516	(16.1627, 16.7595)	(16.0681, 16.8542)
θ_1	**LB**	8.4046	0.2005	(8.0100, 8.7992)	(7.8848, 8.9244)
(8.5)	**BNIP**	8.4046	0.1991	(8.0127, 8.7965)	(7.8883, 8.9208)
	BIP	10.0000	0.0008	(9.9983, 10.0016)	(9.9978, 10.0021)
θ_2	**LB**	4.7788	0.3860	(4.0191, 5.5385)	(3.7780, 5.7796)
(5.0)	**BNIP**	4.7788	0.3834	(4.0242, 5.5334)	(3.7848, 5.7728)
	BIP	4.9459	0.2947	(4.3660, 5.5259)	(4.1820, 5.7098)
θ_3	**LB**	2.0009	0.3968	(1.2200, 2.7818)	(0.9721, 3.0297)
(2.0)	**BNIP**	2.0009	0.3942	(1.2252, 2.7766)	(0.9791, 3.0227)
	BIP	1.6626	0.3808	(0.9132, 2.4119)	(0.6754, 2.6497)

From Tables 5.45-5.64, the following observations were made when the sample sizes are 200 and 300:

The mean estimates of all the estimators (LB, BNIP and BIP) are not too far from the true parameter values. Bayesian estimators (BNIP and BIP) have smaller SE than LB. HPC and MPC have the highest values of SE for sample sizes 200 and 300 than LPC.

The confidence intervals of LB are wider than the Bayesian estimators (BIP and BNIP) for HPC, MPC and LPC at 95% and 99% CI. For instance, at 95% CI, CI of LB, BNIP and BIP when the sample size is 200 for parameter θ_2 are (4.6506, 6.5315), (4.6602, 6.5219) and (4.6542, 5.9115), respectively in LPC (ρ =0.20).

BIP has the smallest SE values for all the parameters considered. The performances of all the estimators become better due to increase in sample sizes compared to sample sizes of 10, 30, 70 and 100; for instance, when the sample size is 70, the SE of LB, BNIP and BIP are 2.1729, 2.1099 and 0.3843, respectively while the SE of LB, BNIP and BIP when the sample size is 300 are 0.5404, 0.8348 and 0.3043 for parameter θ_2 in HPC (ρ =0.90).

Hence, increase in sample sizes has a great effect in reducing SE.

Table 5.65: Summary of Tables 5.56 -5.64 for Standard Error when the sample size, N= 300.

Parameters	Estimators	0.95	0.90	0.80	0.49	0.46	0.36	0.20	0.17	0.15
θ_0	**LB**	0.1720	0.1572	0.1744	0.1854	0.1732	0.1859	0.1845	0.1812	0.1776
	BNIP	0.1708	0.1561	0.1732	0.1841	0.1720	0.1847	0.1833	0.1800	0.1764
	BIP	0.1297	0.1247	0.1482	0.1613	0.1610	0.1651	0.1560	0.1491	0.1516
θ_1	**LB**	0.7995	0.6309	0.4130	0.3067	0.2451	0.2340	0.2092	0.2115	0.2005
	BNIP	0.7941	0.6266	0.4102	0.3046	0.2435	0.2324	0.2078	0.2101	0.1991
	BIP	0.0008	0.0007	0.0008	0.0008	0.0008	0.0008	0.0008	0.0008	0.0008
θ_2	**LB**	1.1662	0.8404	0.6380	0.5210	0.4473	0.4316	0.4143	0.4126	0.3860
	BNIP	1.1584	0.8348	0.6337	0.5175	0.4443	0.4287	0.4115	0.4098	0.3834
	BIP	0.3327	0.3043	0.3068	0.3065	0.3048	0.3017	0.3050	0.3008	0.2947
θ_3	**LB**	1.1761	0.8998	0.6512	0.5025	0.4347	0.4382	0.4210	0.4079	0.3960
	BNIP	1.1682	0.8938	0.6468	0.4991	0.4318	0.4352	0.4182	0.4052	0.3942
	BIP	0.4245	0.3860	0.3952	0.3784	0.3743	0.3911	0.4981	0.3844	0.3808

Table 5.65 shows the summary of SE for multicollinearity (HPC, MPC and LPC) of the estimators across the parameters (θ_0, θ_1, θ_2 and θ_3) when the sample size is 300.

As ρ decreases, SE of the estimators also decreases for parameters θ_1, θ_2 and θ_3. Large SE is also observed in high and moderate positive collinearity. The SE of the estimators are smaller compared to when the sample sizes are 10, 30, 70, 100 and 200.

Table 5.66: Summary of Tables 5.56-5.64 for Mean for sample size, N= 300.

Parameters	Estimators	0.95	0.90	0.80	0.49	0.46	0.36	0.20	0.17	0.15
θ_0	**LB**	16.6654	17.1138	16.5598	17.1785	16.9032	16.3307	17.1668	16.9019	17.2024
(17.00)	**BNIP**	16.6654	17.1138	16.5598	17.1785	16.9032	16.3307	17.1668	16.9019	17.2024
	BIP	16.8260	17.0194	16.6580	16.9271	16.6155	16.7763	16.4520	16.2381	16.4611
θ_1	**LB**	6.9703	8.8058	7.8726	8.6202	8.3735	8.5606	8.4631	8.5005	8.4046
(8.5)	**BNIP**	6.9703	8.8058	7.8726	8.6202	8.3735	8.5606	8.4631	8.5005	8.4046
	BIP	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000
θ_2	**LB**	5.4153	0.8404	5.4873	4.4730	5.3975	5.0745	5.0539	5.3246	4.7788
(5.00)	**BNIP**	5.4153	0.8404	5.4873	4.4304	5.3975	5.0745	5.0539	5.3246	4.7788
	BIP	4.6317	0.3043	4.6711	4.4304	4.8655	4.8544	4.9868	5.2013	4.9459
θ_3	**LB**	4.9680	0.8998	3.5240	1.7868	1.9901	0.7574	1.2353	1.8227	2.0009
(2.00)	**BNIP**	4.9680	0.3860	3.5240	1.7868	1.9901	0.7574	1.2353	1.8227	2.0009
	BIP	0.4720	0.3860	1.1061	0.6372	1.0248	0.4090	1.1284	1.6928	1.6626

Table 5.66 shows the summary of mean for Tables 5-56-5.64 when the sample size is 300. The means of LB and BNIP are the same for all the levels of collinearity across the parameters. All the means of the estimator are positive. The means of parameter θ_0 are not too far from the true parameter. BIP outperformed all other estimators having the closest mean values to the true parameter value.

Table 5.67: High Negative Collinearity, ρ = -0.95 and sample size, N=10

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	15.8201	0.8105	(13.8370, 17.8032)	(12.8154, 18.8248)
(17)	**BNIP**	15.8201	0.6278	(14.4213, 17.2189)	(13.8305, 17.8097)
	BIP	16.7516	0.3626	(15.9738, 17.5293)	(15.6721, 17.8310)
θ_1	**LB**	18.7079	4.0834	(8.7161, 28.6997)	(3.5688, 33.8469)
(8.5)	**BNIP**	18.7079	3.1630	(11.6602, 25.7555)	(8.6834, 28.7323)
	BIP	10.0000	0.0009	(9.9982, 10.0018)	(9.9974, 10.0026)
θ_2	**LB**	18.0901	5.9046	(3.6420, 32.5382)	(-3.8008, 39.9810)
(5.0)	**BNIP**	18.0901	4.5737	(7.8993, 28.2809)	(3.5948, 32.5854)
	BIP	5.6452	0.4236	(4.7366, 6.5537)	(4.3841, 6.9062)
θ_3	**LB**	12.6807	7.0814	(-4.6470, 30.0083)	(13.5732, 38.9346)
(2.0)	**BNIP**	12.6807	5.4853	(0.4588, 24.9026)	(-4.7036, 30.0649)
	BIP	2.9897	0.8118	(1.2486, 4.7309)	(0.5731, 5.4063)

Table 5.68: High Negative Collinearity, ρ = -0.90 and sample size, N=10

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	15.8212	0.5321	(14.5191, 17.1234)	(13.8483, 17.7942)
(17)	**BNIP**	15.8212	0.4122	(14.9028, 16.7397)	(14.5149, 17.1276)
	BIP	16.2791	0.2635	(15.7141, 16.8442)	(15.4948, 17.0634)
θ_1	**LB**	13.1034	1.9080	(8.4346, 17.7721)	(6.0295, 20.1772)
(8.5)	**BNIP**	13.1034	1.4779	(9.8103, 16.3964)	(8.4194, 17.7873)
	BIP	10.0000	0.0006	(9.9986, 10.0014)	(9.9981, 10.0019)
θ_2	**LB**	12.4186	2.9994	(5.0794, 19.7579)	(1.2986, 23.5387)
(5.0)	**BNIP**	12.4186	2.3233	(7.2420, 17.5953)	(5.0554, 19.7819)
	BIP	5.5918	0.3102	(4.9264, 6.2572)	(4.6682, 6.5153)
θ_3	**LB**	4.4901	3.3105	(-3.6105, 12.5906)	(-7.7834, 16.7636)
(2.0)	**BNIP**	4.4901	2.5643	(-1.2236, 10.2037)	(-3.6369, 12.6171)
	BIP	2.7271	0.5860	(1.4702, 3.9839)	(0.9827, 4.4715)

Table 5.69: High Negative Collinearity, ρ = -0.80 and sample size, N=10

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.6465	1.5018	(12.9717, 20.3212)	(11.0787,22.2143)
(17)	**BNIP**	16.6465	1.1633	(14.0545, 19.2384)	(12.9597, 20.3332)
	BIP	15.7885	0.2817	(15.1844, 16.3927)	(14.9500, 16.6271)
θ_1	**LB**	7.9561	4.1433	(-2.1821, 18.0944)	(-7.4048, 23.3171)
(8.5)	**BNIP**	7.9561	3.2094	(0.8052, 15.1071)	(-2.2153, 18.1275)
	BIP	10.0000	0.0007	(9.9985, 10.0015)	(9.9980, 10.0020)
θ_2	**LB**	3.8754	4.2386	(-6.4962, 14.2469)	(-11.8391, 19.5898)
(5.0)	**BNIP**	3.8754	3.2832	(-3.4401, 11.1909)	(-6.5301, 14.2808)
	BIP	5.5119	0.3346	(4.7943, 6.2296)	(4.5158, 6.5080)
θ_3	**LB**	1.0505	6.5888	(-15.0716, 17.1726)	(-23.3769, 25.4779)
(2.0)	**BNIP**	1.0505	5.1036	(-10.3211, 12.4221)	(-15.1243, 17.225)
	BIP	2.5869	0.6501	(1.1927, 3.9812)	(0.6518, 4.5221)

Table 5.70: Moderate Negative Collinearity, ρ = -0.49 and sample size, N=10

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	18.7485	0.5727	(17.3471, 20.1499)	(16.6252, 20.8718)
(17)	**BNIP**	18.7485	0.4436	(17.7600, 19.7369)	(17.3425, 20.1544)
	BIP	15.6463	0.2887	(15.0271, 16.2656)	(14.7868, 16.5058)
θ_1	**LB**	6.1585	0.7693	(4.2761, 8.0409)	(3.30637, 9.0106)
(8.5)	**BNIP**	6.1585	0.5959	(4.8307, 7.4862)	(4.2699, 8.0470)
	BIP	10.0000	0.0007	(9.9985, 10.0015)	(9.9979, 10.0021)
θ_2	**LB**	1.0879	1.4513	(-2.4633, 4.6391)	(-4.2927, 6.4685)
(5.0)	**BNIP**	1.0879	1.1242	(-1.4169, 3.5927)	(-2.4749, 4.6507)
	BIP	5.4831	0.3475	(4.7378, 6.2283)	(4.4487, 6.5174)
θ_3	**LB**	-4.8820	1.7412	(-9.1426, -0.6214)	(-11.3374,1.5735)
(2.0)	**BNIP**	-4.8820	1.3487	(-7.8872, -1.8768)	(-9.1565, -0.6074)
	BIP	2.2833	0.6866	(0.8108, 3.7558)	(0.2395, 4.3271)

Table 5.71: Moderate Negative Collinearity, ρ = - 0.46 and sample size, N=10

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.5479	1.0223	(14.0464, 19.0494)	(12.7578, 20.3380)
(17)	**BNIP**	16.5479	0.7919	(14.7835, 18.3123)	(14.0383, 19.0575)
	BIP	15.4561	0.2867	(14.8413, 16.0710)	(14.6028, 16.3095)
θ_1	**LB**	8.3739	1.3498	(5.0710, 11.6769)	(3.3695, 13.3784)
(8.5)	**BNIP**	8.3739	1.0456	(6.0442, 10.7036)	(5.0602, 11.6876)
	BIP	10.0000	0.0007	(9.9985, 10.0015)	(9.9979, 10.0021)
θ_2	**LB**	4.9129	3.5398	(-3.7485, 13.5744)	(-8.2105, 18.0363)
(5.0)	**BNIP**	4.9129	2.7419	(-1.1964, 11.0222)	(-3.7768, 13.6027)
	BIP	5.5189	0.3498	(4.7687, 6.2691)	(4.4777, 6.5601)
θ_3	**LB**	2.6512	2.7550	(-4.0901, 9.3925)	(-7.5628, 12.8653)
(2.0)	**BNIP**	2.6512	2.1340	(-2.1037, 7.4062)	(-4.1121, 9.4146)
	BIP	2.6077	0.6693	(1.1722, 4.0431)	(0.6153, 4.6000)

Table 5.72: Moderate Negative Collinearity, ρ = - 0.36 and sample size, N=10

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	14.4522	1.5200	(10.7329, 18.1715)	(8.81693, 20.0874)
(17)	**BNIP**	14.4522	1.1774	(11.8288, 17.0755)	(10.7207, 18.1836)
	BIP	15.4811	0.3053	(14.8262, 16.1360)	(14.5721, 16.3901)
θ_1	**LB**	10.8809	1.3927	(7.4730, 14.2888)	(5.71738, 16.0444)
(8.5)	**BNIP**	10.8809	1.0788	(8.4771, 13.2846)	(7.4618, 14.2999)
	BIP	10.0000	0.0007	(9.9985, 10.0015)	(9.9979, 10.0021)
θ_2	**LB**	6.7326	2.8012	(-0.1217, 13.5869)	(-3.6526, 17.1178)
(5.0)	**BNIP**	6.7326	2.1698	(1.8980, 11.5672)	(-0.1440, 13.6093)
	BIP	5.4971	0.3399	(4.7681, 6.2260)	(4.4853, 6.5088)
θ_3	**LB**	4.6547	2.9370	(-2.5318, 11.8412)	(-6.2339, 15.5433)
(2.0)	**BNIP**	4.6547	2.2750	(-4.1677, 5.0494)	(-2.5553, 11.8647)
	BIP	2.6607	0.6456	(1.2761, 4.0452)	(0.7390, 4.5824)

Table 5.73: Low Negative Collinearity, ρ = -0.20 and sample size, N=10

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.1853	1.0335	(13.6563, 18.7143)	(12.3535, 20.0171)
(17)	**BNIP**	16.1853	0.8006	(14.4015, 17.9691)	(13.6480, 18.7225)
	BIP	16.0090	0.3060	(15.3528, 16.6652)	(15.0982, 16.9198)
θ_1	**LB**	10.4408	1.3498	(7.4931, 13.3885)	(5.9747, 14.9070)
(8.5)	**BNIP**	10.4408	0.9331	(8.3617, 12.5199)	(7.4835, 13.3981)
	BIP	10.0000	0.0007	(9.9986, 10.0014)	(9.9980, 10.0020)
θ_2	**LB**	5.8531	2.1701	(0.5430, 11.1632)	(-2.1925, 13.8987)
(5.0)	**BNIP**	5.8531	1.6810	(2.1077, 9.5986)	(0.5256, 11.1806)
	BIP	5.5370	0.3285	(4.8324, 6.2416)	(4.5591, 6.5149)
θ_3	**LB**	0.4409	2.6702	(-6.0929, 6.97460)	(-9.4587, 10.3404)
(2.0)	**BNIP**	0.4409	2.0683	(-4.1677, 5.0494)	(-6.1142, 6.9959)
	BIP	2.3645	0.6364	(0.9995, 3.7295)	(0.4699, 4.2590)

Table 5.74: Low Negative Collinearity, ρ = -0.17 and sample size, N=10

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.6981	1.8274	(13.2266,22.1697)	(10.9231, 24.4732)
(17)	**BNIP**	17.6981	1.4155	(14.5442, 20.8521)	(13.2120, 22.1843)
	BIP	16.5199	0.4727	(15.5061, 17.5336)	(15.1129, 17.9269)
θ_1	**LB**	8.7610	2.4826	(2.6862, 14.8357)	(-0.4432, 17.9651)
(8.5)	**BNIP**	8.7610	0.9331	(4.4762, 13.0457)	(2.6663, 14.8556)
	BIP	10.0000	0.0010	(9.9978, 10.0022)	(9.9969, 10.0031)
θ_2	**LB**	8.0309	3.5764	(-0.7203, 16.7821)	(-5.2284, 21.2902)
(5.0)	**BNIP**	8.0309	1.6810	(1.8583, 14.2035)	(-0.7489, 16.8107)
	BIP	5.6040	0.5079	(4.5146, 6.6935)	(4.0920, 7.1161)
θ_3	**LB**	-1.2739	4.5380	(-12.378, 9.8301)	(-18.0982,15.5503)
(2.0)	**BNIP**	-1.2739	2.0683	(-9.1061, 6.5582)	(-12.4143, 9.8664)
	BIP	2.2854	0.9877	(0.1670, 4.4037)	(-0.6548, 5.2256)

Table 5.75: Low Negative Collinearity, ρ = -0.15 and sample size, N=10

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.8145	0.7140	(16.0674,19.5616)	(15.1674, 20.4616)
(17)	**BNIP**	17.8145	0.5531	(16.5822, 19.0468)	(16.0617, 19.5673)
	BIP	16.5954	0.3293	(15.8891, 17.3017)	(15.6151, 17.5757)
θ_1	**LB**	7.4041	1.4158	(3.9399, 10.8684)	(2.1553, 12.6529)
(8.5)	**BNIP**	7.4041	1.0966	(4.9607, 9.8476)	(3.9286, 10.8797)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9977, 10.0023)
θ_2	**LB**	4.0483	2.4788	(-2.0171, 10.1136)	(-5.1416, 13.2382)
(5.0)	**BNIP**	4.0483	1.9201	(-0.2299, 8.3264)	(-2.0369, 10.1334)
	BIP	5.5025	0.3779	(4.6920, 6.3130)	(4.3776, 6.6275)
θ_3	**LB**	3.3376	2.5347	(-2.8646, 9.5398)	(-6.0597, 12.7348)
(2.0)	**BNIP**	3.3376	1.9634	(-1.0371, 7.7122)	(-2.8849, 9.5600)
	BIP	2.3346	0.7145	(0.8022, 3.8669)	(0.2077, 4.4614)

Table 5.76: Summary of Tables 5.67-5.75 for Standard Error when the sample size, N= 10.

Parameters	Estimators	-0.95	-0.90	-0.80	-0.49	-0.46	-0.36	-0.20	-0.17	-0.15
θ_0	**LB**	0.8105	0.5321	1.5018	0.5727	1.0223	1.5200	1.0335	1.8274	0.7140
	BNIP	0.6278	0.4122	1.1633	0.4436	0.7919	1.1774	0.8006	1.4155	0.5531
	BIP	0.3626	0.2635	0.2817	0.2887	0.2867	0.3053	0.3060	0.4727	0.3293
θ_1	**LB**	4.0834	1.9080	4.1433	0.7693	1.3498	1.3927	1.3498	2.4826	1.4158
	BNIP	3.1630	1.4779	3.2094	0.5959	1.0456	1.0788	0.9331	0.9331	1.0966
	BIP	0.0009	0.0006	0.0007	0.0007	0.0007	0.0007	0.0007	0.0010	0.0008
θ_2	**LB**	5.9046	2.9994	4.2386	1.4513	3.5398	2.8012	2.1701	3.5764	2.4788
	BNIP	4.5737	2.3233	3.2832	1.1242	2.7419	2.1698	1.6810	1.6810	1.9201
	BIP	0.4236	0.3102	0.3346	0.3475	0.3498	0.3399	0.3285	0.5079	0.3779
θ_3	**LB**	7.0814	3.3105	6.5888	1.7412	2.7550	2.9370	2.6702	4.5380	2.5347
	BNIP	5.4853	2.5643	5.1036	1.3487	2.1340	2.2750	2.0683	2.0683	1.9634
	BIP	0.8118	0.5860	0.6501	0.6866	0.6693	0.6456	0.6364	0.9877	0.7145

Table 5.76 shows the summary of SE for multicollinearity (HNC, MNC and LNC) of the estimators across the parameters (θ_0, θ_1, θ_2 and θ_3) when the sample size is 10. There seems to be no fixed pattern in the performance of the estimators for the levels of multicollinearity ($\rho =$ -0.15 to -0.95). It is also observed that as ρ increases, the SE of estimators also decreases for all the parameters.

The Bayesian estimators (BIP and BNIP) have the smallest SE for all the levels of multicollinearity considered. When the sample size, N=10. The SE of BIP for parameter θ_1 for ρ's are almost the same.

Table 5.77: Summary of Tables 5.67-5.75 for Mean when the sample size, N= 10.

Parameters	Estimators	-0.95	-0.90	-0.80	-0.49	-0.46	-0.36	-0.20	-0.17	-0.15
θ_0 (17.00)	**LB**	15.8201	15.8212	16.6465	18.7485	16.5479	14.4522	16.1853	17.6981	16.8145
	BNIP	15.8201	16.2791	16.6465	18.7485	16.5479	14.4522	16.1853	17.6981	16.8145
	BIP	16.7516	16.2791	15.7885	15.6463	15.4561	15.4811	16.0090	16.5199	16.5954
θ_1 (8.5)	**LB**	18.7079	13.1034	7.9561	6.1585	8.3739	10.8809	10.4408	8.7610	7.4041
	BNIP	18.7079	13.1034	7.9561	6.1585	8.3739	10.8809	10.4408	8.7610	7.4041
	BIP	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000
θ_2 (5.00)	**LB**	18.0901	12.4186	3.8754	1.0879	4.9129	6.7326	5.8531	8.0309	4.0483
	BNIP	18.0901	12.4186	3.8754	1.0879	4.9129	6.7326	5.8531	8.0309	4.0483
	BIP	5.6452	5.5918	5.5119	5.4831	5.5189	5.4971	5.5370	5.6040	5.5025
θ_3 (2.00)	**LB**	12.6807	4.4901	1.0505	-4.8820	2.6512	4.6547	0.4409	-1.2739	3.3376
	BNIP	12.6807	4.4901	1.0505	-4.8820	2.6512	4.6547	0.4409	-1.2739	3.3376
	BIP	2.9897	2.7271	2.5869	2.2833	2.6077	2.6607	2.3645	2.2854	2.3346

Table 5.77 summarizes the mean estimates of all the estimators for parameters (θ_0, θ_1, θ_2 and θ_3), when the sample size is 10.

The mean of LB and BNIP are the same for all levels of multicollinearity across the parameters. However, there is evidence to suggest that BIP is the best for estimating parameters of the regression model because the means are closer to the true parameter value for all the levels of multicollinearity.

None of the estimators generated negative average estimates and none generated large positive estimates. The average estimates have showed no consistent pattern for all levels of multicollinearity across the parameters when the sample size is 10.

Table 5.78: High Negative Collinearity, ρ = -0.95 and sample size, N=30

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.3394	0.5739	(16.1598,18.5190)	(15.7448, 18.9340)
(17)	**BNIP**	17.3394	0.5342	(16.2484, 18.4305)	(15.8703, 18.8086)
	BIP	16.4476	0.2264	(15.9874, 16.9077)	(15.8298, 17.0653)
θ_1	**LB**	5.7488	3.0300	(-0.4794, 11.9770)	(-2.6707, 14.1683)
(8.5)	**BNIP**	5.7488	2.8208	(-0.0120, 11.5096)	(-2.0083, 13.5059)
	BIP	10.0000	0.0009	(9.9982, 10.0018)	(9.9976, 10.0024)
θ_2	**LB**	5.2686	4.4026	(-3.7810, 14.3182)	(-6.9648, 17.5021)
(5.0)	**BNIP**	5.2686	4.0986	(-3.1018, 13.6390)	(-6.0024, 16.5396)
	BIP	5.6483	0.4237	(4.7872, 6.5094)	(4.4923, 6.8044)
θ_3	**LB**	-3.8396	4.6462	(-13.39, 5.7108)	(-16.7501, 9.0710)
(2.0)	**BNIP**	-3.8396	4.3254	(-12.6732, 4.9940)	(-15.7344, 8.0552)
	BIP	2.8783	0.7496	(1.3549, 4.4016)	(0.8331, 4.9235)

Table 5.79: High Negative Collinearity, ρ = -0.90 and sample size, N=30

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.8113	0.5450	(15.6910,17.931)	(15.2969, 18.3257)
(17)	**BNIP**	16.8113	0.5074	(15.7751, 17.8475)	(15.4160, 18.2066)
	BIP	15.8980	0.1775	(15.5372, 16.2588)	(15.4136, 16.3824)
θ_1	**LB**	6.9570	1.8625	(3.1287, 10.7854)	(1.7818, 12.1323)
(8.5)	**BNIP**	6.9570	1.7339	(3.4160, 10.4980)	(2.1889, 11.7251)
	BIP	10.0000	0.0007	(9.9986, 10.0014)	(9.9981, 10.0019)
θ_2	**LB**	2.2208	2.5703	(-3.0626, 7.5042)	(-4.9214, 9.3630)
(5.0)	**BNIP**	2.2208	2.3928	(-2.6660, 7.1077)	(-4.3595, 8.8011)
	BIP	5.5552	0.3327	(4.8790, 6.2314)	(4.6474, 6.4631)
θ_3	**LB**	0.9391	2.6238	(-4.4541, 6.3324)	(-6.3516, 8.2298)
(2.0)	**BNIP**	0.9391	2.4426	(-4.0493, 5.9276)	(-5.7780, 7.6563)
	BIP	2.8989	0.5749	(1.7306, 4.0673)	(1.3304, 4.4675)

Table 5.80: High Negative Collinearity, ρ = -0.80 and sample size, N=30

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.3352	0.6604	(14.9778,17.69257)	(14.5002, 18.1701)
(17)	**BNIP**	16.3352	0.6148	(15.0797, 17.5907)	(14.6446, 18.0258)
	BIP	16.4904	0.1976	(16.0890, 16.8919)	(15.9514, 17.0294)
θ_1	**LB**	10.6496	1.6237	(7.3121, 13.9871)	(6.1378, 15.1613)
(8.5)	**BNIP**	10.6496	1.5116	(7.5625, 13.7366)	(6.4928, 14.8064)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9978, 10.0022)
θ_2	**LB**	7.3045	2.4292	(2.3111, 12.2979)	(0.5543, 14.0547)
(5.0)	**BNIP**	7.3045	2.2615	(2.6859, 11.9231)	(1.0853, 13.5236)
	BIP	5.5755	0.3884	(4.7862, 6.3647)	(4.5159, 6.6351)
θ_3	**LB**	3.1868	2.6238	(-2.0559, 8.4295)	(-3.9004, 10.274)
(2.0)	**BNIP**	3.1868	2.3744	(-1.6624, 8.0360)	(-3.3428, 9.7164)
	BIP	2.6335	0.6863	(1.2388, 4.0281)	(0.7611, 4.5059)

Table 5.81: Moderate Negative Collinearity, ρ = -0.49 and sample size, N=30

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	15.7047	0.7552	(14.1525, 17.2570)	(13.6064, 17.8031)
(17)	**BNIP**	15.7047	0.7030	(14.2690, 17.1405)	(13.7714, 17.6380)
	BIP	16.1452	0.2092	(15.7200, 16.5703)	(15.5743, 16.7160)
θ_1	**LB**	10.3071	1.1342	(7.9757, 12.6386)	(7.1554, 13.4589)
(8.5)	**BNIP**	10.3071	1.0559	(8.1507, 12.4636)	(7.4034, 13.2109)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
θ_2	**LB**	6.2458	1.6286	(2.8981, 9.5934)	(1.7203, 10.7712)
(5.0)	**BNIP**	6.2458	1.5162	(3.1494, 9.3422)	(2.0764, 10.4152)
	BIP	5.5441	0.3672	(4.7977, 6.2904)	(4.5421, 6.5461)
θ_3	**LB**	4.7978	1.6644	(1.3766, 8.2191)	(0.1729, 9.4228)
(2.0)	**BNIP**	4.7978	1.5495	(1.6334, 7.9623)	(0.5368, 9.0589)
	BIP	3.0733	0.6555	(1.7413, 4.4054)	(1.2850, 4.8617)

Table 5.82: Moderate Negative Collinearity, ρ = -0.46 and sample size, N=30

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.2145	0.6256	(15.9286, 18.5003)	(15.4762, 18.9527)
(17)	**BNIP**	17.2145	0.5824	(16.0251, 18.4038)	(15.6130, 18.8159)
	BIP	16.2226	0.2284	(15.7584, 16.6867)	(15.5994, 16.8457)
θ_1	**LB**	8.1671	0.7831	(6.5574, 9.7768)	(5.9910, 10.3431)
(8.5)	**BNIP**	8.1671	0.7290	(6.6782, 9.6560)	(6.1622, 10.1719)
	BIP	10.0000	0.0009	(9.9982, 10.0018)	(9.9976, 10.0024)
θ_2	**LB**	5.7829	1.7119	(2.2641, 9.3017)	(1.026, 10.5397)
(5.0)	**BNIP**	5.7829	1.5937	(2.5282, 9.0376)	(1.4003, 10.1654)
	BIP	5.6399	0.4166	(4.7933, 6.4865)	(4.5033, 6.7765)
θ_3	**LB**	0.6887	1.5586	(-2.5150, 3.8924)	(-3.6421, 5.0195)
(2.0)	**BNIP**	0.6887	1.4509	(-2.2746, 3.6519)	(-3.3014, 4.6788)
	BIP	2.6467	0.7243	(1.1747, 4.1187)	(0.6705, 4.6230)

Table 5.83: Moderate Negative Collinearity, ρ = -0.36 and sample size, N=30

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.3045	0.7225	(15.8193, 18.7896)	(15.2968, 19.3121)
(17)	**BNIP**	17.3045	0.6726	(15.9308, 18.6782)	(15.4547, 19.1542)
	BIP	16.2709	0.2710	(15.7201, 16.8217)	(15.5314, 17.0104)
θ_1	**LB**	8.2585	1.0144	(6.1733, 10.3437)	(5.4397, 11.0773)
(8.5)	**BNIP**	8.2585	0.9444	(6.3298, 10.1872)	(5.6614, 10.8555)
	BIP	10.0000	0.0010	(9.9979, 10.0020)	(9.9972, 10.0028)
θ_2	**LB**	4.9714	2.0772	(0.7017, 9.2411)	(-0.8005, 10.7433)
(5.0)	**BNIP**	4.9714	1.9337	(1.022, 8.9206)	(-0.3464, 10.2892)
	BIP	5.5445	0.4863	(4.5563, 6.5327)	(4.217, 6.8712)
θ_3	**LB**	1.3013	1.6926	(-2.1779, 4.7806)	(-3.4020,6.0047)
(2.0)	**BNIP**	1.3013	1.5757	(-1.9168, 4.5194)	(-3.0320, 5.6346)
	BIP	2.3265	0.8464	(0.6064, 4.0466)	(0.0172, 4.6358)

Table 5.84: Low Negative Collinearity, ρ = -0.20 and sample size, N=30

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.5516	0.5799	(16.3596, 18.7436)	(15.9402, 19.1630)
(17)	**BNIP**	17.5516	0.5399	(16.4491, 18.6542)	(16.0670, 19.0362)
	BIP	16.0422	0.2524	(15.5292, 16.5552)	(15.3535, 16.7309)
θ_1	**LB**	7.8554	0.6708	(6.4765, 9.2343)	(5.9913, 9.7195)
(8.5)	**BNIP**	7.8554	0.6245	(6.5800, 9.1308)	(6.1380, 9.5728)
	BIP	10.0000	0.0008	(9.9983, 10.0017)	(9.9977, 10.0023)
θ_2	**LB**	5.1607	1.2645	(2.5615, 7.7600)	(1.64701, 8.6744)
(5.0)	**BNIP**	5.1607	1.1772	(2.7566, 7.5649)	(1.9234, 8.3980)
	BIP	5.5644	0.4034	(4.7445, 6.3843)	(4.4637, 6.6651)
θ_3	**LB**	0.8796	1.3170	(-1.8276, 3.5868)	(-2.7801, 4.5392)
(2.0)	**BNIP**	0.8796	1.2261	(-1.6244, 3.3836)	(-2.4922, 4.2513)
	BIP	1.9842	0.7264	(0.5080, 3.4605)	(0.0023, 3.9662)

Table 5.85: Low Negative Collinearity, ρ = -0.17 and sample size, N=30

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.5178	0.5639	(15.3588, 17.6768)	(14.9510, 18.0846)
(17)	**BNIP**	16.5178	0.5249	(15.4458, 17.5898)	(15.0743, 17.9613)
	BIP	15.8770	0.2158	(15.4385, 16.3155)	(15.2883, 16.4657)
θ_1	**LB**	9.5804	0.6351	(8.2750, 10.8859)	(7.8157, 11.3452)
(8.5)	**BNIP**	9.5804	0.5912	(8.3729, 10.7879)	(7.9545, 11.2063)
	BIP	10.0000	0.0007	(9.9986, 10.0014)	(9.9981, 10.0019)
θ_2	**LB**	4.4683	1.1755	(2.0520, 6.8847)	(1.2018, 7.7348)
(5.0)	**BNIP**	4.4683	1.0944	(2.2333, 6.7033)	(1.4588, 7.4778)
	BIP	5.4192	0.3386	(4.7311, 6.1074)	(4.4953, 6.3432)
θ_3	**LB**	1.1972	1.3588	(-1.5959, 3.9902)	(-2.5785, 4.9729)
(2.0)	**BNIP**	1.1972	1.2650	(-1.3862, 3.7806)	(-2.2815, 4.6758)
	BIP	2.2185	0.6216	(0.9553, 3.4818)	(0.5225, 3.9145)

Table 5.86: Low Negative Collinearity, ρ = -0.15 and sample size, N=30

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.7900	0.6683	(15.4163, 18.1637)	(14.9330, 18.6470)
(17)	**BNIP**	16.7900	0.6221	(15.5194, 18.0606)	(15.0791, 18.5009)
	BIP	15.9441	0.2483	(15.4395, 16.4488)	(15.2666, 16.6216)
θ_1	**LB**	8.4550	0.7479	(6.9176, 9.9924)	(6.3767, 10.5332)
(8.5)	**BNIP**	8.4550	0.6963	(7.0330, 9.8770)	(6.5402, 10.3697)
	BIP	10.0000	0.0009	(9.9982, 10.0018)	(9.9976, 10.0024)
θ_2	**LB**	4.7293	1.7032	(1.2282, 8.2303)	(-0.0036, 9.4621)
(5.0)	**BNIP**	4.7293	1.5856	(1.4910, 7.9676)	(0.3688, 9.0897)
	BIP	5.4862	0.4242	(4.6241, 6.3484)	(4.3287, 6.6438)
θ_3	**LB**	3.6380	1.2977	(0.9705, 6.3055)	(0.0321, 7.2440)
(2.0)	**BNIP**	3.6380	1.2081	(1.1707, 6.1053)	(0.3157, 6.9603)
	BIP	3.2902	0.7047	(1.8581, 4.7223)	(1.3675, 5.2128)

Table 5.87: Summary of Tables 5.78 -5.86 for Standard Error when the sample size, N= 30.

Parameters	Estimators	-0.95	-0.90	-0.80	-0.49	-0.46	-0.36	-0.20	-0.17	-0.15
θ_0	**LB**	0.5739	0.5450	0.6604	0.7552	0.6256	0.7225	0.5799	0.5639	0.6683
	BNIP	0.5342	0.5074	0.6148	0.7030	0.5824	0.6726	0.5399	0.5249	0.6221
	BIP	0.2264	0.1775	0.1976	0.2092	0.2284	0.2710	0.2524	0.2158	0.2483
θ_1	**LB**	3.0300	1.8625	1.6237	1.1342	0.7831	1.0144	0.6708	0.6351	0.7479
	BNIP	2.8208	1.7339	1.5116	1.0559	0.7290	0.9444	0.6245	0.5912	0.6963
	BIP	0.0009	0.0007	0.0008	0.0008	0.0009	0.0010	0.0008	0.0008	0.0009
θ_2	**LB**	4.4026	2.5703	2.4292	1.6286	1.7119	2.0772	1.2645	1.1755	1.7032
	BNIP	4.0986	2.3928	2.2615	1.5162	1.5937	1.9337	1.1772	1.0944	1.5856
	BIP	0.4237	0.3327	0.3884	0.3672	0.4166	0.4863	0.4034	0.3386	0.4242
θ_3	**LB**	4.6462	2.6238	2.6238	1.6644	1.5586	1.6926	1.3170	1.3588	1.2977
	BNIP	4.3254	2.4426	2.3744	1.5495	1.4509	1.5757	1.2261	1.2650	1.2081
	BIP	0.7496	0.5749	0.6863	0.6555	0.7243	0.8464	0.7264	0.6216	0.7047

Table 5.87 gives the summary of SE in Tables 5.78-5.86 when the sample size is 30. ρ = -0.95 gives the highest SE among the levels of multicollinearity for all the estimators across the parmaters.

Bayesian estimators (BNIP and BIP) have the smallest SE for all the ρ's (-0.15 to -0.95) across the parameters. It was also observed that when ρ = -0.17, all the estimators have minimum SE values for all the parameters.

Among the three estimators, BIP outperformed all the estimators at all the levels of multicollinearity having the smallest SE in most cases. Hence, SE has not shown any consistent pattern within the three levels of multicollinearity (high, moderate and low negative collinearity).

Table 5.88: Summary of Tables 5.78-5.86 for Mean when the sample size, N= 30.

Parameters	Estimators	-0.95	--0.90	0.80	-0.49	-0.46	-0.36	-0.20	-0.17	-0.15
θ_0 (17.00)	**LB**	17.3394	16.8980	16.3352	15.7047	17.2145	17.3045	17.5516	16.5178	16.7900
	BNIP	17.3394	16.8980	16.3352	15.7047	17.2145	17.3045	17.5516	16.5178	16.7900
	BIP	16.4476	15.8980	16.4904	16.1452	16.2226	16.2709	16.0422	15.8770	15.9441
θ_1 (8.5)	**LB**	5.7488	6.9570	7.3045	10.3071	8.1671	8.2585	7.8554	9.5804	8.4550
	BNIP	5.7488	6.9570	7.3045	10.3071	8.1671	8.2585	7.8554	9.5804	8.4550
	BIP	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000
θ_2 (5.00)	**LB**	5.2686	2.2208	7.3045	6.2458	5.7829	4.9714	5.1607	4.4683	4.7293
	BNIP	5.2686	2.2208	7.3045	6.2458	5.7829	4.9714	5.1607	4.4683	4.7293
	BIP	5.6483	5.5552	5.5755	5.5441	5.6399	5.5445	5.5644	5.4192	5.4862
θ_3 (2.00)	**LB**	-3.8396	0.9391	3.1868	4.7978	0.6887	1.3013	0.8796	1.1972	3.6380
	BNIP	-3.8396	0.9391	3.1868	4.7978	0.6887	1.3013	0.8796	1.1972	3.6380
	BIP	2.8783	2.8989	2.6335	3.0733	2.6467	2.3265	1.9842	2.2185	3.2902

Table 5.88 shows the summary of mean for Tables 5-78-5.86 for all the levels of multicollinearity when the sample size is 30. The means of LB and BNIP are the same for all levels of multicollinearity across the parameters. The means of BIP are not too far from true parameter values for all the levels of multicollinearity for all the parameters considered in most cases.

Negative means were observed only when ρ= -0.95 for parameter θ_3.

Table 5.89: High Negative Collinearity, ρ = - 0.95 and sample size, N=70

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.7580	0.3552	(16.0488,17.4672)	(15.8158, 17.7002)
(17)	**BNIP**	16.7580	0.3449	(16.0701, 17.4459)	(15.8447, 17.6713)
	BIP	16.6744	0.1458	(16.3840, 16.9649)	(16.2890, 17.0598)
θ_1	**LB**	10.4427	1.6856	(7.0774, 13.8081)	(5.9719, 14.9136)
(8.5)	**BNIP**	10.4427	1.6367	(7.1784, 13.7071)	(6.1089, 14.7766)
	BIP	10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0020)
θ_2	**LB**	6.9878	2.5865	(1.8236, 12.1519)	(0.1273, 13.8482)
(5.0)	**BNIP**	6.9878	2.5115	(1.9787, 11.9968)	(0.3375, 13.6380)
	BIP	5.7206	0.3503	(5.0226, 6.4186)	(4.7945, 6.6468)
θ_3	**LB**	3.9209	2.3526	(-0.7762, 8.6179)	(-2.3190, 10.1608)
(2.0)	**BNIP**	3.9209	2.2844	(-0.6351, 8.4769)	(-2.1279, 9.9697)
	BIP	3.3375	0.5509	(2.2398, 4.4352)	(1.8810, 4.7940)

Table 5.90: High Negative Collinearity, ρ = -0.90 and sample size, N=70

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.7747	0.4014	(15.9732,17.5762)	(15.7099, 17.8394)
(17)	**BNIP**	16.7747	0.3898	(15.9973, 17.5521)	(15.7425, 17.8068)
	BIP	16.4612	0.1429	(16.1764, 16.7460)	(16.0833, 16.8391)
θ_1	**LB**	9.4176	1.3191	(6.7840, 12.0513)	(5.9188, 12.9164)
(8.5)	**BNIP**	9.4176	1.2809	(6.8630, 11.9722)	(6.0260, 12.8092)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
θ_2	**LB**	7.4624	2.0001	(3.4692, 11.4556)	(2.1575, 12.7673)
(5.0)	**BNIP**	7.4624	1.9421	(3.5891, 11.3357)	(2.3200, 12.6048)
	BIP	5.7307	0.3700	(4.9934, 6.4680)	(4.7524, 6.7090)
θ_3	**LB**	1.0001	1.8565	(-2.7065, 4.7067)	(-3.9241, 5.9242)
(2.0)	**BNIP**	1.0001	1.8027	(-2.5952, 4.5954)	(-3.7732, 5.7734)
	BIP	2.8491	0.5832	(1.6871, 4.0111)	(1.3073, 4.3910)

Table 5.91: High Negative Collinearity, ρ = -0.80 and sample size, N=70

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.5411	0.4014	(15.7381,17.3441)	(15.4743, 17.6079)
(17)	**BNIP**	16.5411	0.3905	(15.7622, 17.3200)	(15.5070, 17.5752)
	BIP	16.4612	0.1317	(16.2106, 16.7353)	(16.1249, 16.8210)
θ_1	**LB**	9.7700	1.3191	(7.8621, 11.6779)	(7.2354, 12.3046)
(8.5)	**BNIP**	9.7700	0.9279	(7.9194, 11.6206)	(7.3131, 12.2269)
	BIP	10.0000	0.0008	(9.9983, 10.0017)	(9.9978, 10.0022)
θ_2	**LB**	7.4624	2.0001	(-0.3689, 5.9937)	(-1.4139, 7.0387)
(5.0)	**BNIP**	7.4624	1.5472	(-0.2734, 5.8982)	(-1.2844, 6.9092)
	BIP	5.7307	0.3945	(4.5733, 6.1452)	(4.3164, 6.4022)
θ_3	**LB**	5.4567	1.8565	(2.5995, 8.3139)	(1.6609, 9.2524)
(2.0)	**BNIP**	5.4567	1.3896	(2.6853, 8.2281)	(1.7772, 9.1362)
	BIP	2.8491	0.6353	(1.9996, 4.5314)	(1.5858, 4.9452)

Table 5.92: Moderate Negative Collinearity, ρ = -0.49 and sample size, N=70

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.8032	0.3412	(16.1219,17.4845)	(15.8981, 17.7083)
(17)	**BNIP**	16.8032	0.3313	(16.1424, 17.4640)	(15.9259, 17.6805)
	BIP	16.0960	0.1281	(15.8407, 16.3512)	(15.7573, 16.4347)
θ_1	**LB**	8.8908	0.4567	(7.9791, 9.8026)	(7.6796, 10.1021)
(8.5)	**BNIP**	8.8908	0.4434	(8.0064, 9.7752)	(7.7167, 10.0650)
	BIP	10.0000	0.0007	(9.9985, 10.0015)	(9.9980, 10.0020)
θ_2	**LB**	6.2174	0.9269	(4.3668, 8.0680)	(3.7589, 8.6759)
(5.0)	**BNIP**	6.2174	0.9000	(4.4224, 8.0124)	(3.8342, 8.6006)
	BIP	5.7343	0.3419	(5.0531, 6.4156)	(4.8304, 6.6383)
θ_3	**LB**	0.8514	0.9413	(-1.0280, 2.7308)	(-1.6453, 3.3481)
(2.0)	**BNIP**	0.8514	0.9140	(-0.9716, 2.6743)	(-1.5689, 3.2716)
	BIP	2.3918	0.5590	(1.2781, 3.5056)	(0.9140, 3.8697)

Table 5.93: Moderate Negative Collinearity, ρ = -0.46 and sample size, N=70.

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.3749	0.2976	(16.7807,17.9692)	(16.5856, 18.1643)
(17)	**BNIP**	17.3749	0.2890	(16.7986, 17.9513)	(16.6097, 18.1401)
	BIP	16.2612	0.1208	(16.0205, 16.5019)	(15.9418, 16.5806)
θ_1	**LB**	8.3294	0.3753	(7.5800, 9.0788)	(7.3338, 9.3249)
(8.5)	**BNIP**	8.3294	0.3645	(7.6025, 9.0563)	(7.3643, 9.2944)
	BIP	10.0000	0.0007	(9.9987, 10.0013)	(9.9983, 10.0017)
θ_2	**LB**	4.7574	0.7381	(3.2838, 6.2310)	(2.7998, 6.7151)
(5.0)	**BNIP**	4.7574	0.7167	(3.3281, 6.1868)	(2.8598, 6.6551)
	BIP	5.6639	0.2990	(5.0681, 6.2596)	(4.8734, 6.4543)
θ_3	**LB**	0.8412	0.7173	(-0.5908, 2.2732)	(-1.0612, 2.7437)
(2.0)	**BNIP**	0.8412	0.6965	(-0.5478, 2.2302)	(-1.0029, 2.6854)
	BIP	2.4116	0.4858	(1.4437, 3.3795)	(1.1273, 3.6960)

Table 5.94: Moderate Negative Collinearity, ρ = -0.36 and sample size, N=70

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.1651	0.4025	(16.3615,17.9686)	(16.0976, 18.2324)
(17)	**BNIP**	17.1651	0.3908	(16.3857, 17.9445)	(16.1303, 18.1998)
	BIP	16.0669	0.1553	(15.7575, 16.3764)	(15.6564, 16.4775)
θ_1	**LB**	8.4060	0.4719	(7.4638, 9.3481)	(7.1544, 9.6576)
(8.5)	**BNIP**	8.4060	0.4582	(7.4921, 9.3198)	(7.1927, 9.6193)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
θ_2	**LB**	4.2391	0.8972	(2.4478, 6.0304)	(1.8593, 6.6189)
(5.0)	**BNIP**	4.2391	0.8712	(2.5015, 5.9767)	(1.9322, 6.5460)
	BIP	5.4992	0.3643	(4.7732, 6.2252)	(4.5359, 6.4625)
θ_3	**LB**	2.6000	0.8694	(0.8642, 4.3358)	(0.2940, 4.9059)
(2.0)	**BNIP**	2.6000	0.8442	(0.9163, 4.2836)	(0.3647, 4.8353)
	BIP	3.3056	0.5646	(2.1805, 4.4307)	(1.8127, 4.7985)

Table 5.95: Low Negative Collinearity, ρ = -0.20 and sample size, N=70

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.3060	0.3268	(16.6534,17.9585)	(16.4391, 18.1728)
(17)	**BNIP**	17.3060	0.3173	(16.6730, 17.9389)	(16.4657, 18.1462)
	BIP	16.1139	0.1870	(15.7412, 16.4865)	(15.6194, 16.6083)
θ_1	**LB**	8.2389	0.3626	(7.5149, 8.9628)	(7.2771, 9.2006)
(8.5)	**BNIP**	8.2389	0.3521	(7.5367, 8.9411)	(7.3066, 9.1712)
	BIP	10.0000	0.0007	(9.9985, 10.0015)	(9.9980, 10.0019)
θ_2	**LB**	5.3509	0.7411	(3.8712, 6.8306)	(3.3851, 7.3167)
(5.0)	**BNIP**	5.3509	0.7197	(3.9156, 6.7862)	(3.4453, 7.2565)
	BIP	5.6106	0.3371	(4.9389, 6.2824)	(4.7192, 6.5020)
θ_3	**LB**	0.7785	0.7707	(-0.7603, 2.3174)	(-1.2658, 2.8228)
(2.0)	**BNIP**	0.7785	0.7484	(-0.7141, 2.2712)	(-1.2031, 2.7602)
	BIP	1.6916	0.5648	(0.5662, 2.8170)	(0.1983, 3.1848)

Table 5.96: Low Negative Collinearity, ρ = -0.17 and sample size, N=70

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.2079	0.3077	(16.5935,17.8224)	(16.3916, 18.0242)
(17)	**BNIP**	17.2079	0.2988	(16.6119, 17.8039)	(15.7538, 16.6663)
	BIP	16.2100	0.1726	(15.8662, 16.5539)	(15.6194, 16.6083)
θ_1	**LB**	8.5170	0.3695	(7.7792, 9.2547)	(7.5369, 9.4971)
(8.5)	**BNIP**	8.5170	0.3588	(7.8014, 9.2326)	(9.9980, 10.0020)
	BIP	10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0019)
θ_2	**LB**	4.8439	0.7546	(3.3374, 6.3504)	(2.8425, 6.8453)
(5.0)	**BNIP**	4.8439	0.7327	(3.3826, 6.3052)	(4.5539, 6.3947)
	BIP	5.4743	0.3481	(4.7806, 6.1679)	(4.7192, 6.5020)
θ_3	**LB**	0.9768	0.7338	(-0.4883, 2.4420)	(-0.9696, 2.9232)
(2.0)	**BNIP**	0.9768	0.7125	(-0.4443, 2.3980)	(0.3954, 3.3220)
	BIP	1.8587	0.5535	(0.7559, 2.9615)	(0.1983, 3.1848)

Table 5.97: Low Negative Collinearity, ρ = -0.15 and sample size, N=70

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.0006	0.3749	(16.2520,17.7492)	(16.0061, 17.9951)
(17)	**BNIP**	17.0006	0.3641	(16.2745, 17.7267)	(16.0365, 17.9646)
	BIP	16.1303	0.1914	(15.7490, 16.5116)	(15.6243, 16.6363)
θ_1	**LB**	8.4793	0.4211	(7.6385, 9.3200)	(7.3624, 9.5962)
(8.5)	**BNIP**	8.4793	0.4089	(7.6638, 9.2948)	(7.3966, 9.5619)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9978, 10.0022)
θ_2	**LB**	5.4701	0.8135	(3.8459, 7.0944)	(3.3124, 7.6279)
(5.0)	**BNIP**	5.4701	0.7899	(3.8947, 7.0456)	(3.3785, 7.5618)
	BIP	5.6017	0.3672	(4.8700, 6.3334)	(4.6309, 6.5726)
θ_3	**LB**	2.0570	0.7722	(0.5153, 3.5987)	(0.0089, 4.1051)
(2.0)	**BNIP**	2.0570	0.7125	(0.5616, 3.5524)	(0.0716, 4.0423)
	BIP	2.4171	0.5712	(1.2791, 3.5552)	(0.9070, 3.9272)

From Tables 5.67-5.97, the following were observed:

The mean estimates are in line with the true values of the simulated data for HNC, MNC and LNC. The means of LB and BNIP are the same for all the parameters in HNC, MNC and LNC for sample sizes of 10, 30 and 70. The mean of LB/BNIP are 17.3060, 8.2389, 5.5509 and 0.7785 for parameters θ_0, θ_1, θ_2 and θ_3 respectively in LNC (ρ= 0.20) when the sample size is 70.

The CI of Bayesian estimators at 95% and 99% are more compact than the LB estimator especially the BIP. The standard errors of Bayesian estimators (BNIP and BIP) are smaller than the LB method for HNC, MNC and LNC for all the sample sizes. High values of SE were observed especially in HNC.

In Low Negative Collinearity, it was shown that performance of Likelihood Based (LB) method becomes better. Hence, the Collinearity does not have much effect on the LB for Low Negative Collinearity (LNC). It is also observed that BIP performs very well than other estimators in terms of SE having the minimum SE for all parameters in HNC, MNC and LNC for the sample sizes of 10, 30 and 70 considered, these patterns were also observed in positive collinearity (HPC, MPC and LPC) for all the sample sizes.

However, SE obtained in negative collinearities (HNC, MNC and LNC) for sample sizes of 10, 30 and 70 are smaller than the SE obtained in positive collineairites (HPC, MPC and LPC).

Table 5.98: Summary of Tables 5.89 -5.97 for Standard Error when the sample size, N= 70.

Parameters	Estimators	-0.95	-0.90	-0.80	-0.49	-0.46	-0.36	-0.20	-0.17	-0.15
θ_0	**LB**	0.3552	0.4014	0.4014	0.3412	0.2976	0.4025	0.3268	0.3077	0.3749
	BNIP	0.3449	0.3898	0.3905	0.3313	0.2890	0.3908	0.3173	0.2988	0.3641
	BIP	0.1458	0.1429	0.1317	0.1281	0.1208	0.1553	0.1870	0.1726	0.1914
θ_1	**LB**	1.6856	1.3191	1.3191	0.4567	0.3753	0.4719	0.3626	0.3695	0.4211
	BNIP	1.6367	1.2809	0.9279	0.4434	0.3645	0.4582	0.3521	0.3588	0.4089
	BIP	0.0008	0.0008	0.0008	0.0007	0.0007	0.0008	0.0007	0.0008	0.0008
θ_2	**LB**	2.5865	2.0001	2.0001	0.9269	0.7381	0.8972	0.7411	0.7546	0.8135
	BNIP	2.5115	1.9421	1.5472	0.9000	0.7167	0.8712	0.7197	0.7327	0.7899
	BIP	0.3503	0.3700	0.3945	0.3419	0.2990	0.3643	0.3371	0.3481	0.3672
θ_3	**LB**	2.3526	1.8565	1.8565	0.9413	0.7173	0.8694	0.7707	0.7338	0.7722
	BNIP	2.2844	1.8565	1.3896	0.9140	0.6965	0.8442	0.7484	0.7125	0.7125
	BIP	0.5509	0.5832	0.6353	0.5590	0.4858	0.5646	0.5648	0.5535	0.5712

Table 5.98 gives the summary of SE in Tables 5.89-5.97. When ρ = -0.95, all the estimators have the highest SE for all the parameters except for intercept parameter θ_0. The SE of estimators decreases as the ρ increases across the parameters but increases when ρ =-0.36 and also decreases again when ρ =-0.20. For all levels of multicollinearity, BIP has the smallest SE across the parameters followed by BNIP estimator.

Table 5.99: Summary of Tables 5.89-5.97 for Mean when the sample size, N= 70.

Parameters	Estimators	-0.95	-0.90	0.80	-0.49	-0.46	-0.36	-0.20	-0.17	-0.15
θ_0 (17.00)	**LB**	16.7580	16.7747	16.5411	16.8032	17.3749	17.1651	17.3060	17.2079	17.0006
	BNIP	16.7580	16.7747	16.5411	16.8032	17.3749	17.1651	17.3060	17.2079	17.0006
	BIP	16.6744	16.4612	16.4612	16.0960	16.2612	16.0669	16.1139	16.2100	16.1303
θ_1 (8.5)	**LB**	10.4427	9.4176	9.7700	8.8908	8.3294	8.4060	8.2389	8.5170	8.4793
	BNIP	10.4427	9.4176	9.7700	8.8908	8.3294	8.4060	8.2389	8.5170	8.4793
	BIP	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000
θ_2 (5.00)	**LB**	6.9878	7.4624	7.4624	6.2174	4.7574	4.2391	5.3509	4.8439	5.4701
	BNIP	6.9878	7.4624	7.4624	6.2174	4.7574	4.2391	5.3509	4.8439	5.4701
	BIP	5.7206	5.7307	5.7307	5.7343	5.6639	5.4992	5.6106	5.4743	5.6017
θ_3 (2.00)	**LB**	3.9209	1.0001	5.4567	0.8514	0.8412	2.6000	0.7785	0.9768	2.0570
	BNIP	3.9209	1.0001	5.4567	0.8514	0.8412	2.6000	0.7785	0.9768	2.0570
	BIP	3.3375	2.8491	2.8491	2.3918	2.4116	3.3056	1.6916	1.8587	2.4171

Table 5.99 shows the summary of mean for Tables 5.88-5.96 when the sample size is 70, BIP has positive means values for all the levels of collinearity across the parameters. As the ρ reduces, the mean estimates tend toward the true parameter values. The mean values of LB and BNIP are the same for all the levels of collinearity across the parameters considered while the negative mean values were not observed.

Table 5.100: High Negative Collinearity, ρ = - 0.95 and sample size, N=100

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.9052	0.2942	(16.3213,17.4891)	(16.1322, 17.6783)
(17)	**BNIP**	16.9052	0.2882	(16.3334, 17.4770)	(16.1484, 17.6620)
	BIP	16.7236	0.1217	(16.4824, 16.9649)	(16.4044, 17.0429)
θ_1	**LB**	9.7087	1.4293	(6.8715, 12.5458)	(5.9524, 13.4649)
(8.5)	**BNIP**	9.7087	1.4004	(6.9303, 12.4871)	(6.0313, 13.3861)
	BIP	10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0020)
θ_2	**LB**	7.9382	2.0529	(3.8633, 12.0132)	(2.5433, 13.3332)
(5.0)	**BNIP**	7.9382	2.0114	(3.9477, 11.9288)	(2.6565, 13.2200)
	BIP	5.8249	0.3490	(5.1327, 6.5170)	(4.9090, 6.7407)
θ_3	**LB**	1.3896	2.1540	-2.8861, 5.6653)	(-4.2712, 7.0505)
(2.0)	**BNIP**	1.3896	2.1105	(-2.7976, 5.5768)	(-4.1524, 6.9316)
	BIP	3.2876	0.4855	(2.2335, 4.3417)	(1.8928, 4.6824)

Table 5.101: High Negative Collinearity, ρ = -0.90 and sample size, N=100

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.0726	0.3788	(16.3207,17.8245)	(16.0771, 18.0681)
(17)	**BNIP**	17.0726	0.3712	(16.3362, 17.8090)	(16.0980, 18.0472)
	BIP	16.3849	0.1218	(16.1434, 16.6263)	(16.0654, 16.7043)
θ_1	**LB**	8.0771	1.2659	(5.5642, 10.5900)	(4.7502, 11.4040)
(8.5)	**BNIP**	8.0771	1.2404	(5.6162, 10.5379)	(9.9979, 10.0021)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9980, 10.0020)
θ_2	**LB**	5.0934	1.7204	(1.6785, 8.5084)	(0.5722, 9.6147)
(5.0)	**BNIP**	5.0934	1.6857	(1.7492, 8.4377)	(4.8791, 6.8026)
	BIP	5.8408	0.3665	(5.1140, 6.5677)	(4.9090, 6.7407)
θ_3	**LB**	1.5857	1.6326	(-1.6550, 4.8264)	(-2.7048, 5.8763)
(2.0)	**BNIP**	1.5857	1.5996	(-1.5879, 4.7593)	(-2.6147, 5.7862)
	BIP	3.5419	0.5492	(2.4528, 4.6310)	(2.1008, 4.9830)

Table 5.102: High Negative Collinearity, ρ = -0.80 and sample size, N=100

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.3103	0.2924	(16.7300,17.8906)	(16.5420, 18.0786)
(17)	**BNIP**	17.3103	0.2865	(16.7420, 17.8786)	(16.5581, 18.0625)
	BIP	16.7372	0.1013	(16.5362, 16.9381)	(16.4713, 17.0031)
θ_1	**LB**	8.6181	0.6866	(7.2552, 9.9811)	(6.8137, 10.4226)
(8.5)	**BNIP**	8.6181	0.6728	(7.2834, 9.9529)	(6.8515, 10.3847)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
θ_2	**LB**	4.9165	1.1460	(2.6418, 7.1912)	(1.9049, 7.9281)
(5.0)	**BNIP**	4.9165	1.1228	(2.6889, 7.1442)	(1.9682, 7.8649)
	BIP	5.8117	0.3535	(5.1106, 6.5128)	(4.8841, 6.7394)
θ_3	**LB**	3.3383	1.2196	(0.9175, 5.7592)	(0.1333, 6.5434)
(2.0)	**BNIP**	3.3383	1.5996	(0.9676, 5.7090)	(0.2006, 6.4761)
	BIP	3.8285	0.5592	(2.7196, 4.9374)	(2.3613, 5.2958)

Table 5.103: Moderate Negative Collinearity, ρ = -0.49 and sample size, N=100

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.9422	0.3327	(16.2818,17.6025)	(16.0679, 17.8164)
(17)	**BNIP**	16.9422	0.3259	(16.2955, 17.5888)	(16.0863, 17.7981)
	BIP	16.1598	0.1162	(15.9294, 16.3902)	(15.8549, 16.4647)
θ_1	**LB**	8.8014	0.4631	(7.8822, 9.7206)	(7.5845, 10.0183)
(8.5)	**BNIP**	8.8014	0.4537	(7.9013, 9.7015)	(7.6100, 9.9928)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
θ_2	**LB**	4.1510	0.7740	(2.6147, 5.6874)	(1.9049, 6.1851)
(5.0)	**BNIP**	4.1510	0.7583	(2.6465, 5.6556)	(2.1597, 6.1424)
	BIP	5.4171	0.3470	(4.7290, 6.1053)	(4.5066, 6.3277)
θ_3	**LB**	2.7507	0.8606	(1.0425, 4.4589)	(0.4891, 5.0122)
(2.0)	**BNIP**	2.7507	0.8432	(1.0779, 4.4235)	(0.5366, 4.9647)
	BIP	3.1613	0.5411	(2.0882, 4.2344)	(1.7414, 4.5813)

Table 5.104: Moderate Negative Collinearity, ρ = -0.46 and sample size, N=100

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.1603	0.3307	(16.5039,17.8167)	(16.2912, 18.0294)
(17)	**BNIP**	17.1603	0.3240	(16.5175, 17.8031)	(16.3095, 18.0111)
	BIP	16.1462	0.1207	(15.9069, 16.3855)	(15.8295, 16.4629)
θ_1	**LB**	8.3225	0.4742	(7.3811, 9.2638)	(7.0762, 9.5688)
(8.5)	**BNIP**	8.3225	0.4647	(7.4006, 9.2444)	(7.1024, 9.5426)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
θ_2	**LB**	4.8893	0.7640	(3.3727, 6.4058)	(2.8814, 6.8971)
(5.0)	**BNIP**	4.8893	0.7486	(3.4041, 6.3744)	(2.9236, 6.8550)
	BIP	5.6639	0.3458	(4.9782, 6.3496)	(4.7566, 6.5712)
θ_3	**LB**	1.4494	0.7189	(0.0225, 2.8764)	(-0.4398, 3.3386)
(2.0)	**BNIP**	1.4494	0.7043	(0.0520, 2.8468)	(-0.4001, 3.2989)
	BIP	2.6235	0.5065	(1.6190, 3.6280)	(1.2944, 3.9526)

Table 5.105: Moderate Negative Collinearity, ρ = -0.36 and sample size, N=100

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.7556	0.3270	(16.1065,17.4048)	(15.8962, 17.6151)
(17)	**BNIP**	16.7556	0.3204	(16.1199, 17.3913)	(15.9142, 17.5970)
	BIP	15.9068	0.1411	(15.6271, 16.1866)	(15.5367, 16.2770)
θ_1	**LB**	8.6386	0.3737	(7.8967, 9.3804)	(7.6564, 9.6207)
(8.5)	**BNIP**	8.6386	0.3662	(7.9121, 9.3650)	(7.6770, 9.6001)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9978, 10.0021)
θ_2	**LB**	5.9511	0.7308	(4.5005, 7.4017)	(4.0306, 7.8717)
(5.0)	**BNIP**	5.9511	0.7160	(4.5305, 7.3717)	(4.0709, 7.8313)
	BIP	5.8648	0.3554	(5.1600, 6.5697)	(4.9322, 6.7975)
θ_3	**LB**	1.8460	0.7310	(0.3951, 3.2969)	(-0.0750, 3.7670)
(2.0)	**BNIP**	1.8460	0.7162	(0.4251, 3.2669)	(-0.0346, 3.7266)
	BIP	2.6764	0.5423	(1.6009, 3.7519)	(1.2534, 4.0995)

Table 5.106: Low Negative Collinearity, ρ = -0.20 and sample size, N=100

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.7840	0.3640	(16.0615,17.5065)	(15.8274, 17.7406)
(17)	**BNIP**	16.7840	0.3566	(16.0764, 17.4915)	(15.8475, 17.7205)
	BIP	15.9159	0.1644	(15.5899, 16.2420)	(15.4845, 16.3473)
θ_1	**LB**	8.6384	0.4242	(7.7963, 9.4805)	(7.5235, 9.7533)
(8.5)	**BNIP**	8.6384	0.4157	(7.8138, 9.4630)	(7.5470, 9.7299)
	BIP	10.0000	0.0009	(9.9983, 10.0017)	(9.9977, 10.0023)
θ_2	**LB**	4.8901	0.7853	(3.3312, 6.4489)	(2.8262, 6.9539)
(5.0)	**BNIP**	4.8901	0.7695	(3.3635, 6.4166)	(2.8696, 6.9106)
	BIP	5.4865	0.3817	(4.7295, 6.2435)	(4.4848, 6.4881)
θ_3	**LB**	3.0525	0.7704	(1.5234, 4.5817)	(1.0280, 5.0771)
(2.0)	**BNIP**	3.0525	0.7548	(1.5550, 4.5501)	(1.0705, 5.0346)
	BIP	3.3589	0.5627	(2.2431, 4.4747)	(1.8825, 4.8354)

Table 5.107: Low Negative Collinearity, ρ = -0.17 and sample size, N=100

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.3024	0.2841	(16.7386, 17.8663)	(16.5559, 18.0490)
(17)	**BNIP**	17.3024	0.2783	(16.7502, 17.8546)	(16.5716, 18.0333)
	BIP	16.3263	0.1785	(15.7944, 16.4152)	(15.8581, 16.7946)
θ_1	**LB**	8.2539	0.3386	(7.5818, 8.9261)	(7.3640, 9.1438)
(8.5)	**BNIP**	8.2539	0.3318	(7.5957, 8.9121)	(7.3827, 9.1251)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9978, 10.0021)
θ_2	**LB**	6.0653	0.6519	(4.7713, 7.3593)	(4.3521, 7.7785)
(5.0)	**BNIP**	6.0653	0.6387	(4.7981, 7.3325)	(4.3881, 7.7425)
	BIP	5.7778	0.3569	(5.0701, 6.4855)	(4.8414, 6.7142)
θ_3	**LB**	0.3993	0.6705	(-0.9317, 1.7303)	(-1.3629, 2.1615)
(2.0)	**BNIP**	0.3993	0.6570	(-0.9042, 1.7027)	(-1.3259, 2.1245)
	BIP	1.4617	0.5549	(0.3613, 2.5621)	(0.0056, 2.9178)

Table 5.108: Low Negative Collinearity, ρ = -0.15 and sample size, N=100

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.0024	0.2841	(16.3643, 17.6405)	(16.1576, 17.8471)
(17)	**BNIP**	17.0024	0.3150	(16.3775, 17.6272)	(16.1753, 17.8294)
	BIP	16.0939	0.1678	(15.7944, 16.4152)	(15.6535, 16.5342)
θ_1	**LB**	8.6171	0.3611	(7.9003, 9.3340)	(7.6680, 9.56620)
(8.5)	**BNIP**	8.6171	0.3538	(7.9151, 9.3191)	(7.6880, 9.5463)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
θ_2	**LB**	4.5558	0.6568	(3.2521, 5.8594)	(2.8298, 6.28192)
(5.0)	**BNIP**	4.5558	0.6435	(3.2791, 5.8326)	(2.8660, 6.2457)
	BIP	5.3677	0.3448	(4.6839, 6.0516)	(4.4629, 6.2726)
θ_3	**LB**	2.7728	0.7162	(1.3512, 4.1944)	(0.8907, 4.6549)
(2.0)	**BNIP**	2.7728	0.7017	(1.3807, 4.1650)	(0.9302, 4.6154)
	BIP	2.9253	0.5430	(1.8484, 4.0022)	(1.5004, 4.3502)

Table 5.109: Summary of Tables 5.100 -5.108 for Standard Error for sample size, N= 100.

Parameters	Estimators	-0.95	-0.90	-0.80	-0.49	-0.46	-0.36	-0.20	-0.17	-0.15
θ_0	**LB**	0.2942	0.3788	0.2924	0.3327	0.3307	0.3270	0.3640	0.2841	0.2841
	BNIP	0.2882	0.3712	0.2865	0.3259	0.3240	0.3204	0.3566	0.2783	0.3150
	BIP	0.1217	0.1218	0.1013	0.1162	0.1207	0.1411	0.1644	0.1785	0.1678
θ_1	**LB**	1.4293	1.2659	0.6866	0.4631	0.4742	0.3737	0.4242	0.3386	0.3611
	BNIP	1.4004	1.2404	0.6728	0.4537	0.4647	0.3662	0.4157	0.3318	0.3538
	BIP	0.0008	0.0008	0.0008	0.0008	0.0008	0.0008	0.0009	0.0008	0.0008
θ_2	**LB**	2.0529	1.7204	1.1460	0.7740	0.7640	0.7308	0.7853	0.6519	0.6568
	BNIP	2.0114	1.6857	1.1228	0.7583	0.7486	0.7160	0.7695	0.6387	0.6435
	BIP	0.3490	0.3665	0.3535	0.3470	0.3458	0.3554	0.3817	0.3569	0.3448
θ_3	**LB**	2.1540	1.6326	1.2196	0.8606	0.7189	0.7310	0.7704	0.6705	0.7162
	BNIP	2.1105	1.5996	1.5996	0.8432	0.7043	0.7162	0.7548	0.6570	0.7017
	BIP	0.4855	0.5492	0.5592	0.5411	0.5065	0.5423	0.5627	0.5549	0.5430

Table 5.109 gives a summary of SE of Tables 5.1000-5.108 for all the levels of collinearity.

Highest SE of estimate was observed mostly when $\rho = -0.95$, except for parameter θ_0 while the lowest SE of estimate was observed when $\rho = -0.17$ for all the estimators across the parameters. BIP has the smallest SE compared to other estimators (LB and BNIP) for all the level of collinearity across the parameters. LB has the highest Se for all levels of collinearity across the parameters considered.

Hence, BIP outperformed other estimators in terms of SE.

Table 5.110: Summary of Tables 5.100-5.108 for Mean for sample size, N= 100.

Parameters	Estimators	-0.95	-0.90	0.80	-0.49	-0.46	-0.36	-0.20	-0.17	-0.15
θ_0 (17.00)	**LB**	16.9052	17.0726	17.3103	16.9422	17.1603	16.7556	16.7840	17.3024	17.0024
	BNIP	16.9052	17.0726	17.3103	16.9422	17.1603	16.7556	16.7840	17.3024	17.0024
	BIP	16.7236	16.3849	16.7372	16.1598	16.1462	15.9068	15.9159	16.3263	16.1303
θ_1 (8.5)	**LB**	9.7087	8.0771	8.6181	8.8014	8.3225	8.6386	8.6384	8.2539	8.6171
	BNIP	9.7087	8.0771	8.6181	8.8014	8.3225	8.6386	8.6384	8.2539	8.6171
	BIP	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000
θ_2 (5.00)	**LB**	7.9382	5.0934	4.9165	4.1510	4.8893	5.9511	4.8901	6.0653	4.5558
	BNIP	7.9382	5.0934	4.9165	4.1510	4.8893	5.9511	4.8901	6.0653	4.5558
	BIP	5.8249	5.8408	5.8117	5.4171	5.6639	5.8648	5.4865	5.7778	5.3677
θ_3 (2.00)	**LB**	1.3896	1.5857	3.3383	2.7507	1.4494	1.8460	3.0525	0.3993	2.7728
	BNIP	1.3896	1.5857	3.3383	2.7507	1.4494	1.8460	3.0525	0.3993	2.7728
	BIP	3.2876	3.5419	3.8285	3.1613	2.6235	2.6235	3.3589	1.4617	2.9253

Table 5.110 gives a summary of means of Tables 5.100-5.108 for all the levels of collinearity. Negative means of estimate was not observed at all the levels of collinearity using the estimators. Most of the means are not too far from the true parameter values except for parameter θ_3.

Table 5.111: High Negative Collinearity, ρ = -0.95 and sample size, N=200

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.1002	0.2250	(16.6565,17.5439)	(16.5150, 17.6854)
(17)	**BNIP**	17.1002	0.2227	(16.6610, 17.5394)	(16.5210, 17.6794)
	BIP	16.6208	0.0980	(16.4276, 16.8141)	(16.3660, 16.8757)
θ_1	**LB**	7.6913	1.1145	(5.4933, 9.8893)	(4.7923, 10.5904)
(8.5)	**BNIP**	7.6913	1.1033	(5.5157, 9.8670)	(4.8220, 10.5607)
	BIP	10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0020)
θ_2	**LB**	4.3923	1.5496	(1.3363, 7.4482)	(0.3616, 8.4229)
(5.0)	**BNIP**	4.3923	1.5340	(1.3674, 7.4171)	(0.4029, 8.3816)
	BIP	5.7822	0.3454	(5.1012, 6.4633)	(4.8841, 6.6804)
θ_3	**LB**	0.5354	1.6601	(-2.7385, 3.8093)	(-3.7827, 4.8535)
(2.0)	**BNIP**	0.5354	1.6434	(-2.7052, 3.7760)	(-3.7384, 4.8092)
	BIP	3.3070	0.4855	(2.3498, 4.2641)	(2.0447, 4.5692)

Table 5.112: High Negative Collinearity, ρ = -0.90 and sample size, N=200

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.9794	0.2127	(16.5600,17.3989)	(16.4262, 17.5327)
(17)	**BNIP**	16.9794	0.2105	(16.5643, 17.3946)	(16.4319, 17.5270)
	BIP	16.5188	0.0851	(16.3510, 16.6866)	(16.2975, 16.7401)
θ_1	**LB**	8.4246	0.7201	(7.0044, 9.8448)	(6.5514, 10.2978)
(8.5)	**BNIP**	8.4246	0.7129	(7.0189, 9.8304)	(6.5706, 10.2786)
	BIP	10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0020)
θ_2	**LB**	3.8926	1.0992	(1.7248, 6.0605)	(1.0334, 6.7519)
(5.0)	**BNIP**	3.8926	1.0882	(1.7469, 6.0384)	(1.0627, 6.7226)
	BIP	5.6385	0.3306	(4.9867, 6.2902)	(4.7789, 6.4980)
θ_3	**LB**	2.2542	1.1106	(0.0639, 4.4445)	(-0.6347, 5.1431)
(2.0)	**BNIP**	2.2542	1.0994	(0.0862, 4.4222)	(-0.6051, 5.1135)
	BIP	3.3768	0.4439	(2.5016, 4.2520)	(2.2227, 4.5310)

Table 5.113: High Negative Collinearity, ρ = -0.80 and sample size, N=200

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.8596	0.2324	(16.4012,17.3179)	(16.2551, 17.4641)
(17)	**BNIP**	16.8596	0.2301	(16.4059, 17.3133)	(16.2612, 17.4579)
	BIP	16.3869	0.0733	(16.2424, 16.5313)	(16.1964, 16.5773)
θ_1	**LB**	8.8478	0.5476	(7.7679, 9.9277)	(7.4235, 10.2721)
(8.5)	**BNIP**	8.8478	0.5421	(7.7789, 9.9167)	(7.4381, 10.2575)
	BIP	10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0020)
θ_2	**LB**	4.4669	0.7794	(2.9298, 6.0040)	(2.4395, 6.4943)
(5.0)	**BNIP**	4.4669	0.7716	(2.9454, 5.9884)	(2.4603, 6.4735)
	BIP	5.5551	0.3172	(4.9296, 6.1806)	(4.7302, 6.3799)
θ_3	**LB**	2.0436	0.8415	(0.3840, 3.7033)	(-0.1454, 4.2326)
(2.0)	**BNIP**	2.0436	0.8331	(0.4009, 3.6864)	(-0.1229, 4.2102)
	BIP	3.0250	0.4245	(2.1881, 3.8619)	(1.9213, 4.1287)

Table 5.114: Moderate Negative Collinearity, ρ = -0.49 and sample size, N=200

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.2385	0.2220	(16.8007,17.6764)	(16.6610, 17.816)
(17)	**BNIP**	17.2385	0.2198	(16.8051, 17.6719)	(16.6669, 17.8101)
	BIP	16.1350	0.0981	(15.9416, 16.3283)	(15.8800, 16.3900)
θ_1	**LB**	8.0898	0.3228	(7.4533, 8.7263)	(7.2503, 8.9294)
(8.5)	**BNIP**	8.0898	0.3195	(7.4598, 8.7199)	(7.2589, 8.9208)
	BIP	10.0000	0.0009	(9.9983, 10.0017)	(9.9978, 10.0022)
θ_2	**LB**	4.8951	0.5817	(3.7479, 6.0424)	(3.3819, 6.4083)
(5.0)	**BNIP**	4.8951	0.5759	(3.7595, 6.0307)	(3.3974, 6.3928)
	BIP	5.8310	0.3412	(5.1583, 6.5037)	(4.9439, 6.7181)
θ_3	**LB**	2.0941	0.6348	(0.8422, 3.3461)	(0.4428, 3.7454)
(2.0)	**BNIP**	2.0941	0.6285	(0.8549, 3.3334)	(0.4598, 3.7285)
	BIP	3.2888	0.4983	(2.3064, 4.2712)	(1.9932, 4.5844)

Table 5.115: Moderate Negative Collinearity, ρ = -0.46 and sample size, N=200

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.0192	0.2140	(16.5973,17.4412)	(16.4627, 17.5758)
(17)	**BNIP**	17.0192	0.2118	(16.6016, 17.4369)	(16.4684, 17.5701)
	BIP	16.0736	0.0964	(15.8835, 16.2637)	(15.8229, 16.3243)
θ_1	**LB**	8.5127	0.2931	(7.9347, 9.0907)	(7.7503, 9.2750)
(8.5)	**BNIP**	8.5127	0.2901	(7.9405, 9.0848)	(7.7581, 9.2672)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
θ_2	**LB**	3.8866	0.5542	(2.7936, 4.9796)	(2.4450, 5.3282)
(5.0)	**BNIP**	3.8866	0.5486	(2.8047, 4.9685)	(2.4598, 5.3134)
	BIP	5.3567	0.3240	(4.7180, 5.9955)	(4.5143, 6.1991)
θ_3	**LB**	2.6477	0.5825	(1.4989, 3.7964)	(1.1325, 4.1628)
(2.0)	**BNIP**	2.6477	0.5766	(1.5106, 3.7847)	(1.1481, 4.1473)
	BIP	3.3771	0.4559	(2.4783, 4.2758)	(2.1918, 4.5623)

Table 5.116: Moderate Negative Collinearity, ρ = -0.36 and sample size, N=200

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.1723	0.2268	(16.7251,17.6195)	(16.5825, 17.7621)
(17)	**BNIP**	17.1723	0.2245	(16.7297, 17.6150)	(16.5885, 17.7561)
	BIP	16.0520	0.1064	(15.8423, 16.2618)	(15.7755, 16.3286)
θ_1	**LB**	8.2606	0.2847	(7.6990, 8.8222)	(7.5199, 9.0013)
(8.5)	**BNIP**	8.2606	0.2819	(7.7047, 8.8164)	(7.5275, 8.9937)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9978, 10.0021)
θ_2	**LB**	4.8536	0.5073	(3.8531, 5.8541)	(3.5340, 6.1732)
(5.0)	**BNIP**	4.8536	0.5022	(3.8633, 5.8440)	(3.5476, 6.1597)
	BIP	5.6744	0.3219	(5.0397, 6.3092)	(4.8373, 6.5115)
θ_3	**LB**	2.3758	0.5563	(1.2786, 3.4729)	(0.9286, 3.8229)
(2.0)	**BNIP**	2.3758	0.5507	(1.2897, 3.4618)	(0.9435, 3.8080)
	BIP	3.2812	0.4624	(2.3695, 4.1930)	(2.0788, 4.4836)

Table 5.117: Low Negative Collinearity, ρ = -0.20 and sample size, N=200

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.2281	0.2087	(16.8164,17.6397)	(16.5825, 17.7621)
(17)	**BNIP**	17.2281	0.2066	(16.8206, 17.6355)	(16.6907, 17.7654)
	BIP	16.1995	0.1248	(15.9535, 16.4455)	(15.8751, 16.5239)
θ_1	**LB**	8.6311	0.2337	(8.1701, 9.0920)	(7.5199, 9.0013)
(8.5)	**BNIP**	8.6311	0.2314	(8.1748, 9.0873)	(8.0293, 9.2328)
	BIP	10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0020)
θ_2	**LB**	3.8447	0.4772	(2.9036, 4.7859)	(3.5340, 6.1732)
(5.0)	**BNIP**	3.8447	0.4724	(2.9131, 4.7763)	(2.6161, 5.0734)
	BIP	5.0807	0.3068	(4.4758, 5.6856)	(4.2830, 5.8784)
θ_3	**LB**	2.0391	0.4683	(1.1154, 2.9628)	(0.9286, 3.8229)
(2.0)	**BNIP**	2.0391	0.4636	(1.1248, 2.9533)	(0.8333, 3.2448)
	BIP	2.6046	0.4111	(1.7941, 3.4152)	(1.5357, 3.6735)

Table 5.118: Low Negative Collinearity, ρ = -0.17 and sample size, N=200

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.1721	0.2174	(16.7434,17.6008)	(16.6067, 17.7375)
(17)	**BNIP**	17.1721	0.2152	(16.7477, 17.5964)	(16.6124, 17.7317)
	BIP	16.1460	0.1376	(15.8747, 16.4172)	(15.7882, 16.5037)
θ_1	**LB**	8.3021	0.2522	(7.8047, 8.7994)	(7.6461, 8.9581)
(8.5)	**BNIP**	8.3021	0.2497	(7.8098, 8.7944)	(7.6528, 8.9514)
	BIP	10.0000	0.0009	(9.9983, 10.0017)	(9.9978, 10.0022)
θ_2	**LB**	5.2155	0.4953	(4.2386, 6.1923)	(3.9271, 6.5039)
(5.0)	**BNIP**	5.2155	0.4903	(4.2486, 6.1824)	(3.9403, 6.4907)
	BIP	5.6098	0.3338	(4.9518, 6.2679)	(4.7420, 6.4777)
θ_3	**LB**	1.5060	0.5152	(0.4900, 2.5219)	(0.1660, 2.8460)
(2.0)	**BNIP**	1.5060	0.5100	(0.5003, 2.5116)	(0.1797, 2.8322)
	BIP	2.1203	0.4678	(1.1979, 3.0427)	(0.9039, 3.3367)

Table 5.119: Low Negative Collinearity, ρ = -0.15 and sample size, N=200

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.9454	0.2126	(16.5262,17.3647)	(16.3924, 17.4984)
(17)	**BNIP**	16.9454	0.2105	(16.5304, 17.3604)	(16.3981, 17.4928)
	BIP	16.2263	0.1282	(15.9734, 16.4791)	(15.8928, 16.5597)
θ_1	**LB**	8.8222	0.2407	(8.3475, 9.2969)	(8.1961, 9.4484)
(8.5)	**BNIP**	8.8222	0.2383	(8.3524, 9.2921)	(8.2025, 9.4419)
	BIP	10.0000	0.0009	(9.9984, 10.0016)	(9.9979, 10.0020)
θ_2	**LB**	5.6640	0.5041	(4.6700, 6.6590)	(4.3529, 6.9751)
(5.0)	**BNIP**	5.6640	0.4990	(4.6801, 6.6480)	(4.3664, 6.9617)
	BIP	5.6985	0.3157	(5.0760, 6.3210)	(4.8776, 6.5194)
θ_3	**LB**	1.1365	0.4761	(0.1975, 2.0754)	(-0.1019, 2.3749)
(2.0)	**BNIP**	1.1365	0.4713	(0.2071, 2.0658)	(-0.0892, 2.3622)
	BIP	1.7847	0.4192	(0.9581, 2.6112)	(0.6946, 2.8747)

Table 5.120: Summary of Tables 5.111-5.119 for Standard Error for sample size, N= 200.

Parameters	Estimators	-0.95	-0.90	-0.80	-0.49	-0.46	-0.36	-0.20	-0.17	-0.15
θ_0	**LB**	0.2250	0.2127	0.2324	0.2220	0.2140	0.2268	0.2087	0.2174	0.2126
	BNIP	0.2227	0.2105	0.2301	0.2198	0.2118	0.2245	0.2066	0.2152	0.2105
	BIP	0.0980	0.0851	0.0733	0.0981	0.0964	0.1064	0.1248	0.1376	0.1282
θ_1	**LB**	1.1145	0.7201	0.5476	0.3228	0.2931	0.2847	0.2337	0.2522	0.2407
	BNIP	1.1033	0.7129	0.5421	0.3195	0.2901	0.2819	0.2314	0.2497	0.2383
	BIP	0.0008	0.0008	0.0008	0.0009	0.0008	0.0008	0.0008	0.0009	0.0009
θ_2	**LB**	1.5496	1.0992	0.7794	0.5817	0.5542	0.5073	0.4772	0.4953	0.5041
	BNIP	1.5340	1.0882	0.7716	0.5759	0.5486	0.5022	0.4724	0.4903	0.4990
	BIP	0.3454	0.3306	0.3172	0.3412	0.3240	0.3219	0.3068	0.3338	0.3157
θ_3	**LB**	1.6601	1.1106	0.8415	0.6348	0.5825	0.5563	0.4683	0.5152	0.4761
	BNIP	1.6434	1.0994	0.8331	0.6285	0.5766	0.5507	0.4636	0.5100	0.4713
	BIP	0.4855	0.4439	0.4245	0.4983	0.4559	0.4624	0.4111	0.4678	0.4192

Table 5.120 gives the summary of SE in Tables 5.111-5.119 when the sample size is 200. As ρ increases, the SE of the estimators also decreases for all the parameters. It is also observed that the SE are smaller compared to sample sizes of 10, 30, 70 an 100 across the parameters.

Among all the estimators considered, BIP outperformed all other estimators having the smallest SE in all the cases of collinearity considered (-0.15 to -0.95).

Table 5.121: Summary of Tables 5.91-5.99 for Mean for sample size, N= 200.

Parameters	Estimators	-0.95	-0.90	0.80	-0.49	-0.46	-0.36	-0.20	-0.17	-0.15
θ_0 (17.00)	**LB**	17.1002	16.9794	16.8596	17.2385	17.0192	17.1723	17.2281	17.1721	16.9454
	BNIP	17.1002	16.9794	16.8596	17.2385	17.0192	17.1723	17.2281	17.1721	16.9454
	BIP	16.6208	16.5188	16.3869	16.1350	16.0736	16.0520	16.1995	16.1460	16.2263
θ_1 (8.5)	**LB**	7.6913	8.4246	8.8478	8.0898	8.5127	8.2606	8.6311	8.3021	8.8222
	BNIP	7.6913	8.4246	8.8478	8.0898	8.5127	8.2606	8.6311	8.3021	8.8222
	BIP	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000
θ_2 (5.00)	**LB**	4.3923	3.8926	4.4669	4.8951	3.8866	4.8536	3.8447	5.2155	5.6640
	BNIP	4.3923	3.8926	4.4669	4.8951	3.8866	4.8536	3.8447	5.2155	5.6640
	BIP	5.7822	5.6385	5.5551	5.8310	5.3567	5.6744	5.0807	5.6098	5.6985
θ_3 (2.00)	**LB**	0.5354	2.2542	2.0436	2.0941	2.6477	2.3758	2.0391	1.5060	1.1365
	BNIP	0.5354	2.2542	2.0436	2.0941	2.6477	2.3758	2.0391	1.5060	1.1365
	BIP	3.3070	3.3768	3.0250	3.2888	3.3771	3.2812	2.6046	2.1203	1.7847

Table 5.121 summarizes the mean estimates of all the estimators for parameters (θ_0, θ_1, θ_2 and θ_3), when the sample size is 200.

The means of LB and BNIP are the same for all the levels of collinearity considered across the parameters. All the means are positive and not too far from the true parameter values. For parameter θ_3, the means of BIP are far from the true parameter value except for ρ= -0.20, -0.17 and -0.15.

Table 5.122: High Negative Collinearity, ρ = -0.95 and sample size, N=300

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.0489	0.1783	(16.6980, 17.3999)	(16.5866, 17.5113)
(17)	**BNIP**	17.0489	0.1771	(16.7003, 17.3975)	(16.6917, 17.6420)
	BIP	16.7771	0.0858	(16.6082, 16.9460)	(16.5546, 16.9996)
θ_1	**LB**	8.9103	0.8281	(7.2806, 10.5401)	(6.7634, 11.0573)
(8.5)	**BNIP**	8.9103	0.8226	(7.2915, 10.5291)	(6.7779, 11.0428)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
θ_2	**LB**	4.9761	1.3195	(2.3793, 7.5728)	(1.5552, 8.3970)
(5.0)	**BNIP**	4.9761	1.3107	(4.2441, 5.8637)	(1.5784, 8.3737)
	BIP	5.8495	0.3455	(5.1697, 6.5293)	(4.9541, 6.7449)
θ_3	**LB**	2.9407	1.2601	(0.4608, 5.4207)	(-0.3262, 6.2076)
(2.0)	**BNIP**	2.9407	1.2517	(0.4123, 2.0582)	(-0.3040, 6.1855)
	BIP	3.8656	0.4380	(3.0036, 4.7276)	(2.7302, 5.0011)

Table 5.123: High Negative Collinearity, ρ = -0.90 and sample size, N=300

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.1179	0.1825	(16.7587,17.4771)	(16.6447, 17.5911)
(17)	**BNIP**	17.1179	0.1813	(16.7611, 17.4747)	(16.6479, 17.5879)
	BIP	16.5510	0.0726	(16.4081, 16.6938)	(16.3627, 16.7392)
θ_1	**LB**	8.0066	0.6321	(6.7626, 9.2507)	(6.3678, 9.6455)
(8.5)	**BNIP**	8.0066	0.6279	(6.7710, 9.2423)	(6.3789, 9.6344)
	BIP	10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0020)
θ_2	**LB**	4.4146	0.8800	(2.6827, 6.1464)	(2.1332, 6.6960)
(5.0)	**BNIP**	4.4146	0.8741	(2.6944, 6.1347)	(2.1486, 6.6805)
	BIP	5.8495	0.3198	(5.2201, 6.4788)	(5.0204, 6.6785)
θ_3	**LB**	1.2559	0.9258	(-0.5661, 3.0779)	(-1.1443, 3.6562)
(2.0)	**BNIP**	1.2559	0.9196	(-0.5538, 3.0657)	(-1.1281, 3.6399)
	BIP	3.4357	0.4102	(2.6285, 4.2429)	(2.3724, 4.4990)

Table 5.124: High Negative Collinearity, ρ = -0.80 and sample size, N=300

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.3138	0.1914	(16.9372,17.6904)	(16.8177, 17.8100)
(17)	**BNIP**	17.3138	0.1901	(16.9397, 17.6879)	(16.8210, 17.8066)
	BIP	16.3371	0.0658	(16.2075, 16.4666)	(16.1664, 16.5077)
θ_1	**LB**	7.6331	0.4487	(6.7501, 8.5161)	(6.4698, 8.7963)
(8.5)	**BNIP**	7.6331	0.4457	(6.7560, 8.5101)	(6.4777, 8.7884)
	BIP	10.0000	0.0009	(9.9983, 10.0017)	(9.9978, 10.0022)
θ_2	**LB**	4.4530	0.6863	(3.1023, 5.8037)	(2.6737, 6.2323)
(5.0)	**BNIP**	4.4530	0.6817	(3.1114, 5.7946)	(2.6857, 6.2202)
	BIP	6.0515	0.3315	(5.3992, 6.7039)	(5.1922, 6.9108)
θ_3	**LB**	1.2668	0.7205	(-0.1511, 2.6847)	(-0.6010, 3.1346)
(2.0)	**BNIP**	1.2668	0.7156	(-0.1415, 2.6751)	(-0.5884, 3.1220)
	BIP	3.7052	0.4348	(2.8497, 4.5607)	(2.5783, 4.8321)

Table 5.125: Moderate Negative Collinearity, ρ = -0.49 and sample size, N=300

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.9782	0.1545	(16.6741,17.2823)	(16.5776, 17.3788)
(17)	**BNIP**	16.9782	0.1535	(16.6761, 17.2802)	(16.5803, 17.3760)
	BIP	16.0837	0.0705	(15.9450, 16.2225)	(15.9009, 16.2665)
θ_1	**LB**	8.4816	0.2216	(8.0454, 8.9177)	(7.9070, 9.0561)
(8.5)	**BNIP**	8.4816	0.2201	(8.0484, 8.9147)	(7.9109, 9.0522)
	BIP	10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0019)
θ_2	**LB**	5.1194	0.4166	(4.2996, 5.9392)	(4.0394, 6.1993)
(5.0)	**BNIP**	5.1194	0.4138	(4.3051, 5.9336)	(4.0467, 6.1920)
	BIP	5.9368	0.2747	(5.3963, 6.4773)	(5.2248, 6.6487)
θ_3	**LB**	2.0330	0.4127	(1.2208, 2.8451)	(0.9631, 3.1028)
(2.0)	**BNIP**	2.0330	0.4099	(1.2263, 2.8396)	(0.9704, 3.0956)
	BIP	3.0300	0.3583	(2.3249, 3.7350)	(2.1012, 3.9587)

Table 5.126: Moderate Negative Collinearity, ρ = -0.46 and sample size, N=300

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.8575	0.1795	(16.5043,17.2107)	(16.3923, 17.3228)
(17)	**BNIP**	16.8575	0.1783	(16.5067, 17.2083)	(16.3954, 17.3196)
	BIP	16.1425	0.0751	(15.9948, 16.2903)	(15.9480, 16.3371)
θ_1	**LB**	8.8981	0.2458	(8.4143, 9.3818)	(8.2608, 9.5354)
(8.5)	**BNIP**	8.8981	0.2442	(8.4175, 9.3786)	(8.2651, 9.5310)
	BIP	10.0000	0.0008	(9.9984, 10.0015)	(9.9979, 10.0020)
θ_2	**LB**	4.6093	0.4461	(3.7313, 5.4873)	(3.4529, 5.7659)
(5.0)	**BNIP**	4.6093	0.4431	(3.7372, 5.4813)	(3.4605, 5.7580)
	BIP	5.4809	0.2856	(4.9188, 6.0429)	(4.7405, 6.2213)
θ_3	**LB**	2.2096	0.4587	(1.3069, 3.1123)	(1.0204, 3.3988)
(2.0)	**BNIP**	2.2096	0.4556	(1.3130, 3.1062)	(1.0285, 3.3907)
	BIP	2.9434	0.3713	(2.2127, 3.6742)	(1.9809, 3.9060)

Table 5.127: Moderate Negative Collinearity, ρ = -0.36 and sample size, N=300

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.2150	0.1775	(16.8657,17.5642)	(16.7549, 17.6750)
(17)	**BNIP**	17.2150	0.1763	(16.8681, 17.5618)	(16.7580, 17.6719)
	BIP	16.0777	0.0849	(15.9108, 16.2447)	(15.8578, 16.2977)
θ_1	**LB**	8.2925	0.2233	(7.8530, 8.7319)	(7.7135, 8.8714)
(8.5)	**BNIP**	8.2925	0.2218	(7.8560, 8.7289)	(7.7175, 8.8674)
	BIP	10.0000	0.0008	(9.9984, 10.0015)	(9.9979, 10.0020)
θ_2	**LB**	4.8902	0.4233	(4.0572, 5.7233)	(3.7928, 5.9876)
(5.0)	**BNIP**	4.8902	0.4205	(4.0628, 5.7176)	(3.8003, 5.9802)
	BIP	5.7195	0.2916	(5.1458, 6.2932)	(4.9637, 6.4752)
θ_3	**LB**	1.5154	0.3899	(0.7480, 2.2828)	(0.5045, 2.5263)
(2.0)	**BNIP**	1.5154	0.3873	(0.7532, 2.2776)	(0.5113, 2.5195)
	BIP	2.6599	0.3490	(1.9732, 3.3467)	(1.7553, 3.5645)

Table 5.128: Low Negative Collinearity, ρ = -0.20 and sample size, N=300

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.9102	0.1833	(16.5494,17.2710)	(16.4349, 17.3855)
(17)	**BNIP**	16.9102	0.1821	(16.5518, 17.2685)	(16.4381, 17.3822)
	BIP	15.7450	0.1132	(15.5222, 15.9678)	(15.4515, 16.0385)
θ_1	**LB**	8.1574	0.2076	(7.7489, 8.5659)	(7.6193, 8.6955)
(8.5)	**BNIP**	8.1574	0.2062	(7.7517, 8.5631)	(7.6229, 8.6918)
	BIP	10.0000	0.0009	(9.9983, 10.0016)	(9.9978, 10.0022)
θ_2	**LB**	5.0531	0.3980	(4.2699, 5.8364)	(4.0213, 6.0850)
(5.0)	**BNIP**	5.0531	0.3953	(4.2751, 5.8311)	(4.0283, 6.0780)
	BIP	5.6629	0.3033	(5.0661, 6.2597)	(4.8767, 6.4490)
θ_3	**LB**	2.3635	0.4009	(1.5745, 3.1525)	(1.3241, 3.4028)
(2.0)	**BNIP**	2.3635	0.3982	(1.5798, 3.1471)	(1.3311, 3.3958)
	BIP	2.9987	0.3879	(2.2355, 3.7619)	(1.9933, 4.0041)

Table 5.129: Low Negative Collinearity, ρ = -0.17 and sample size, N=300

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	17.2646	0.1960	(16.8789,17.6503)	(16.7565, 17.7727)
(17)	**BNIP**	17.2646	0.1947	(16.8815, 17.6477)	(16.7599, 17.7693)
	BIP	15.9510	0.1278	(15.6995, 16.2025)	(15.6197, 16.2823)
θ_1	**LB**	7.9770	0.2181	(7.5478, 8.4062)	(7.4116, 8.5425)
(8.5)	**BNIP**	7.9770	0.2166	(7.5507, 8.4034)	(7.4154, 8.5386)
	BIP	10.0000	0.0009	(9.9982, 10.0018)	(9.9976, 10.0023)
θ_2	**LB**	4.8610	0.4198	(4.0348, 5.6873)	(3.7726, 5.9494)
(5.0)	**BNIP**	4.8610	0.4170	(4.0404, 5.6817)	(3.7800, 5.9421)
	BIP	5.5270	0.3247	(4.8881, 6.1660)	(4.6854, 6.3687)
θ_3	**LB**	1.7794	0.4282	(0.9366, 2.6222)	(0.6692, 2.8897)
(2.0)	**BNIP**	1.7794	0.4254	(0.9423, 2.6165)	(0.6767, 2.8822)
	BIP	2.4966	0.4199	(1.6703, 3.3230)	(1.4081, 3.5851)

Table 5.130: Low Negative Collinearity, ρ = -0.15 and sample size, N=300

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0	**LB**	16.9567	0.1664	(16.6292,17.2842)	(16.5253,17.38812)
(17)	**BNIP**	16.9567	0.1653	(16.6314, 17.2820)	(16.5282, 17.3852)
	BIP	16.1131	0.1092	(15.8983, 16.3279)	(15.8302, 16.3961)
θ_1	**LB**	8.5423	0.2029	(8.1431, 8.9416)	(8.0164, 9.0682)
(8.5)	**BNIP**	8.5423	0.2015	(8.1458, 8.9389)	(8.0200, 9.0647)
	BIP	10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0020)
θ_2	**LB**	5.1829	0.3944	(4.4066, 5.9592)	(4.1603, 6.2055)
(5.0)	**BNIP**	5.1829	0.3918	(4.4119, 5.9540)	(4.1672, 6.1986)
	BIP	5.5460	0.2889	(4.9774, 6.1146)	(4.7970, 6.2949)
θ_3	**LB**	1.9835	0.3782	(1.2392, 2.7279)	(1.0030, 2.9641)
(2.0)	**BNIP**	1.9835	0.3757	(1.2442, 2.7228)	(1.0097, 2.9574)
	BIP	2.3129	0.3600	(1.6046, 3.0213)	(1.3799, 3.2460)

From Tables 5.100-5.5.130, the following observations were made when the sample sizes are 100, 200 and 300:

BIP has the smallest SE values for all the parameters considered in HNC, MNC and LNC. The values of SE obtained for sample sizes of 100, 200 and 300 are smaller compared to the SE of sample sizes, 10, 30 and 70 obtained in tables 5.68 – 5.97 show that the increase in sample sizes has great effect on multicollinearity.

The mean estimates of LB and BNIP are closer to the true parameter values compared to smaller sample sizes of 10, 30 and 70. For instance, when the sample size is 10, the mean is 16.5411 while the mean for sample size of 300 is 17.3138 for parameter θ_0 in HNC ($\rho = -0.80$) Also for sample size of 10, the mean is 16.1833 while for sample size of 300, the mean is 16.9102 for parameter θ_0 in LNC ($\rho = -0.20$).

Bayesian estimators (BNIP and BIP) have a smaller SE than LB for the sample sizes (100, 200 and 300) considered. HNC and MNC have the highest values of SE for sample sizes 100, 200 and 300 compared to LNC.

The CI also supported the evidence provided by SE in terms of performance of the estimators; CI of LB has a wider CI at both 95% and 99% than the Bayesian estimators (BIP and BNIP) for HNC, MNC and LNC. For instance, the CI at 95% are (8.1431≤ CI ≤8.9416), (8.1458≤ CI ≤8.9389) and (9.9984≤ CI ≤10.0016) for LB, BNIP and BIP, respectively for sample size, N=300 in LNC ($\rho = -0.15$) for parameter θ_1. Hence, Bayesian estimators outperformed Likelihood based estimator in large sample for all levels of negative collinearity considered (HNC, MNC and LNC).

It is also observed that CI of large samples is more compact for all the estimators compared to CI obtained in small sample sizes, this means that increase in sample sizes can be a solution to the problem of multicollineairty.

Table 5.131: Summary of Tables 5.122-5.130 for Standard Error for sample size, N= 300.

Parameters	Estimators	-0.95	-0.90	-0.80	-0.49	-0.46	-0.36	-0.20	-0.17	-0.15
	LB	0.1783	0.1825	0.1914	0.1545	0.1795	0.1775	0.1833	0.1960	0.1664
θ_0	**BNIP**	0.1771	0.1813	0.1901	0.1535	0.1783	0.1763	0.1821	0.1947	0.1653
	BIP	0.0858	0.0726	0.0658	0.0705	0.0751	0.0849	0.1132	0.1278	0.1092
	LB	0.8281	0.6321	0.4487	0.2216	0.2458	0.2233	0.2076	0.2181	0.2029
θ_1	**BNIP**	0.8226	0.6279	0.4457	0.2201	0.2442	0.2218	0.2062	0.2166	0.2015
	BIP	0.0008	0.0008	0.0009	0.0008	0.0008	0.0008	0.0009	0.0009	0.0008
	LB	1.3195	0.8800	0.6863	0.4166	0.4461	0.4233	0.3980	0.4198	0.3944
θ_2	**BNIP**	1.3107	0.8741	0.6817	0.4138	0.4431	0.4205	0.3953	0.4170	0.3918
	BIP	0.3455	0.3198	0.3315	0.2747	0.2856	0.2916	0.3033	0.3247	0.2889
	LB	1.2601	0.9258	0.7205	0.4127	0.4587	0.3899	0.4009	0.4282	0.3782
θ_3	**BNIP**	1.2517	0.9196	0.7156	0.4099	0.4556	0.3873	0.3982	0.4254	0.3757
	BIP	0.4380	0.4102	0.4348	0.3583	0.3713	0.3490	0.3879	0.4199	0.3600

Table 5.131 shows the summary of SE for multicollinearity (HNC, MNC and LNC) of the estimators across the parameters (θ_0, θ_1, θ_2 and θ_3) when the sample size is 300.

As ρ increases, SE of the estimators also decreases for parameters θ_1, θ_2 and θ_3. Large SE is also observed in high and moderate negative collinearity. The SE of the estimators are smaller compared to when the sample sizes are 10, 30, 70, 100 and 200.

Table 5.132: Summary of Tables 5.100-5.108 for Mean for sample size, N= 300.

Parameters	Estimators	-0.95	-0.90	0.80	-0.49	-0.46	-0.36	-0.20	-0.17	-0.15
θ_0 (17.00)	**LB**	17.0489	17.1179	17.3138	16.9782	16.8575	17.2150	16.9102	17.1721	16.9567
	BNIP	17.0489	17.1179	17.3138	16.9782	16.8575	17.2150	16.9102	17.1721	16.9567
	BIP	16.7771	16.5510	16.3371	16.0837	16.1425	16.0777	15.7450	15.9510	16.1131
θ_1 (8.5)	**LB**	8.9103	8.0066	7.6331	8.4816	8.8981	8.2925	8.1574	7.9770	8.5423
	BNIP	8.9103	8.0066	7.6331	8.4816	8.8981	8.2925	8.1574	7.9770	8.5423
	BIP	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000
θ_2 (5.00)	**LB**	4.9761	4.4146	4.4530	5.1194	4.6093	4.8902	5.0531	4.8610	5.1829
	BNIP	4.9761	4.4146	4.4530	5.1194	4.6093	4.8902	5.0531	4.8610	5.1829
	BIP	5.8495	5.8495	6.0515	5.9368	5.4809	5.7195	5.6629	5.5270	5.5460
θ_3 (2.00)	**LB**	2.9407	1.2559	1.2668	2.0330	2.2096	1.5254	2.3635	1.7794	1.9835
	BNIP	2.9407	1.2559	1.2668	2.0330	2.2096	1.5254	2.3635	1.7794	1.9835
	BIP	3.8656	3.4357	3.7052	3.0300	2.9434	2.6599	2.9987	2.4966	2.3129

Table 5.132 shows the summary of mean for tables 5.120-5.128 when the sample size is 300. The means of LB and BNIP are the same for all the levels of collinearity across the parameters. All the means of the estimator are positive. The means of parameter $\theta_0$are not too far from the true parameter. LIB and BNIP outperformed all other estimators having the closest mean values to the true parameter value.

5.3Effects of Replications on the Estimators in the Presence of Multicollinearity using Posterior Simulation (MCI) Method

Table 5.133: High Positive Collinearity, ρ = 0.90 and sample size, N=10

		Mean	SE	95% CI	99% CI
MCI(1000)		16.5029	0.3780	(15.7185, 17.2873)	(15.4142, 17.5916)
MCI(10000)	θ_0= 17	16.5214	0.3936	(15.7370, 17.3058)	(15.4327, 17.6101)
MCI(100000)		16.5191	0.3947	(15.7347, 17.3035)	(15.4304, 17.6079)
Analytical		16.5191	0.3657	(15.7347, 17.3035)	(15.4303, 17.6078)
MCI(1000)		10.0000	0.0008	(9.9984, 10.0016)	(9.9978, 10.0023)
MCI(10000)	θ_1=8.5	10.0000	0.0008	(9.9984, 10.0016)	(9.9978, 10.0022)
MCI(100000)		10.0000	0.0008	(9.9984, 10.0016)	(9.9978 ,10.0022)
Analytical		10.0000	0.0007	(9.9984, 10.0016)	(9.9978, 10.0022)
MCI(1000)		5.3709	0.3771	(4.5821, 6.1596)	(4.2761, 6.4656)
MCI(10000)	θ_2=5.0	5.3682	0.3966	(4.5795, 6.1570)	(4.2735, 6.4630)
MCI(100000)		5.3801	0.3977	(4.5913, 6.1688)	(4.2853, 6.4748)
Analytical		5.3799	0.3677	(4.5912, 6.1687)	(4.2852, 6.4747)
MCI(1000)		1.8813	0.7348	(0.4108, 3.3518)	(-0.1597, 3.9223)
MCI(10000)	θ_3=2.0	1.8395	0.7355	(0.3690, 3.3100)	(-0.2014, 3.8805)
MCI(100000)		1.8321	0.7412	(0.3616, 3.3026)	(-0.2088, 3.8731)
Analytical		1.8349	0.6856	(0.3644, 3.3054)	(-0.2061, 3.8759)

Table 5.134:Moderate Positive Collinearity, ρ = 0.49 and sample size, N=10

		Mean	SE	95% CI	99% CI
MCI(1000)		15.6825	0.6003	(14.4488, 16.9162)	(13.9702, 17.3948)
MCI(10000)	θ_0= 17	15.6325	0.6343	(14.3988, 16.8661)	(13.9202, 17.3447)
MCI(100000)		15.6376	0.6228	(14.4039, 16.8713)	(13.9253, 17.3499)
Analytical		15.6370	0.5752	(14.4033, 16.8706)	(13.9247, 17.3492)
MCI(1000)		10.0000	0.0012	(9.9977, 10.0024)	(9.9968, 10.0033)
MCI(10000)	θ_1=8.5	10.0000	0.0012	(9.9976, 10.0023)	(9.9967, 10.0033)
MCI(100000)		10.0000	0.0012	(9.9977, 10.0024)	(9.9967, 10.0033)
Analytical		10.0000	0.0011	(9.9976, 10.0023)	(9.9967, 10.0033)
MCI(1000)		5.3132	0.5723	(4.1563, 6.4701)	(3.7074, 6.9189)
MCI(10000)	θ_2=5.0	5.3405	0.5849	(4.1836, 6.4974)	(3.7348, 6.9462)
MCI(100000)		5.3391	0.5808	(4.1822, 6.4960)	(3.7334, 6.9448)
Analytical		5.3373	0.5394	(4.1804, 6.4942)	(3.7316, 6.9430)
MCI(1000)		1.6205	1.0888	(-0.5775, 3.8184)	(-1.4302, 4.6711)
MCI(10000)	θ_3=2.0	1.6214	1.1220	(-0.5766, 3.8193)	(-1.4293, 4.6720)
MCI(100000)		1.6386	1.1047	(-0.5594, 3.8365)	(-1.4121, 4.6892)
Analytical		1.6390	1.0248	(-0.5590, 3.8370)	(-1.4117, 4.6896)

Table 5.135: Low Positive Collinearity, ρ = 0.20 and sample size, N=10

		Mean	SE	95% CI	99% CI
MCI(1000)		16.0117	0.3999	(15.2636, 16.7598)	(14.9733, 17.0500)
MCI(10000)	θ_0= 17	16.0254	0.3766	(15.2773, 16.7735)	(14.9870, 17.0637)
MCI(100000)		16.0183	0.3771	(15.2702, 16.7664)	(14.9800, 17.0566)
Analytical		16.0203	0.3488	(15.2722, 16.7684)	(14.9820, 17.0587)
MCI(1000)		10.0000	0.0008	(9.9985, 10.0015)	(9.9979, 10.0021)
MCI(10000)	θ_1=8.5	10.0000	0.0008	(9.9985, 10.0015)	(9.9979, 10.0021)
MCI(100000)		10.0000	0.0008	(9.9985, 10.0015)	(9.9979, 10.0021)
Analytical		10.0000	0.0007	(9.9985, 10.0015)	(9.9979, 10.0021)
MCI(1000)		5.4598	0.3887	(4.7161, 6.2034)	(4.4276, 6.4919)
MCI(10000)	θ_2=5.0	5.4380	0.3713	(4.6944, 6.1817)	(4.4059, 6.4702)
MCI(100000)		5.4437	0.3734	(4.7000, 6.1874)	(4.4115, 6.4758)
Analytical		5.4410	0.3467	(4.6974,	(4.4089,

				6.1847)	6.4732)
MCI(1000)		2.2723	0.7118	(0.8941, 3.6506)	(0.3593, 4.1853)
MCI(10000)	θ_3=2.0	2.2880	0.6916	(0.9097, 3.6663)	(0.3750, 4.2010)
MCI(100000)		2.2941	0.6936	(0.9158, 3.6724)	(0.3811, 4.2071)
Analytical		2.2905	0.6426	(0.9122, 3.6688)	(0.3775, 4.2035)

Table 5.136: High Positive Collinearity, ρ = 0.90 and sample size, N=30

		Mean	SE	95% CI	99% CI
MCI(1000)		16.5855	0.3234	(15.9572, 17.2137)	(15.7420, 17.4289)
MCI(10000)	θ_0= 17	16.5785	0.3199	(15.9476, 17.2041)	(15.7324, 17.4193)
MCI(100000)		16.5771	0.3183	(15.9489, 17.2053)	(15.7336, 17.4206)
Analytical		16.5785	0.3091	(15.9503, 17.2068)	(15.7351, 17.4220)
MCI(1000)	θ_1=8.5	10.0000	0.0010	(9.9982, 10.0018)	(9.9975, 10.0024)

MCI(10000)		10.0000	0.0009	(9.9982, 10.0018)	(9.9975, 10.0025)
MCI(100000)		10.0000	0.0009	(9.9982, 10.0018)	(9.9975, 10.0025)
Analytical		10.0000	0.0009	(9.9982, 10.0018)	(9.9975, 0.0025)
MCI(1000)		5.3663	0.4526	(4.4848, 6.2478)	(4.1828, 6.5497)
MCI(10000)	θ_2=5.0	5.3574	0.4457	(4.4804, 6.2434)	(4.1785, 6.5454)
MCI(100000)		5.3592	0.4466	(4.4777, 6.2407)	(4.1758, 6.5427)
Analytical		5.3574	0.4338	(4.4759, 6.2389)	(4.1740, 6.5409)
MCI(1000)		1.3186	0.7907	(-0.2022, 2.8394)	(-0.7231, 3.3604)
MCI(10000)	θ_3=2.0	1.3367	0.7786	(-0.1846, 2.8570)	(-0.7055, 3.3780)
MCI(100000)		1.3382	0.7724	(-0.1826, 2.8590)	(-0.7035, 3.3800)
Analytical		1.3367	0.7483	(-0.1841, 2.8575)	(-0.7051, 3.3785)

Table 5.137: Moderate Positive Collinearity, ρ = 0.49 and sample size, N=30

		Mean	SE	95% CI	99% CI

MCI(1000)		16.1713	0.2727	(15.6254, 16.7172)	(15.4384, 16.9042)
MCI(10000)	θ_0= 17	16.1621	0.2793	(15.6161, 16.7080)	(15.4291, 16.8950)
MCI(100000)		16.1629	0.2769	(15.6170, 16.7088)	(15.4300, 16.8958)
Analytical		16.1627	0.2686	(15.6168, 16.7086)	(15.4298, 16.8956)
MCI(1000)		10.0000	0.0007	(9.9987, 10.0014)	(9.9982, 10.0018)
MCI(10000)	θ_1=8.5	10.0000	0.0007	(9.9987, 10.0013)	(9.9982, 10.0018)
MCI(100000)		10.0000	0.0007	(9.9987, 10.0013)	(9.9982, 10.0018)
Analytical		10.0000	0.0007	(9.9987, 10.0013)	(9.9982, 10.0018)
MCI(1000)		5.2617	0.3143	(4.6226, 5.9008)	(4.4036, 6.1197)
MCI(10000)	θ_2=5.0	5.2577	0.3272	(4.6186, 5.8968)	(4.3997, 6.1157)
MCI(100000)		5.2549	0.3242	(4.6158, 5.8940)	(4.3968, 6.1129)
Analytical		5.2558	0.3145	(4.6167, 5.8949)	(4.3977, 6.1138)
MCI(1000)		1.5526	0.5804	(0.4153, 2.6899)	(0.0257, 3.0795)
MCI(10000)	θ_3=2.0	1.5742	0.5760	(0.4369, 2.7115)	(0.0473, 3.1011)
MCI(100000)		1.5742	0.5760	(0.4369, 2.7115)	(0.0473, 3.1011)
Analytical		1.5739	0.5596	(0.4366, 2.7112)	(0.0470, 3.1008)

Table 5.138: Low Positive Collinearity, ρ = 0.20 and sample size, N=30

		Mean	SE	95% CI	99% CI
MCI(1000)		16.3854	0.2844	(15.8317, 16.9391)	(15.6420, 17.1288)
MCI(10000)	θ_0= 17	16.3835	0.2808	(15.8298, 16.9372)	(15.6401, 17.1269)
MCI(100000)		16.3869	0.2792	(15.8331, 16.9406)	(15.6434, 17.1303)
Analytical		16.3871	0.2725	(15.8334, 16.9408)	(15.6437, 17.1305)
MCI(1000)		10.0000	0.0008	(9.9985, 10.0016)	(9.9979, 10.0021)
MCI(10000)	θ_1=8.5	10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
MCI(100000)		10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
Analytical		10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
MCI(1000)		5.4554	0.3878	(4.7013, 6.2095)	(4.4430, 6.4678)
MCI(10000)	θ_2=5.0	5.4305	0.3781	(4.6764, 6.1846)	(4.4181, 6.4429)
MCI(100000)		5.4285	0.3827	(4.6744, 6.1826)	(4.4160, 6.4409)

Analytical		5.4291	0.3711	(4.6750, 6.1832)	(4.4167, 6.4415)
MCI(1000)		2.4485	0.6340	(1.1901, 3.7069)	(0.7590, 4.1380)
MCI(10000)	θ_3=2.0	2.4689	0.6326	(1.2105, 3.7274)	(0.7794, 4.1584)
MCI(100000)		2.4622	0.6361	(1.2038, 3.7206)	(0.7727, 4.1517)
Analytical		2.4619	0.6192	(1.2035, 3.7203)	(0.7724, 4.1514)

The following observations were made from Tables 5.133-5.138;

Bayesian Analytical method has the minimum standard error in the case of HPC, MPC and LPC for sample sizes of 10 and 30 compared to Bayesian posterior simulation method; for instance in Table 5.133, SE for parameters θ_0 , θ_1 , θ_2 and θ_3 are 0.3657, 0.007, 0.3677 and 0.6856 respectively. The standard error of Bayesian posterior simulation method (MCI) reduces with increase number of replications as observed in Tables 5.134, 5.135, 5.136 and 5.138.

The mean of analytical and Bayesian posterior simulation approaches for parameter θ_1 are the same for all the three levels of collinearity for sample size of 10 while the mean of Bayesian posterior simulation (1000, 10000 and 100000) and Bayesian analytical are not too far from the true parameter values. The posterior means all fall within the credible intervals.

Bayesian Analytical method has narrower CI at both 95% and 99% than Bayesian Posterior simulation (1000, 10000 and 100000 replications) for all the parameters considered in HPC, MPC and LPC. As the replications of Bayesian Posterior simulation method (MCI) increases, the estimate tend towards the estimates of Bayesian Analytical method. Thus, increase in replications has great effect on the estimator in the presence of multicollinearity.

Table 5.139: High Positive Collinearity, ρ = 0.90 and sample size, N=70

		Mean	SE	95% CI	99% CI
MCI(1000)		16.2645	0.2413	(15.7872, 16.7419)	(15.6311, 16.8980
MCI(10000)	θ_0= 17	16.2662	0.2427	(15.7888, 16.7436)	(15.6328, 16.8997)
MCI(100000)		16.2704	0.2438	(15.7930, 16.7478)	(15.6370, 16.9039)
Analytical		16.2691	0.2396	(15.7917, 16.7465)	(15.6356, 16.9025)
MCI(1000)		10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0022)
MCI(10000)	θ_1=8.5	10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0022)
MCI(100000)		10.0000	0.0008	(9.9984, 10.0016)	(9.9978, 10.0021)
Analytical		10.0000	0.0008	(9.9984, 10.0016)	(9.9978, 10.0021)
MCI(1000)	θ_2=5.0	5.2308	0.3927	(4.4651, 5.9964)	(4.2148, 6.2467)

MCI(10000)		5.2307	0.3888	(4.4651, 5.9964)	(4.2148, 6.2467)
MCI(100000)		5.2300	0.3886	(4.4643, 5.9956)	(4.2140, 6.2459)
Analytical		5.2293	0.3843	(4.4636, 5.9949)	(4.2133, 6.2452)
MCI(1000)		1.1821	0.6139	(-0.0039, 2.3680)	(-0.3916, 2.7557)
MCI(10000)	θ_3=2.0	1.1743	0.6079	(-0.0116, 2.3602)	(-0.3993, 2.7479)
MCI(100000)		1.1703	0.6021	(-0.0156, 2.3562)	(-0.4033, 2.7439)
Analytical		1.1752	0.5952	(-0.0107, 2.3611)	(-0.3984, 2.7488)

Table 5.140: Moderate Positive Collinearity, ρ = 0.49 and sample size, N=70

		Mean	SE	95% CI	99% CI
MCI(1000)		16.2790	0.2684	(15.7548, 16.8031)	(15.5835, 16.9745)
MCI(10000)	θ_0= 17	16.2918	0.2680	(15.7677, 16.8160)	(15.5963, 16.9873)
MCI(100000)		16.2917	0.2670	(15.7675, 16.8158)	(15.5962, 16.9872)
Analytical		16.2896	0.2667	(15.7917, 16.7465)	(15.5941, 16.9851)
MCI(1000)	θ_1=8.5	10.0000	0.0008	(9.9983, 10.0017)	(9.9978, 10.0022)

MCI(10000)		10.0000	0.0009	(9.9983, 10.0017)	(9.9978, 10.0022)
MCI(100000)		10.0000	0.0008	(9.9983, 10.0017)	(9.9978, 10.0022)
Analytical		10.0000	0.0008	(9.9983, 10.0017)	(9.9978, 10.0022)
MCI(1000)		5.1290	0.3616	(4.3683, 5.8897)	(4.1197, 6.1383)
MCI(10000)	θ_2=5.0	5.1214	0.3869	(4.3608, 5.8821)	(4.1121, 6.1307)
MCI(100000)		5.1198	0.3875	(4.3592, 5.8805)	(4.1105, 6.1292)
Analytical		5.1215	0.3870	(4.3608, 5.8822)	(4.1122, 6.1308)
MCI(1000)		1.7422	0.6271	(0.5249, 2.9596)	(0.1269, 3.3575)
MCI(10000)	θ_3=2.0	1.7162	0.6265	(0.4989, 2.9336)	(0.1009, 3.3315)
MCI(100000)		1.7178	0.6193	(0.5005, 2.9352)	(0.1025, 3.3331)
Analytical		1.1752	0.6194	(0.5038, 2.9385)	(0.1058, 3.3364)

Table 5.141: Low Positive Collinearity, ρ = 0.20 and sample size, N=70

		Mean	SE	95% CI	99% CI
MCI(1000)		16.3002	0.2526	(15.8031, 16.7973)	(15.6406, 16.9598)
MCI(10000)	θ_0= 17	16.3043	0.2544	(15.8072, 16.8014)	(15.6447, 16.9639)
MCI(100000)		16.3015	0.2531	(15.8044, 16.7986)	(15.6419, 16.9611)
Analytical		16.3027	0.2495	(15.8056, 16.7998)	(15.6431, 16.9623)
MCI(1000)		10.0000	0.0009	(9.9983, 10.0017)	(9.9977, 10.0023)
MCI(10000)	θ_1=8.5	10.0000	0.0009	(9.9983, 10.0017)	(9.9977, 10.0023)
MCI(100000)		10.0000	0.0009	(9.9983, 10.0017)	(9.9977, 10.0023)
Analytical		10.0000	0.0009	(9.9983, 10.0017)	(9.9977, 10.0023)
MCI(1000)		5.2815	0.4011	(4.4919, 6.0712)	(4.2338, 6.3293)
MCI(10000)	θ_2=5.0	5.2796	0.4029	(4.4900, 6.0692)	(4.2318, 6.3274)
MCI(100000)		5.2797	0.4015	(4.4901, 6.0693)	(4.2320, 6.3275)
Analytical		5.2790	0.3963	(4.4893, 6.0686)	(4.2312, 6.3267)
MCI(1000)		1.6298	0.6426	(0.3795, 2.8801)	(-0.0292, 3.2888)
MCI(10000)	θ_3=2.0	1.6172	0.6388	(0.3669, 2.8675)	(-0.0418, 3.2762)
MCI(100000)		1.6284	0.6368	(0.3781, 2.8787)	(-0.0306, 3.2874)

Analytical	1.6263	0.6275	(0.3760, 2.8766)	(-0.0327, 3.2853)

Table 5.142: High Positive Collinearity, ρ = 0.90 and sample size, N=100

		Mean	SE	95% CI	99% CI
MCI(1000)		16.6682	0.2320	(16.2081, 17.1282)	(16.0594, 17.2769)
MCI(10000)	θ_0= 17	16.6648	0.2334	(16.2119, 17.1320)	(16.0632, 17.2807)
MCI(100000)		16.6676	0.2350	(16.2075, 17.1276)	(16.0588, 17.2763)
Analytical		16.6678	0.2334	(16.2078, 17.1279)	(16.0591, 17.2766)
MCI(1000)		10.0000	0.0009	(9.9983, 10.0016)	(9.9977, 10.0022)
MCI(10000)	θ_1=8.5	10.0000	0.0008	(9.9983, 10.0017)	(9.9978, 10.0022)
MCI(100000)		10.0000	0.0009	(9.9983,10.0017)	(9.9978, 10.0022)
Analytical		10.0000	0.0008	(9.9983, 10.0017)	(9.9978, 10.0022)
MCI(1000)		4.92249	0.3871	(4.1713, 5.6786)	(3.9277, 5.9222)
MCI(10000)	θ_2=5.0	4.9284	0.3830	(4.1680, 5.6754)	(3.9245, 5.9189))
MCI(100000)		4.9275	0.3833	(4.1739, 5.6812)	(3.9303, 5.9248)
Analytical		4.9277	0.3825	(4.1740, 5.6813)	(3.9304, 5.9249)
MCI(1000)	θ_3=2.0	1.2325	0.5660	(0.0968, 2.3681)	(-0.2702, 2.7352)
MCI(10000)		1.2372	0.5742	(0.0898, 2.3611)	(-0.2772, 2.7282)

MCI(100000)		1.2349	0.5789	(0.0992, 2.3705)	(-0.2678, 2.7376)
Analytical		1.2330	0.5727	(0.0974, 2.3687)	(-0.2697, 2.7357)

Table 5.143: Moderate Positive Collinearity, ρ = 0.49 and sample size, N=100

		Mean	SE	95% CI	99% CI
MCI(1000)		16.2675	0.6116	(15.1201, 17.4149)	(14.7493, 17.7857)
MCI(10000)	θ_0= 17	16.2741	0.5824	(15.1267, 17.4215)	(14.7559, 17.7923)
MCI(100000)		16.2835	0.5844	(15.1361, 17.4309)	(14.7653, 17.8017)
Analytical		16.2830	0.5786	(15.1356, 17.4304)	(14.7648, 17.8012)
MCI(1000)		9.9998	0.0022	(9.9955, 10.0041)	(9.9942, 10.0055)
MCI(10000)	θ_1=8.5	10.0000	0.0022	(9.9957, 10.0043)	(9.9943, 10.0057)
MCI(100000)		10.0000	0.0022	(9.9957, 10.0043)	(9.9943, 10.0057)
Analytical		10.0000	0.0022	(9.9957, 10.0043)	(9.9943, 10.0057)
MCI(1000)		4.9701	0.9351	(3.0954, 6.8449)	(2.4896, 7.4507)
MCI(10000)	θ_2=5.0	4.9869	0.9471	(3.1122, 6.8616)	(2.5063, 7.4675)
MCI(100000)		4.9835	0.9565	(3.1088, 6.8582)	(2.5029, 7.4641)
Analytical		4.9845	0.9454	(3.1098, 6.8592)	(2.5039, 7.4651)
MCI(1000)	θ_3=2.0	1.4114	1.4859	(-1.4584, 4.2812)	(-2.3859, 5.2086)

MCI(10000)		1.3920	1.4454	(-1.4778, 4.2617)	(-2.4053, 5.1892)
MCI(100000)		1.3886	1.4637	(-1.4812, 4.2583)	(-2.4087, 5.1858)
Analytical		1.3869	1.4472	(-1.4829, 4.2567)	(-2.4104, 5.1842)

Table 5.144: Low Positive Collinearity, ρ = 0.20 and sample size, N=100

		Mean	SE	95% CI	99% CI
MCI(1000)		16.2152	0.2158	(15.7792, 16.6512)	(15.6383, 16.7921)
MCI(10000)	θ_0= 17	16.2144	0.2217	(15.7784, 16.6504)	(15.6375, 16.7913)
MCI(100000)		16.2141	0.2221	(15.7781, 16.6501)	(15.6372, 16.7910)
Analytical		16.2146	0.2199	(15.7786, 16.6506)	(15.6377, 16.7915)
MCI(1000)		10.0000	0.0008	(9.9984, 10.0016)	(9.9978, 10.0022)
MCI(10000)	θ_1=8.5	10.0000	0.0008	(9.9983, 10.0016)	(9.9978, 10.0021)
MCI(100000)		10.0000	0.0008	(9.9984, 10.0016)	(9.9978, 10.0022)
Analytical		10.0000	0.0008	(9.9984, 10.0016)	(9.9978, 10.0022)
MCI(1000)		5.2801	0.3725	(4.5667, 5.9936)	(4.3361, 6.2242)
MCI(10000)	θ_2=5.0	5.2877	0.3655	(4.5742, 6.0011)	(4.3437, 6.2317)
MCI(100000)		5.2872	0.3630	(4.5737, 6.0006)	(4.3431, 6.2312)
Analytical		5.2852	0.3598	(4.5717, 5.9986)	(4.3411, 6.2292)

MCI(1000)		2.1154	0.5659	(1.0195, 3.2113)	(0.6653, 3.5655)
MCI(10000)	θ_3=2.0	2.1246	0.5550	(1.0287, 3.2205)	(0.6745, 3.5747)
MCI(100000)		2.1228	0.5582	(1.0269, 3.2187)	(0.6727, 3.5729)
Analytical		2.1230	0.5526	(1.0271, 3.2189)	(0.6729, 3.5731)

Table 5.145: High Positive Collinearity, ρ = 0.90 and sample size, N=200

		Mean	SE	95% CI	99% CI
MCI(1000)		16.8721	0.1639	(16.5536, 17.1906)	(16.4521, 17.2921)
MCI(10000)	θ_0= 17	16.8697	0.1609	(16.5512, 17.1882)	(16.4497, 17.2897)
MCI(100000)		16.8702	0.1630	(16.5517, 17.1887)	(16.4502, 17.2902)
Analytical		16.8698	0.1615	(16.5513, 17.1883)	(16.4498, 17.2898)
MCI(1000)		10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
MCI(10000)	θ_1=8.5	10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
MCI(100000)		10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
Analytical		10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
MCI(1000)		4.9436	0.3343	(4.2620, 5.6252)	(4.0447, 5.8424)
MCI(10000)	θ_2=5.0	4.9523	0.3514	(4.2707, 5.6339)	(4.0535, 5.8512)
MCI(100000)		4.9495	0.3480	(4.2679, 5.6311)	(4.0507, 5.8484)

Analytical		4.9483	0.3457	(4.2667, 5.6299)	(4.0494, 5.8471)
MCI(1000)		0.4844	0.4704	(-0.4250, 1.3938)	(-0.7149, 1.6837)
MCI(10000)	θ_3=2.0	0.4811	0.4631	(-0.4283, 1.3905)	(-0.7182, 1.6804)
MCI(100000)		0.4825	0.4651	(-0.4269, 1.3919)	(-0.7168, 1.6818]
Analytical		0.4838	0.4612	(-0.4256, 1.3932)	(-0.7155, 1.6831)

Table 5.146: Moderate Positive Collinearity, ρ = 0.49 and sample size, N=200

		Mean	SE	95% CI	99% CI
MCI(1000)		16.6693	0.1974	(16.3005, 17.0381	(16.1830, 17.1557)
MCI(10000)	θ_0= 17	16.6721	0.1911	(16.3033, 17.0409)	(16.1858, 17.1584)
MCI(100000)		16.6718	0.1883	(16.3030, 17.0406)	(16.1855, 17.1581)
Analytical		16.6711	0.1870	(16.3023, 17.0398)	(16.1847, 17.1574)
MCI(1000)		10.0000	0.0008	(9.9984, 10.0016)	(9.9978, 10.0022)
MCI(10000)	θ_1=8.5	10.0000	0.0008	(9.9983, 10.0016)	(9.9978, 10.0021)
MCI(100000)		10.0000	0.0008	(9.9983, 10.0016)	(9.9978, 10.0021)
Analytical		10.0000	0.0008	9.9983, 10.0016)	(9.9978, 10.0021)
MCI(1000)	θ_2=5.0	4.7865	0.3430	(4.1359, 5.4370)	(3.9286, 5.6443)
MCI(10000)		4.7652	0.3275	(4.1147, 5.4157)	(3.9073, 5.6230)

MCI(100000)		4.7691	0.3311	(4.1186, 5.4196)	(3.9112, 5.6270)
Analytical		4.7703	0.3299	(4.1198, 5.4208)	(3.9124, 5.6281)
MCI(1000)		1.0855	0.4676	(0.1761, 1.9948)	(-0.1137, 2.2847)
MCI(10000)	θ_3=2.0	1.0966	0.4679	(0.1873, 2.0060)	(-0.1026, 2.2958)
MCI(100000)		1.0956	0.4637	(0.1863, 2.0050)	(-0.1036, 2.2948)
Analytical		1.0956	0.4612	(0.1863, 2.0050)	(-0.1036, 2.2948)

Table 5.147: Low Positive Collinearity, ρ = 0.20 and sample size, N=200

		Mean	SE	95% CI	99% CI
MCI(1000)		16.4221	0.1769	(16.0712, 16.7731)	(15.9593, 16.8849)
MCI(10000)	θ_0= 17	16.4239	0.1790	(16.0730, 16.7749)	(15.9611, 16.8868)
MCI(100000)		16.4243	0.1797	(16.0734, 16.7753)	(15.9615, 16.8871)
Analytical		16.4234	0.1780	(16.0724, 16.7743)	(15.9606, 16.8862)
MCI(1000)		10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
MCI(10000)	θ_1=8.5	10.0000	0.0008	(9.9984 10.0016)	(9.9979, 10.0021)
MCI(100000)		10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
Analytical		10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
MCI(1000)	θ_2=5.0	5.2854	0.3255	(4.6568, 5.9140)	(4.4564, 6.1144)

MCI(10000)		5.2807	0.3188	(4.6520 5.9093)	(4.4516, 6.1097)
MCI(100000)		5.2828	0.3212	(4.6542, 5.9114)	(4.4538, 6.1118)
Analytical		5.2829	0.3188	(4.6542, 5.9115)	(4.4539, 6.1119)
MCI(1000)		1.4854	0.4416	(0.6243, 2.3464)	(0.3499, 2.6209)
MCI(10000)	θ_3=2.0	1.4720	0.4381	(0.6110, 2.3331)	(4.4516, 6.1097)
MCI(100000)		1.4713	0.4389	(0.6102, 2.3323)	(0.3358, 2.6068)
Analytical		1.4727	0.4367	(0.6117, 2.3338)	(0.3372, 2.6083)

Table 5.148: High Positive Collinearity, ρ = 0.90 and sample size, N=300

		Mean	SE	95% CI	99% CI
MCI(1000)		17.0142	0.1300	(16.7689, 17.2595)	(16.6910, 17.3374)
MCI(10000)	θ_0= 17	17.0190	0.1242	(16.7737, 17.2643)	(16.6959, 17.3422)
MCI(100000)		17.0192	0.1258	(16.7739, 17.2646)	(16.6961, 17.3424)
Analytical		17.0194	0.1247	(16.7741, 17.2647)	(16.6962, 17.3425)
MCI(1000)		10.0000	0.0007	(9.9985, 10.0014)	(9.9980, 10.0019)
MCI(10000)	θ_1=8.5	10.0000	0.0007	(9.9985, 10.0015)	(9.9981, 10.0019)
MCI(100000)		10.0000	0.0007	(9.9985, 10.0015)	(9.9981, 10.0019)
Analytical		10.0000	0.0007	(9.9985, 10.0015)	(9.9981, 10.0019)

MCI(1000)		4.7073	0.3102	(4.1085, 5.3061)	(3.9186, 5.4960)
MCI(10000)	θ_2=5.0	4.7040	0.3063	(4.1052, 5.3028)	(3.9153, 5.4927)
MCI(100000)		4.6983	0.3043	(4.0996, 5.2971)	(3.9096, 5.4871)
Analytical		4.6994	0.3043	(4.1006, 5.2981)	(3.9106, 5.4881)
MCI(1000)		-0.0918	0.4007	(-0.8513, 0.6677)	(-1.0922, 0.9086)
MCI(10000)	θ_3=2.0	-0.0935	0.3888	(-0.8529, 0.6660)	(-1.0939, 0.9070)
MCI(100000)		-0.0926	0.3876	(-0.8521, 0.6668)	[-1.0931, 0.9078)
Analytical		-0.0930	0.3860	(-0.8525, 0.6665)	(-1.0935, 0.9074)

Table 5.149: Moderate Positive Collinearity, ρ = 0.49 and sample size, N=300

		Mean	SE	95% CI	99% CI
MCI(1000)		16.9316	0.1649	(16.6142, 17.2489)	(16.5135, 17.3496)
MCI(10000)	θ_0= 17	16.9276	0.1608	(16.6102, 17.2450)	(16.5096, 17.3456)
MCI(100000)		16.9276	0.1618	(16.6092, 17.2439)	(16.5085, 17.3446)
Analytical		16.9271	0.1613	(16.6098, 17.2445)	(16.5091, 17.3452)
MCI(1000)	θ_1=8.5	10.0000	0.0008	(9.9984,	(9.9979,

				10.0016)	10.0021)
MCI(10000)		10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
MCI(100000)		10.0000	0.0008	(9.9984, 10.0016)	(9.9978, 10.0021)
Analytical		10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
MCI(1000)		4.4278	0.3175	(3.8248, 5.0309)	(3.6334, 5.2223)
MCI(10000)	θ_2=5.0	4.4304	0.3082	(3.8273, 5.0335)	(3.6360, 5.2248)
MCI(100000)		4.4304	0.3080	(3.8272, 5.0334)	(3.6359, 5.2247)
Analytical		4.4304	0.3065	(3.8273, 5.0335)	(3.6360, 5.2248)
MCI(1000)		0.6348	0.3724	(-0.1098, 1.3794)	(-0.3460, 1.6157)
MCI(10000)	θ_3=2.0	0.6373	0.3739	(-0.1073, 1.3819)	(-0.3436, 1.6181)
MCI(100000)		0.6379	0.3792	(-0.1067, 1.3825)	(-0.3429, 1.6187)
Analytical		0.6372	0.3784	(-0.1074, 1.3818)	(-0.3436, 1.6180)

Table 5.150: Low Positive Collinearity, ρ = 0.20 and sample size, N=300

		Mean	SE	95% CI	99% CI
MCI(1000)		16.9316	0.1642	(16.1446, 16.7588)	(16.0472, 16.8562)
MCI(10000)	θ_0= 17	16.4494	0.1564	(16.1424, 16.7565)	(16.0450, 16.8539)
MCI(100000)		16.4524	0.1570	(16.1453, 16.7594)	(16.0479, 16.8568)
Analytical		16.4520	0.1560	(16.1449, 16.7591)	(16.0475, 16.8565)
MCI(1000)		10.0000	0.0008	(9.9983, 10.0016)	(9.9978, 10.0022)
MCI(10000)	θ_1=8.5	10.0000	0.0009	(9.9983, 10.0016)	(9.9978, 10.0022)
MCI(100000)		10.0000	0.0009	(9.9983, 10.0016)	(9.9978, 10.0022)
Analytical		10.0000	0.0008	(9.9983, 10.0016)	(9.9978, 10.0022)
MCI(1000)		4.4278	0.3098	(4.3873, 5.5877)	(4.1968, 5.7781)
MCI(10000)	θ_2=5.0	4.9882	0.3075	(4.3879, 5.5884)	(4.1975, 5.7788)
MCI(100000)		4.9874	0.3048	(4.3872, 5.5877)	(4.1968, 5.7781)
Analytical		4.9868	0.3050	(4.3866, 5.5870)	(4.1962, 5.7774)
MCI(1000)		1.1323	0.3949	(0.3490, 1.9157)	(0.1004, 2.1642)
MCI(10000)	θ_3=2.0	0.6373	0.3739	(-0.1073, 1.3819)	(-0.3436, 1.6181)
MCI(100000)		1.1267	0.4000	(0.3433,	(0.0948,

				1.9100)	2.1586)
Analytical		1.1284	0.3981	(0.3450, 1.9117)	(0.0965, 2.1603)

The Bayesian analytical method has a narrower CI than the Bayesian Posterior simulation (1000, 10000 and 100000) at both 95% and 99% for all the degrees of collinearity considered as shown in tables 5.139-5.150.The SE of Bayesian posterior simulation for parameter $\theta_1$almost has similar results with Bayesian analytical method in tables 5.139-5.150.

As the replications of Bayesian Posterior simulation method increases, the SE estimates tend towards the estimates of Bayesian analytical method. The mean of Bayesian posterior simulation using (1000, 10000 and 100000) and Bayesian analytical methods are almost the same with the initial values of the simulated data.

The performances of analytical and posterior simulation using MCI improved due to increase in sample sizes, asymptotic effects was noticed as the sample size gets to70. The CI for HPC, MPC and LPC are also almost the same for the two methods (analytical and posterior simulation); for instance in HPC, at 99% CI when the parameter is θ_3 for sample size, N=70 are (-0.3916$\leq CI \leq$2.7557), (-0.3993$\leq CI \leq 2.7479$), (-0.4033$\leq CI \leq 2.7439$) and (-0.3984$\leq CI \leq 2.7488$) for MCI (1000), MCI (10000), MCI (100000) and analytical methods respectively.

Table 5.151: High Negative Collinearity, ρ = -0.90 and sample size, N=10

		Mean	SE	95% CI	99% CI
MCI(1000)		16.2575	0.2929	(15.6924, 16.8225)	(15.4732, 17.0417)
MCI(10000)	θ_0= 17	16.2765	0.2901	(15.7115, 16.8416)	(15.4923, 17.0608)
MCI(100000)		16.2803	0.2848	(15.7152, 16.8453)	(15.4960, 17.0645)
Analytical		16.2791	0.2635	(15.7141, 16.8442)	(15.4948, 17.0634)
MCI(1000)		10.0000	0.0007	(9.9986, 10.0014)	(9.9981, 10.0019)
MCI(10000)	θ_1=8.5	10.0000	0.0007	(9.9987, 10.0014)	(9.9981, 10.0019)
MCI(100000)		10.0000	0.0007	(9.9986, 10.0014)	(9.9981, 10.0019)
Analytical		10.0000	0.0006	(9.9986, 10.0014)	(9.9981, 10.0019)
MCI(1000)		5.5906	0.3448	(4.9252, 6.2560)	(4.6671, 6.5142)
MCI(10000)	θ_2=5.0	5.5910	0.3386	(4.9256, 6.2564)	(4.6674, 6.5145)
MCI(100000)		5.5908	0.3349	(4.9254, 6.2562)	(4.6673, 6.5144)
Analytical		5.5918	0.3102	(4.9264, 6.2572)	(4.6682, 6.5153)

MCI(1000)		2.7390	0.6550	(1.4822, 3.9959)	(0.9946, 4.4834)
MCI(10000)	θ_3=2.0	2.7207	0.6344	(1.4639, 3.9776)	(0.9763, 4.4652)
MCI(100000)		2.7263	0.6332	(1.4694, 3.9831)	(0.9819, 4.4707)
Analytical		2.7271	0.5860	(1.4702, 3.9839)	(0.9827, 4.4715)

Table 5.152: Moderate Negative Collinearity, ρ = -0.49 and sample size, N=10

		Mean	SE	95% CI	99% CI
MCI(1000)		15.6463	0.3082	(15.0389, 16.2774)	(14.7986, 16.5176)
MCI(10000)	θ_0= 17	15.6489	0.3106	(15.0296, 16.2681)	(14.7894, 16.5084)
MCI(100000)		16.5191	0.3947	(15.0267, 16.2652)	(14.7864, 16.5054)
Analytical		15.6463	0.3126	(15.0271, 16.2656)	(14.7868, 16.5058)
MCI(1000)		10.0000	0.0008	(9.9985, 10.0016)	(9.9980, 10.0021)
MCI(10000)	θ_1=8.5	10.0000	0.0008	(9.9985, 10.0015)	(9.9979, 10.0021)
MCI(100000)		10.0000	0.0008	(9.9985, 10.0015)	(9.9979, 10.0021)
Analytical		10.0000	0.0008	(9.9985, 10.0015)	(9.9979, 10.0021)
MCI(1000)	θ_2=5.0	5.4831	0.3829	(4.7494, 6.2399)	(4.4603, 6.5290)
MCI(10000)		5.4862	0.3739	(4.7410, 6.2315)	(4.4519, 6.5206)

MCI(100000)		5.3801	0.3775	(4.7395, 6.2300)	(4.4504, 6.5191)
Analytical		5.4831	0.3475	(4.7378, 6.2283)	(4.4487, 6.5174)
MCI(1000)		2.2833	0.7107	(0.7804, 3.7255)	(0.2091, 4.2967)
MCI(10000)	θ_3=2.0	2.2819	0.7463	(0.8093, 3.7544)	(0.2381, 4.3257)
MCI(100000)		1.8321	0.7440	(0.8131, 3.7582)	(0.2418, 4.3294)
Analytical		2.2833	0.6866	(0.8108, 3.7558)	(0.2395, 4.3271)

Table 5.153: Low Negative Collinearity, ρ = -0.20 and sample size, N=10

		Mean	SE	95% CI	99% CI
MCI(1000)		15.9929	0.3385	(15.0981, 16.9197)	(15.0821, 16.9037)
MCI(10000)	θ_0= 17	16.0089	0.3308	(15.3527, 16.6652)	(15.0981, 16.9197)
MCI(100000)		16.0117	0.3308	(15.3555, 16.6679)	(15.1009, 16.9225)
Analytical		16.0090	0.3060	(15.3528, 16.6652)	(15.0982, 16.9198)
MCI(1000)		10.0000	0.0007	(9.9980, 10.0020)	(9.9980, 10.0020)
MCI(10000)	θ_1=8.5	10.0000	0.0007	(9.9986, 10.0014)	(9.9980, 10.0020)
MCI(100000)		10.0000	0.0007	(9.9986, 10.0014)	(9.9980, 10.0020)
Analytical		10.0000	0.0007	(9.9986, 10.0014)	(9.9980, 10.0020)
MCI(1000)	θ_2=5.0	5.5337	0.3513	(4.5581, 6.5139)	(4.5558, 6.5116)
MCI(10000)		5.5360	0.3526	(4.8314, 6.2406)	(4.5581, 6.5139)

MCI(100000)		5.5357	0.3538	(4.8311, 6.2403)	(4.5578, 6.5136)
Analytical		5.5370	0.3285	(4.8324, 6.2416)	(4.5591, 6.5149)
MCI(1000)		2.3815	0.6989	(0.4681, 4.2572)	(0.4869, 4.2760)
MCI(10000)	θ_3=2.0	2.3626	0.6837	(0.9976, 3.7276)	(0.4681, 4.2572)
MCI(100000)		2.3611	0.6870	(0.9961, 3.7261)	(0.4666, 4.2557)
Analytical		2.3645	0.6364	(0.9995, 3.7295)	(0.4699, 4.2590)

Table 5.154: High Negative Collinearity, ρ = -0.90 and sample size, N=30

		Mean	SE	95% CI	99% CI
MCI(1000)		15.8906	0.1793	(15.5299, 16.2514)	(15.4063, 16.3750)
MCI(10000)	θ_0= 17	15.8971	0.3308	(15.5363, 16.2579)	(15.4127, 16.3815)
MCI(100000)		15.8969	0.1829	(15.5361, 16.2577)	(15.1009, 16.9225)
Analytical		15.8980	0.1775	(15.5372, 16.2588)	(15.4125, 16.3813)
MCI(1000)		10.0000	0.0007	(9.9986, 10.0014)	(9.9981, 10.0019)
MCI(10000)	θ_1=8.5	10.0000	0.0007	(9.9986, 10.0014)	(9.9980, 10.0020)
MCI(100000)		10.0000	0.0007	(9.9986, 10.0014)	(9.9981, 10.0019)
Analytical		10.0000	0.0007	(9.9986, 10.0014)	(9.9981, 10.0019)
MCI(1000)	θ_2=5.0	5.5558	0.3566	(4.8796, 6.2320)	(4.6480, 6.4636)

MCI(10000)		5.5570	0.3501	(4.8808, 6.2332)	(4.6492, 6.4649)
MCI(100000)		5.5548	0.3422	(4.8786, 6.2310)	(4.6470, 6.4627)
Analytical		5.5552	0.3327	(4.8790, 6.2314)	(4.6474, 6.4631)
MCI(1000)		2.8724	0.5932	(1.7040, 4.0407)	(1.3038, 4.4410)
MCI(10000)	θ_3=2.0	2.8986	0.5924	(1.7302, 4.0670)	(1.3300, 4.4672)
MCI(100000)		2.9007	0.5910	(1.7323, 4.0690)	(1.3321, 4.4692)
Analytical		2.8989	0.5749	(1.7306, 4.0673)	(1.3304, 4.4675)

Table 5.155: Moderate Negative Collinearity, ρ = -0.49 and sample size, N=30

		Mean	SE	95% CI	99% CI
MCI(1000)		16.1476	0.2133	(15.7225, 16.5728)	(15.5768, 16.7184)
MCI(10000)	θ_0= 17	16.1482	0.2158	(15.7231, 16.5734)	(15.5774, 16.7191)
MCI(100000)		16.1463	0.2150	(15.7211, 16.5715)	(15.5755, 16.7171)
Analytical		16.1452	0.2092	(15.7200, 16.5703)	(15.5743, 16.7160)
MCI(1000)		10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
MCI(10000)	θ_1=8.5	10.0000	0.0008	(9.9985, 10.0016)	(9.9979, 10.0021)
MCI(100000)		10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
Analytical		10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)

MCI(1000)		5.5390	0.3761	(4.7927, 6.2854)	(4.5370, 6.5410)
MCI(10000)	θ_2=5.0	5.5459	0.3805	(4.7996, 6.2923)	(4.5439, 6.5479)
MCI(100000)		5.5427	0.3794	(4.7964, 6.2890)	(4.5407, 6.5447)
Analytical		5.5441	0.3672	(4.7977, 6.2904)	(4.5421, 6.5461)
MCI(1000)		3.1043	0.6565	(1.7722, 4.4364)	(1.3159, 4.8927)
MCI(10000)	θ_3=2.0	3.0700	0.6820	(1.7379, 4.4020)	(1.2816, 4.8584)
MCI(100000)		3.0731	0.6760	(1.7410, 4.4052)	(1.2847, 4.8615)
Analytical		3.0733	0.6555	(1.7413, 4.4054)	(1.2850, 4.8617)

Table 5.156: Low Negative Collinearity, ρ = -0.20 and sample size, N=30

		Mean	SE	95% CI	99% CI
MCI(1000)		16.0382	0.2578	(15.5252, 16.5512)	(15.3495, 16.7269)
MCI(10000)	θ_0= 17	16.0427	0.2605	(15.5297, 16.5556)	(15.3540, 16.7313)
MCI(100000)		16.0428	0.2597	(15.5298, 16.5557)	(15.3541, 16.7314)
Analytical		16.0422	0.2524	(15.5292, 16.5552)	(15.3535, 16.7309)
MCI(1000)		10.0000	0.0009	(9.9983, 10.0017)	(9.9977, 10.0023)
MCI(10000)	θ_1=8.5	10.0000	0.0009	(9.9983, 10.0017)	(9.9977, 10.0023)
MCI(100000)		10.0000	0.0009	(9.9983, 10.0017)	(9.9977, 10.0023)

Analytical		10.0000	0.0008	(9.9983, 10.0017)	(9.9977, 10.0023)
MCI(1000)		5.5788	0.4238	(4.7589, 6.3987)	(4.4781, 6.6795)
MCI(10000)	θ_2=5.0	5.5642	0.4264	(4.7443, 6.3841)	(4.4635, 6.6649)
MCI(100000)		5.5654	0.4151	(4.7456, 6.3853)	(4.4647, 6.6662)
Analytical		5.5644	0.4034	(4.7445, 6.3843)	(4.4637, 6.6651)
MCI(1000)		1.9820	0.7431	(0.5057, 3.4582)	(0.0000, 3.9639)
MCI(10000)	θ_3=2.0	1.9858	0.7496	(0.5096, 3.4621)	(0.0039, 3.9678)
MCI(100000)		1.9821	0.7501	(0.5059, 3.4584)	(0.0002, 3.9641)
Analytical		1.9842	0.7264	(0.5080, 3.4605)	(0.0023, 3.9662)

In table 5.151, Bayesian analytical method has the minimum standard error for parameters θ_0, θ_1, θ_2 and $\theta_3$with SE of 0.2635, 0.006, 0.3102 and 0.5860 respectively while the standard error of Bayesian posterior simulation method (MCI) reduces with increase number of replications.

Table 5.152 shows the mean estimates of parameters θ_0 , θ_1 , θ_2 and θ_3 that ranges from (15.5191-15.6489), (10.0000), (5.3801-5.4862) and (1.8321-2.2833) respectively, it also shows that the Bayesian analytical method has the minimum SE for all the parameters. θ_0, θ_2 and θ_3 with a SE of 0.3126, 0.3475 and 0.6866, respectively expect for parameter θ_1 that has the same SE of 0.0008 with Bayesian Posterior Simulation method (1000, 10000 and 100000). It was also observed that Bayesian analytical method has a compact CI at 95% and 99% compared to Bayesian posterior simulation method.

In Table 5.153, the mean of Bayesian analytical and posterior simulation methods are not too far from the true parameters values, they have the mean that ranges from (15.9929-16.0117), (10.0000), (5.5337-5.5370) and (2.3611-2.3815) for parameters θ_0 , θ_1 , θ_2 and θ_3 respectively. Bayesian analytical method has a compact CI for all parameters expect for $\theta_1$that has the same CI with posterior simulation method with replications MCI (1000, 10000 and 100000).

From Tables 5.154-5.156, Bayesian analytical method has the minimum Standard Error (SE) for all the parameters considered and compact CI and as the replication increases, the SE tends towards the Bayesian analytical values.

Table 5.157: High Negative Collinearity, ρ = -0.90 and sample size, N=70

		Mean	SE	95% CI	99% CI
MCI(1000)		16.4577	0.1446	(16.1729, 16.7424)	(16.0798, 16.8355)
MCI(10000)	θ_0= 17	16.4602	0.1445	(16.1754, 16.7449)	(16.0823, 16.8380)
MCI(100000)		16.4621	0.1445	(16.1774, 16.7469)	(16.0843, 16.8400)
Analytical		16.4612	0.1429	(16.1764,	(16.0833,

				16.7460)	16.8391)
MCI(1000)		10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
MCI(10000)	θ_1=8.5	10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
MCI(100000)		10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
Analytical		10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
MCI(1000)		5.7241	0.3653	(4.9868, 6.4613)	(4.7458, 6.7023)
MCI(10000)	θ_2=5.0	5.7262	0.3767	(4.9890, 6.4635)	(4.7480, 6.7045)
MCI(100000)		5.7300	0.3764	(4.9928, 6.4673)	(4.7517, 6.7083)
Analytical		5.7307	0.3700	(4.9934, 6.4680)	(4.7524, 6.7090)
MCI(1000)		2.8514	0.5680	(1.6894, 4.0134)	(1.3095, 4.3932)
MCI(10000)	θ_3=2.0	2.8511	0.5950	(1.6891, 4.0131)	(1.3092, 4.3929)
MCI(100000)		2.8515	0.5896	(1.6895, 4.0135)	(1.3096, 4.3933)
Analytical		2.8491	0.5832	(1.6871, 4.0111)	(1.3073, 4.3910)

Table 5.158: Moderate Negative Collinearity, ρ = -0.49 and sample size, N=70

		Mean	SE	95% CI	99% CI
MCI(1000)		16.1020	0.1298	(15.8468, 16.3573)	(15.7633, 16.4407)
MCI(10000)	θ_0= 17	16.0976	0.2605	(15.8423, 16.3528)	(15.7589, 16.4363)
MCI(100000)		16.0954	0.1295	(15.8401, 16.3506)	(15.7567, 16.4341)
Analytical		16.0960	0.1281	(15.8407, 16.3512)	(15.7573, 16.4347)
MCI(1000)		10.0000	0.0007	(9.9985, 10.0015)	(9.9980, 10.0020)
MCI(10000)	θ_1=8.5	10.0000	0.0009	(9.9985, 10.0015)	(9.9980, 10.0020)
MCI(100000)		10.0000	0.0007	(9.9983, 10.0017)	(9.9980, 10.0020)
Analytical		10.0000	0.0007	(9.9985, 10.0015)	(9.9980, 10.0020)
MCI(1000)		5.7297	0.3519	(5.0484, 6.4109)	(4.8257, 6.6337)
MCI(10000)	θ_2=5.0	5.7320	0.3447	(5.0508, 6.4133)	(4.8281, 6.6360)
MCI(100000)		5.7340	0.3455	(5.0527, 6.4152)	(4.8300, 6.6380)
Analytical		5.7343	0.3419	(5.0531, 6.4156)	(4.8304, 6.6383)
MCI(1000)		2.3741	0.5687	(1.2604, 3.4879)	(0.8963, 3.8520)
MCI(10000)	θ_3=2.0	2.3888	0.5701	(1.2751, 3.5026)	(0.9110, 3.8667)
MCI(100000)		2.3939	0.5656	(1.2801, 3.5077)	(0.9161, 3.8718)
Analytical		2.3918	0.5590	(1.2781, 3.5056)	(0.9140, 3.8697)

Table 5.159: Low Negative Collinearity, ρ = -0.20 and sample size, N=70

		Mean	SE	95% CI	99% CI
MCI(1000)		16.1142	0.1956	(15.7416, 16.4868)	(15.6198, 16.6087)
MCI(10000)	θ_0= 17	16.1099	0.1920	(15.7373, 16.4826)	(15.6155, 16.6044)
MCI(100000)		16.1132	0.1897	(15.7406, 16.4859)	(15.6188, 16.6077)
Analytical		16.1139	0.1870	(15.7412, 16.4865)	(15.6194, 16.6083)
MCI(1000)		10.0001	0.0009	(9.9986, 10.0015)	(9.9981, 10.0020)
MCI(10000)	θ_1=8.5	10.0000	0.0007	(9.9985, 10.0015)	(9.9980, 10.0019)
MCI(100000)		10.0000	0.0007	(9.9985, 10.0015)	(9.9980, 10.0019)
Analytical		10.0000	0.0007	(9.9985, 10.0015)	(9.9980, 10.0019)
MCI(1000)		5.6229	0.3540	(4.9512, 6.2947)	(4.7316, 6.5143)
MCI(10000)	θ_2=5.0	5.6134	0.3454	(4.9417, 6.2852)	(4.7221, 6.5048)
MCI(100000)		5.6101	0.3394	(4.9383, 6.2819)	(4.7187, 6.5015)
Analytical		5.6106	0.3371	(4.9389, 6.2824)	(4.7192, 6.5020)
MCI(1000)		1.6730	0.5890	(0.5477, 2.7984)	(0.1798, 3.1663)
MCI(10000)	θ_3=2.0	1.6985	0.5768	(0.5731, 2.8239)	(0.2052, 3.1918)
MCI(100000)		1.6937	0.5747	(0.5684, 2.8191)	(0.2005, 3.1870)
Analytical		1.6916	0.5648	(0.5662, 2.8170)	(0.1983, 3.1848)

Table 5.160: High Negative Collinearity, ρ = -0.90 and sample size, N=100

		Mean	SE	95% CI	99% CI
MCI(1000)		16.3884	0.1239	(16.1470, 16.6299)	(16.0690, 16.7079)
MCI(10000)	θ_0= 17	16.3828	0.1241	(16.1413, 16.6242)	(16.0633, 16.7023)
MCI(100000)		16.3846	0.1232	(16.1432, 16.6261)	(16.0651, 16.7041)
Analytical		16.3849	0.1218	(16.1434, 16.6263)	(16.0654, 16.7043)
MCI(1000)		10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
MCI(10000)	θ_1=8.5	10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
MCI(100000)		10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
Analytical		10.0000	0.0008	(9.9984, 10.0016)	(9.9980, 10.0020)
MCI(1000)		5.8492	0.3603	(5.1224, 6.5761)	(4.8874, 6.8110)
MCI(10000)	θ_2=5.0	5.8403	0.3731	(5.1135, 6.5672)	(4.8786, 6.8021)
MCI(100000)		5.8424	0.3714	(5.1156, 6.5693)	(4.8807, 6.8042)
Analytical		5.8408	0.3665	(5.1140, 6.5677)	(4.9090, 6.7407)
MCI(1000)		3.5400	0.5572	(2.4508, 4.6291)	(2.0988, 4.9811)
MCI(10000)	θ_3=2.0	3.5346	0.5572	(2.4455, 4.6237)	(2.0935, 4.9757)
MCI(100000)		3.5397	0.5528	(2.4506, 4.6289)	(2.0986, 4.9808)
Analytical		3.5419	0.5492	(2.4528, 4.6310)	(2.1008, 4.9830)

Table 5.161: Moderate Negative Collinearity, ρ = -0.49 and sample size, N=100

		Mean	SE	95% CI	99% CI
MCI(1000)		16.1553	0.1232	(15.9248, 16.3857)	(15.8554, 16.4651)
MCI(10000)	θ_0= 17	16.1602	0.1176	(15.9298, 16.3906)	(15.3540, 16.7313)
MCI(100000)		16.1600	0.1169	(15.9296, 16.3904)	(15.8551, 16.4648)
Analytical		16.1598	0.1162	(15.9294, 16.3902)	(15.8549, 16.4647)
MCI(1000)		10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
MCI(10000)	θ_1=8.5	10.0000	0.0008	(9.9984, 10.0016)	(9.9977, 10.0023)
MCI(100000)		10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
Analytical		10.0000	0.0008	(9.9984, 10.0016)	(9.9979, 10.0021)
MCI(1000)		5.4096	0.3531	(4.7215, 6.0978)	(4.4991, 6.3202)
MCI(10000)	θ_2=5.0	5.4123	0.3483	(4.7443, 6.3841)	(4.5017, 6.3228)
MCI(100000)		5.4180	0.3501	(4.7298, 6.1062)	(4.5074, 6.3286)
Analytical		5.4171	0.3470	(4.7290, 6.1053)	(4.5066, 6.3277)
MCI(1000)		3.1911	0.5522	(2.1180, 4.2642)	(1.7712, 4.6110)
MCI(10000)	θ_3=2.0	3.1543	0.5481	(2.0812, 4.2274)	(1.7344, 4.5742)
MCI(100000)		3.1634	0.5459	(2.0903, 4.2365)	(1.7435, 4.5833)
Analytical		3.1613	0.5411	(2.0882, 4.2344)	(1.7414, 4.5813)

Table 5.162: Low Negative Collinearity, ρ = -0.20 and sample size, N=100

		Mean	SE	95% CI	99% CI
MCI(1000)		15.9160	0.1635	(15.5899, 16.2420)	(15.4845, 16.3474)
MCI(10000)	θ_0= 17	15.9165	0.1649	(15.5905, 16.2426)	(15.4851, 16.3479)
MCI(100000)		15.9173	0.1658	(15.5912, 16.2433)	(15.4858, 16.3487)
Analytical		16.3263	0.1785	(15.7944, 16.4152)	(15.8581, 16.7946)
MCI(1000)		10.0000	0.0009	(9.9983, 10.0017)	(9.9977, 10.0023)
MCI(10000)	θ_1=8.5	10.0000	0.0009	(9.9983, 10.0017)	(9.9977, 10.0023)
MCI(100000)		10.0000	0.0009	(9.9983, 10.0017)	(9.9977, 10.0023)
Analytical		10.0000	0.0008	(9.9984, 10.0016)	(9.9978, 10.0021)
MCI(1000)		5.4768	0.3726	(4.7197, 6.2338)	(4.4751, 6.4784)
MCI(10000)	θ_2=5.0	5.4893	0.3842	(4.7323, 6.2463)	(4.4876, 6.4910)
MCI(100000)		5.4839	0.3857	(4.7269, 6.2409)	(4.4822, 6.4856)
Analytical		5.7778	0.3569	(5.0701, 6.4855)	(4.8414, 6.7142)
MCI(1000)		3.3480	0.5571	(2.2322, 4.4638)	(1.8716, 4.8244)
MCI(10000)	θ_3=2.0	3.3545	0.5667	(2.2387, 4.4703)	(1.8780, 4.8309)
MCI(100000)		3.3577	0.5681	(2.2419, 4.4735)	(1.8812, 4.8341)
Analytical		1.4617	0.5549	(0.3613, 2.5621)	(0.0056, 2.9178)

Table 5.163: High Negative Collinearity, ρ = -0.90 and sample size, N=200

		Mean	SE	95% CI	99% CI
MCI(1000)		16.5211	0.0886	(16.3533, 16.6890)	(16.2998, 16.7425)
MCI(10000)	θ_0= 17	16.5181	0.0857	(16.3502, 16.6859)	(16.2967, 16.7394)
MCI(100000)		16.5190	0.0855	(16.3512, 16.6868)	(16.2977, 16.7403)
Analytical		16.5188	0.0851	(16.3510, 16.6866)	(16.2975, 16.7401)
MCI(1000)		10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0020)
MCI(10000)	θ_1=8.5	10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0020)
MCI(100000)		10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0020)
Analytical		10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0020)
MCI(1000)		5.6236	0.3162	(4.9718, 6.2753)	(4.7640, 6.4831)
MCI(10000)	θ_2=5.0	5.6339	0.3326	(4.9821, 6.2857)	(4.7744, 6.4934)
MCI(100000)		5.6381	0.3309	(4.9863, 6.2899)	(4.7785, 6.4976)
Analytical		5.6385	0.3306	(4.9867, 6.2902)	(4.7789, 6.4980)
MCI(1000)		3.3871	0.4508	(2.5119, 4.2623)	(2.2329, 4.5412)
MCI(10000)	θ_3=2.0	3.3767	0.4476	(2.5015, 4.2519)	(2.2225, 4.5309)
MCI(100000)		3.3773	0.4451	(2.5021, 4.2525)	(2.2231, 4.5314)
Analytical		3.3768	0.4439	(2.5016, 4.2520)	(2.2227, 4.5310)

Table 5.164: Moderate Negative Collinearity, ρ = -0.49 and sample size, N=200

		Mean	SE	95% CI	99% CI
MCI(1000)		16.1366	0.0980	(15.9432, 16.3299)	(15.8816, 16.3916)
MCI(10000)	θ_0= 17	16.1354	0.0980	(15.9420, 16.3288)	(15.8804, 16.3904)
MCI(100000)		16.1347	0.0989	(15.9413, 16.3280)	(15.8797, 16.3897)
Analytical		16.1350	0.0981	(15.9416, 16.3283)	(15.8800, 16.3900)
MCI(1000)		10.0000	0.0008	(9.9983, 10.0017)	(9.9977, 10.0022)
MCI(10000)	θ_1=8.5	10.0000	0.0009	(9.9983, 10.0017)	(9.9978, 10.0022)
MCI(100000)		10.0000	0.0009	(9.9983, 10.0017)	(9.9978, 10.0022)
Analytical		10.0000	0.0009	(9.9983, 10.0017)	(9.9978, 10.0022)
MCI(1000)		5.8386	0.3321	(5.1659, 6.5113)	(4.9515, 6.7257)
MCI(10000)	θ_2=5.0	5.8300	0.3420	(5.1574, 6.5027)	(4.9429, 6.7171)
MCI(100000)		5.8322	0.3432	(5.1595, 6.5049)	(4.9451, 6.7193)
Analytical		5.8310	0.3439	(5.1583, 6.5037)	(4.9439, 6.7181)
MCI(1000)		3.2667	0.4995	(2.2843, 4.2491)	(1.9711, 4.5623)
MCI(10000)	θ_3=2.0	3.2889	0.5014	(2.3065, 4.2713)	(1.9933, 4.5845)
MCI(100000)		3.2876	0.5000	(2.3052, 4.2700)	(1.9920, 4.5832)
Analytical		3.2888	0.4983	(2.3064, 4.2712)	(1.9932, 4.5844)

Table 5.165: Low Negative Collinearity, ρ = -0.20 and sample size, N=200

		Mean	SE	95% CI	99% CI
MCI(1000)		16.1946	0.1225	(15.9486, 16.4406)	(15.8702, 16.5190)
MCI(10000)	θ_0= 17	16.1998	0.1248	(15.9538, 16.4458)	(15.8754, 16.5242)
MCI(100000)		16.1987	0.1250	(15.9527, 16.4447)	(15.8743, 16.5231)
Analytical		16.1995	0.1248	(15.9535, 16.4455)	(15.8751, 16.5239)
MCI(1000)		10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0020)
MCI(10000)	θ_1=8.5	10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0020)
MCI(100000)		10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0020)
Analytical		10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0020)
MCI(1000)		5.0921	0.3128	(4.4872, 5.6970)	(4.2944, 5.8898)
MCI(10000)	θ_2=5.0	5.0790	0.3071	(4.4741, 5.6839)	(4.2813, 5.8767)
MCI(100000)		5.0816	0.3083	(4.4767, 5.6864)	(4.2839, 5.8792)
Analytical		5.0807	0.3068	(4.4758, 5.6856)	(4.2830, 5.8784)
MCI(1000)		2.6149	0.4074	(1.8044, 3.4254)	(1.5460, 3.6838)
MCI(10000)	θ_3=2.0	2.6072	0.4124	(1.7966, 3.4177)	(1.5383, 3.6761)
MCI(100000)		2.6067	0.4127	(1.7961, 3.4172)	(1.5378, 3.6756)
Analytical		2.6046	0.4111	(1.7941,	(1.5357,

				3.4152)	3.6735)

Table 5.166: High Negative Collinearity, ρ = -0.90 and sample size, N=300

		Mean	SE	95% CI	99% CI
MCI(1000)		16.5490	0.0744	(16.4061, 16.6919)	(16.3607, 16.7372)
MCI(10000)	θ_0= 17	16.5512	0.0726	(16.4083, 16.6941)	(16.3630, 16.7395)
MCI(100000)		16.5512	0.0731	(16.4083, 16.6941)	(16.3630, 16.7394)
Analytical		16.5510	0.0726	(16.4081, 16.6938)	(16.3627, 16.7392)
MCI(1000)		10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0020)
MCI(10000)	θ_1=8.5	10.0000	0.0008	(9.9984, 10.0015)	(9.9980, 10.0020)
MCI(100000)		10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0020)
Analytical		10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0020)
MCI(1000)		5.8346	0.3229	(5.2052, 6.4639)	(5.0055, 6.6636)
MCI(10000)	θ_2=5.0	5.8515	0.3195	(5.2221, 6.4808)	(5.0224, 6.6805)
MCI(100000)		5.8492	0.3210	(5.2199, 6.4786)	(5.0202, 6.6783)
Analytical		5.8495	0.3198	(5.2201, 6.4788)	(5.0204, 6.6785)
MCI(1000)	θ_3=2.0	3.4289	0.4289	(2.6216, 4.2361)	(2.3655, 4.4922)
MCI(10000)		3.4313	0.4129	(2.6240, 4.2385)	(2.3679, 4.4946)

MCI(100000)		3.4370	0.4107	(2.6298, 4.2443)	(2.3737, 4.5004)
Analytical		3.4357	0.4102	(2.6285, 4.2429)	(2.3724, 4.4990)

Table 5.167: Moderate Negative Collinearity, ρ = -0.49 and sample size, N=300

		Mean	SE	95% CI	99% CI
MCI(1000)		16.0860	0.0728	(15.9472, 16.2247)	(15.9032, 16.2687)
MCI(10000)	θ_0= 17	16.0842	0.0714	(15.9454, 16.2229)	(15.9014, 16.2669)
MCI(100000)		16.0833	0.0705	(15.9445, 16.2220)	(15.9005, 16.2660)
Analytical		16.0837	0.0705	(15.9450, 16.2225)	(15.9009, 16.2665)
MCI(1000)		10.0000	0.0007	(9.9985, 10.0015)	(9.9980, 10.0019)
MCI(10000)	θ_1=8.5	10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0019)
MCI(100000)		10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0019)
Analytical		10.0000	0.0008	(9.9985, 10.0015)	(9.9980, 10.0019)
MCI(1000)		5.9329	0.2750	(5.3924, 6.4733)	(5.2209, 6.6448)
MCI(10000)	θ_2=5.0	5.9368	0.2765	(5.3963, 6.4773)	(5.2248, 6.6487)
MCI(100000)		5.9385	0.2753	(5.3980, 6.4790)	(5.2265, 6.6504)
Analytical		5.9368	0.2747	(5.3963, 6.4773)	(5.2248, 6.6487)

MCI(1000)		3.0294	0.3611	(2.3244, 3.7345)	(2.1007, 3.9582)
MCI(10000)	θ_3=2.0	3.0236	0.3588	(2.3186, 3.7287)	(2.0949, 3.9524)
MCI(100000)		3.0305	0.3604	(2.3254, 3.7355)	(2.1017, 3.9592)
Analytical		3.0300	0.3583	(2.3249, 3.7350)	(2.1012, 3.9587)

Table 5.168: Low Negative Collinearity, ρ = -0.20 and sample size, N=300

		Mean	SE	95% CI	99% CI
MCI(1000)		15.7434	0.1128	(15.5206, 15.9662)	(15.4499, 16.0369)
MCI(10000)	θ_0= 17	15.7439	0.1140	(15.5211, 15.9667)	(15.4505, 16.0374)
MCI(100000)		16.1987	0.1250	(15.5225, 15.9681)	(15.4518, 16.0388)
Analytical		15.7450	0.1132	(15.5222, 15.9678)	(15.4515, 16.0385)
MCI(1000)		10.0000	0.0009	(9.9983, 10.0016)	(9.9978, 10.0022)
MCI(10000)	θ_1=8.5	10.0000	0.0009	(9.9983, 10.0017)	(9.9978, 10.0022)
MCI(100000)		10.0000	0.0008	(9.9983, 10.0016)	(9.9978, 10.0022)
Analytical		10.0000	0.0009	(9.9983, 10.0016)	(9.9978, 10.0022)
MCI(1000)		5.6714	0.3064	(5.0746, 6.2682)	(4.8853, 6.4575)
MCI(10000)	θ_2=5.0	5.6655	0.3033	(5.0687, 6.2623)	(4.8793, 6.4516)
MCI(100000)		5.0816	0.3083	(5.0665, 6.2601)	(4.8772, 6.4495)

Analytical		5.6629	0.3033	(5.0661, 6.2597)	(4.8767, 6.4490)
MCI(1000)		3.0028	0.3916	(2.2395, 3.7660)	(1.9974, 4.0081)
MCI(10000)	θ_3=2.0	2.9987	0.3901	(2.2355, 3.7619)	(1.9933, 4.0041)
MCI(100000)		2.6067	0.4127	(2.2349, 3.7614)	(1.9928, 4.0035)
Analytical		2.9987	0.3879	(2.2355, 3.7619)	(1.9933, 4.0041)

The mean of Bayesian posterior simulation (1000, 10000 and 100000) and Bayesian analytical methods in Tables 5.157-5.168 are in line with the true parameter values specified. In Table 5.164, for MNC, the Bayesian posterior simulation has minimum SE for parameters θ_0, θ_1, θ_2.

Both the Bayesian analytical and posterior simulation methods almost have the same CI at 95% and 99%, this is due to increase in sample sizes.

However, Bayesian analytical method still has smaller standard error and narrower CI than the posterior simulation method (MCI (1000), MCI (10000) and MCI (100000)) at both 95% and 99% in most cases. This shows that Bayesian analytical method is better than the numerical approach. As the replications of Bayesian posterior simulation method (MCI) increases, the estimates tend towards the estimates of Bayesian Analytical method.

5.4 Performances of the Estimators when there is No Multicollinearity.

Table 5.169: No Collinearity with sample size, N=70

Parameters	Estimators	Mean	Standard Error	95% CI	99% CI
θ_0 (17)	**LB**	17.1923	0.3170	(16.5594,17.8252)	(16.3515, 18.0332)
	BNIP	17.1923	0.3078	(16.5784,17.8062)	(16.3772,18.0074)
	BIP	16.5785	0.1893	(15.8195,16.5738)	(15.6962,16.6971)
θ_1 (8.5)	**LB**	8.6799	0.3597	(7.9618, 13.0079)	(7.7259,9.6339)
	BNIP	8.6799	0.3492	(7.9834, 9.3765)	(7.7552, 9.6047)
	BIP	10.0000	0.0007	(9.9985, 10.0015)	(9.9980, 10.0020)
θ_2 (5.0)	**LB**	4.4809	0.7011	(3.0810, 5.8807)	(2.6212, 6.3405)
	BNIP	4.4809	0.6808	(3.1231, 5.8386)	(2.6782, 6.2835)
	BIP	5.3574	0.3339	(4.6546, 5.9852)	(4.4371, 6.2026)
θ_3 (2.0)	**LB**	1.3409	0.7166	(-0.0897, 2.7716)	(-0.5597, 3.2415)
	BNIP	1.3409	0.6958	(-0.0468, 2.7286)	(-0.5015, 3.1833)

BIP	1.3367	0.5350	(0.9025, 3.0345)	(0.5540, 3.3830)

,

Table 5.169 shows the performances of three estimators when there is no multicollinearity in a regression model. It was also observed that the Bayesian estimators proposed (BIP and BNIP) also had better performance in terms of Standard Error (SE) and Credible Interval (CI) in the absence of multicollinearity. The mean estimates of the estimators are almost close to the true parameter values

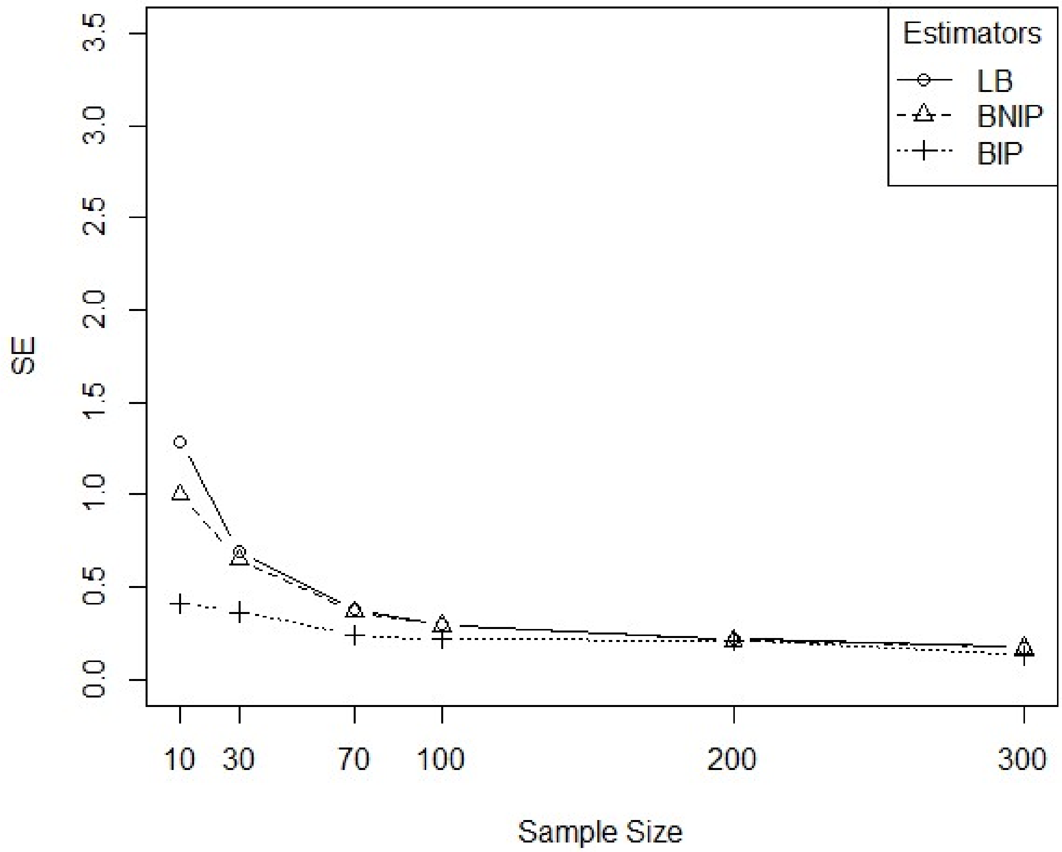

Figure 5.1: Plot of Standard Error for theta 0 of HPC (0.95)

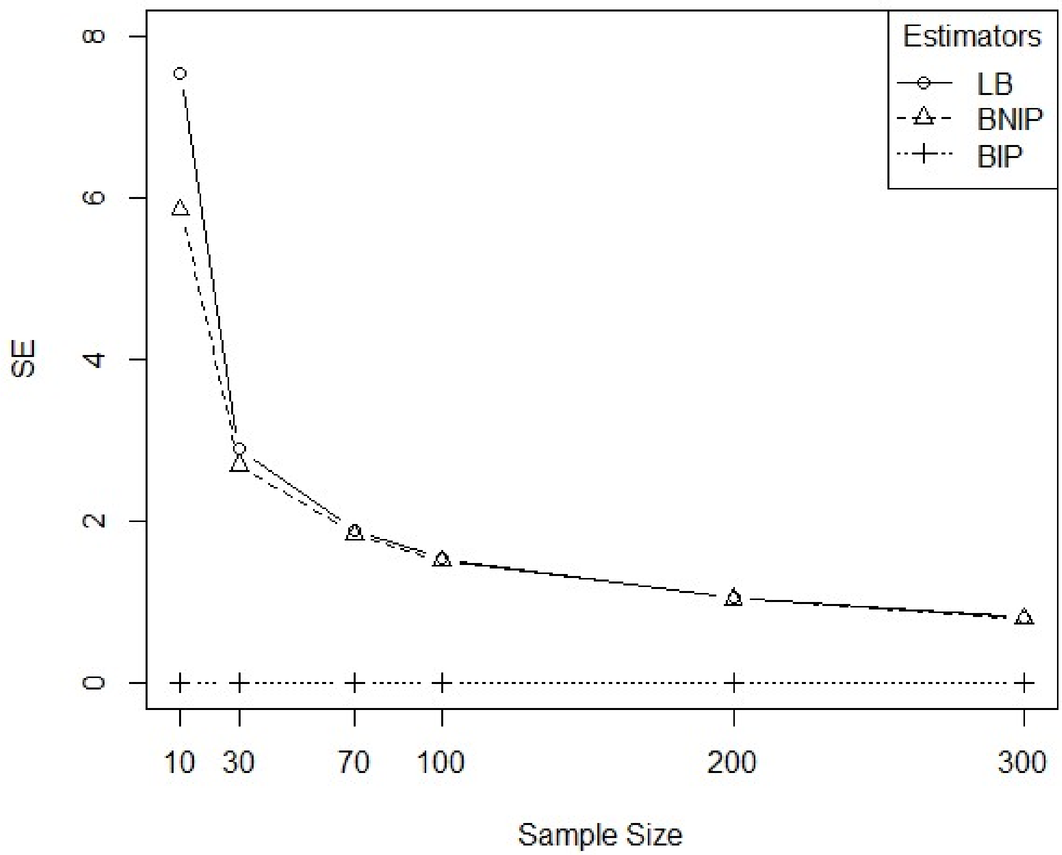

Figure 5.2: Plot of Standard Error for theta 1 of HPC (0.95)

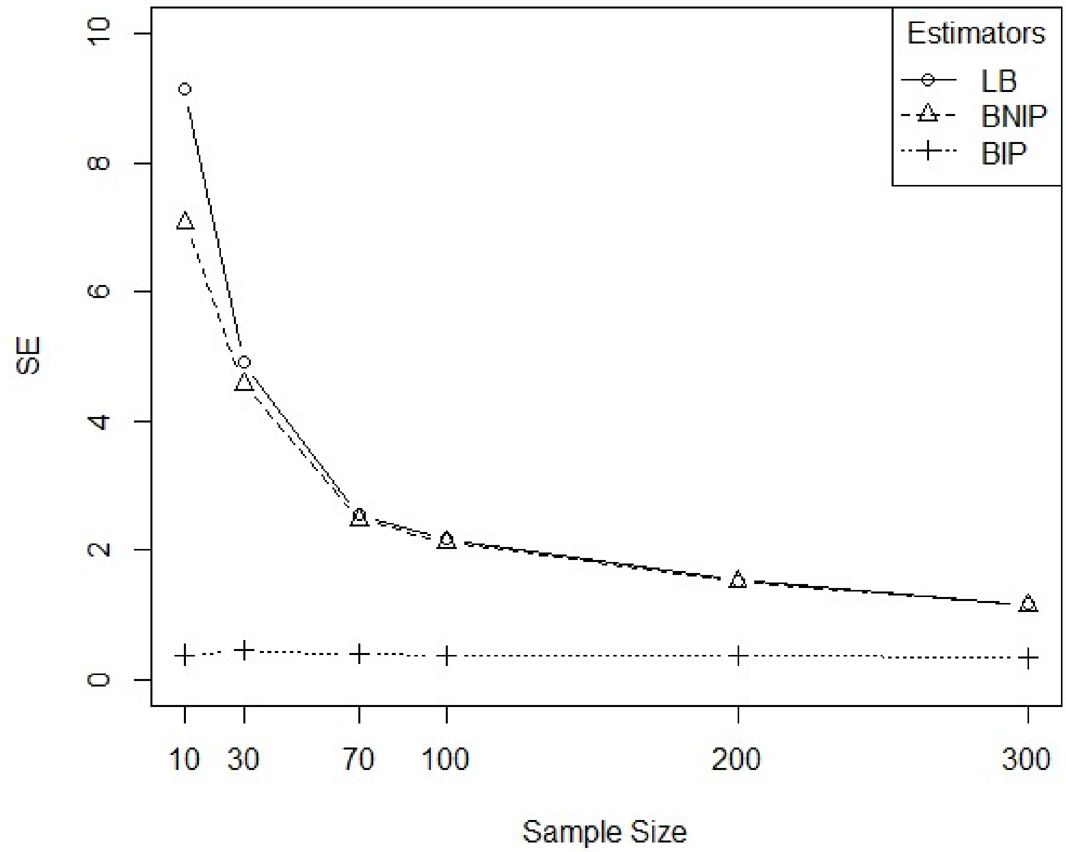

Figure 5.3: Plot of Standard Error for theta 2 of HPC (0.95)

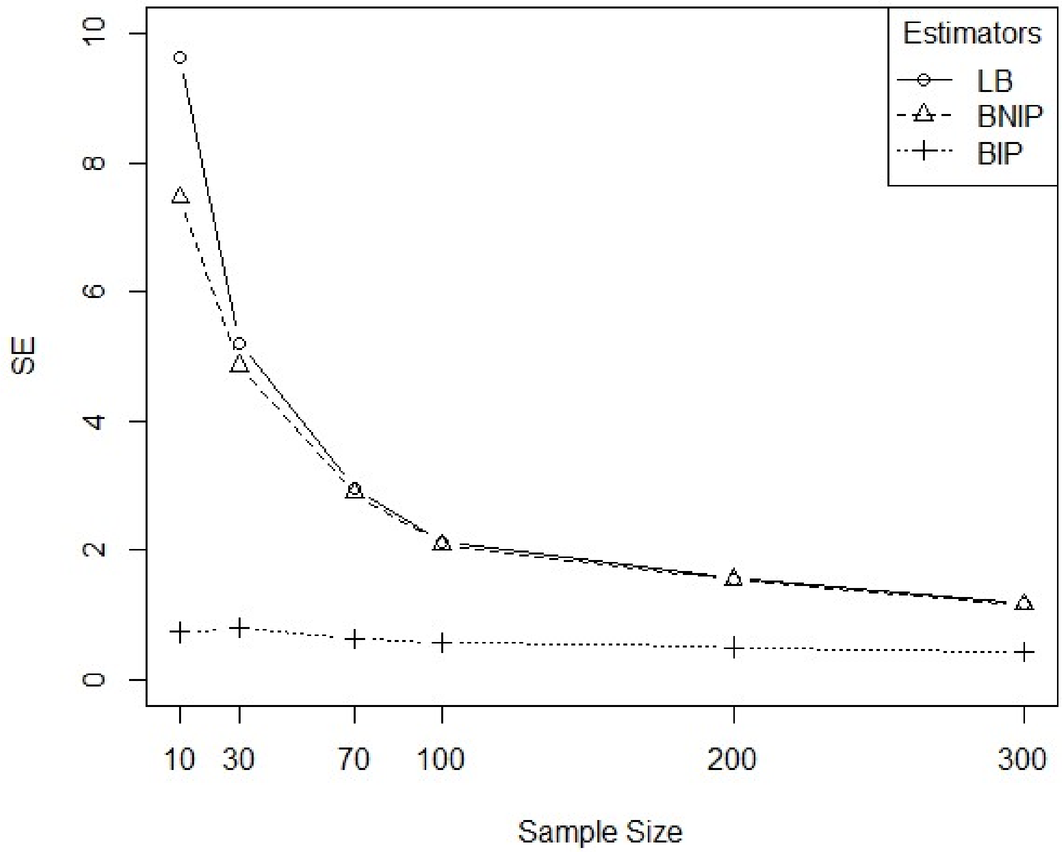

Figure 5.4: Plot of Standard Error for theta 3 of HPC (0.95)

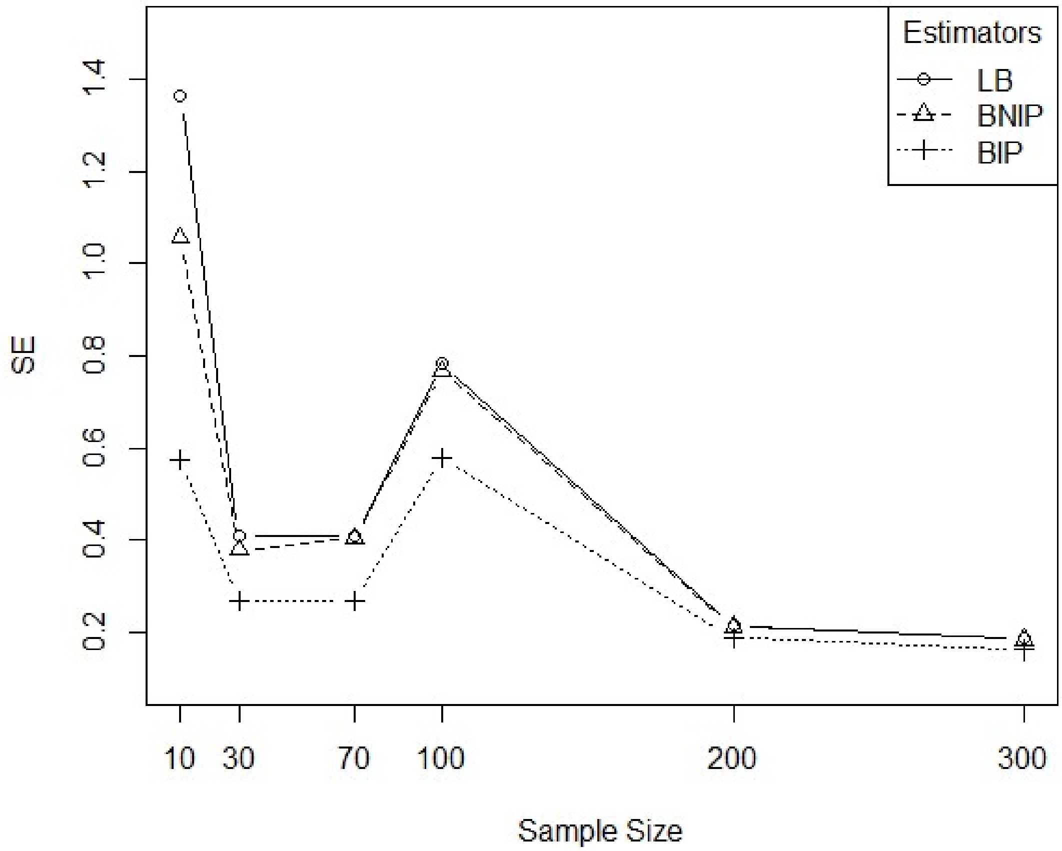

Figure 5.5: Plot of Standard Error for theta 0 of MPC (0.49)

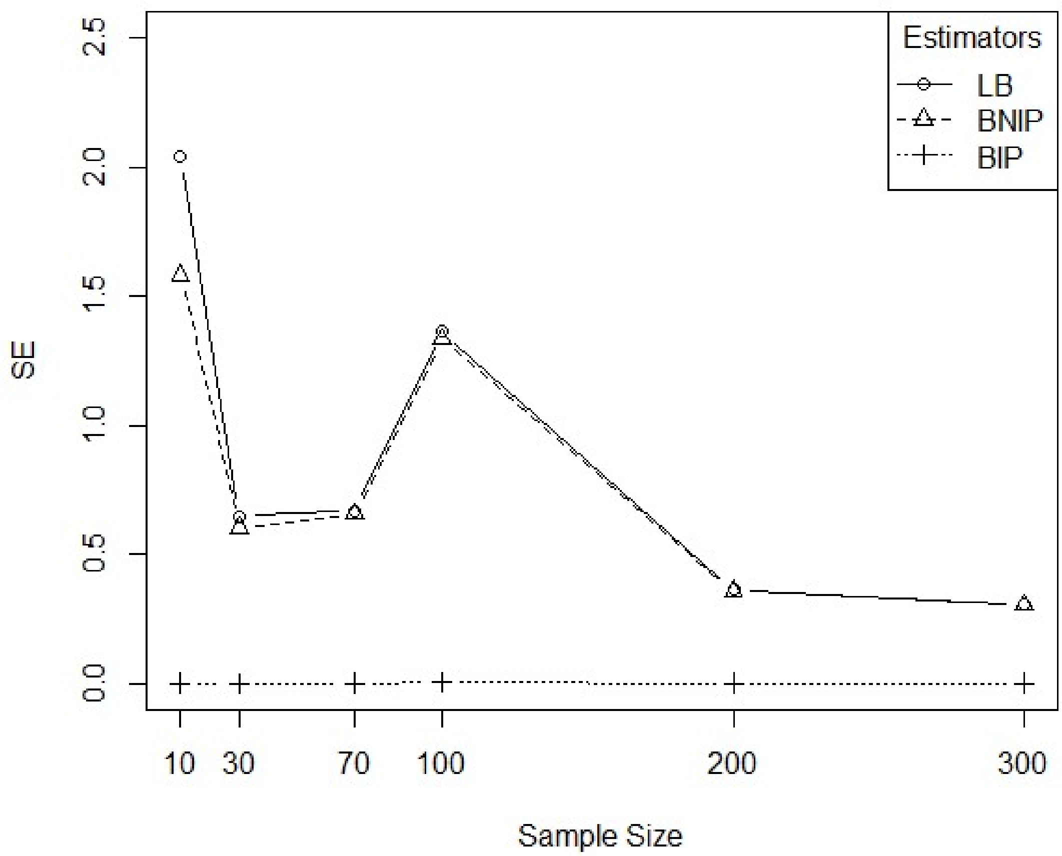

Figure 5.6: Plot of Standard Error for theta 1 of MPC (0.49)

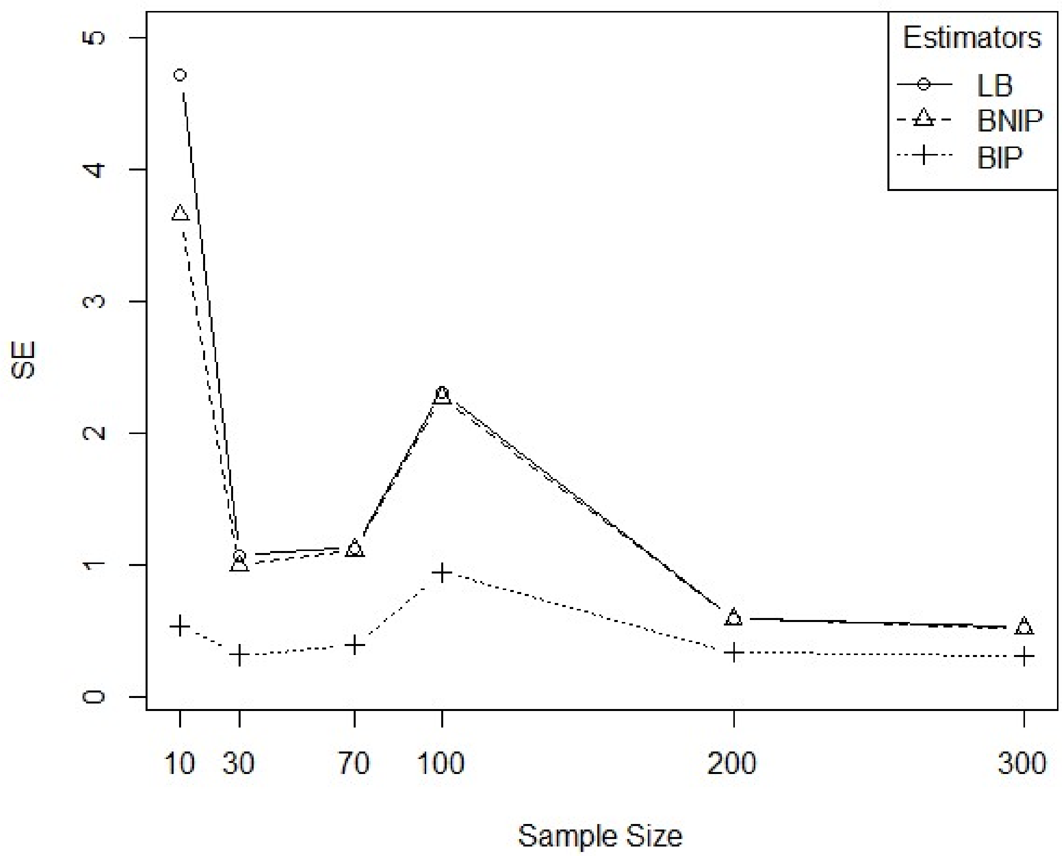

Figure 5.7: Plot of Standard Error for theta 2 of MPC (0.49)

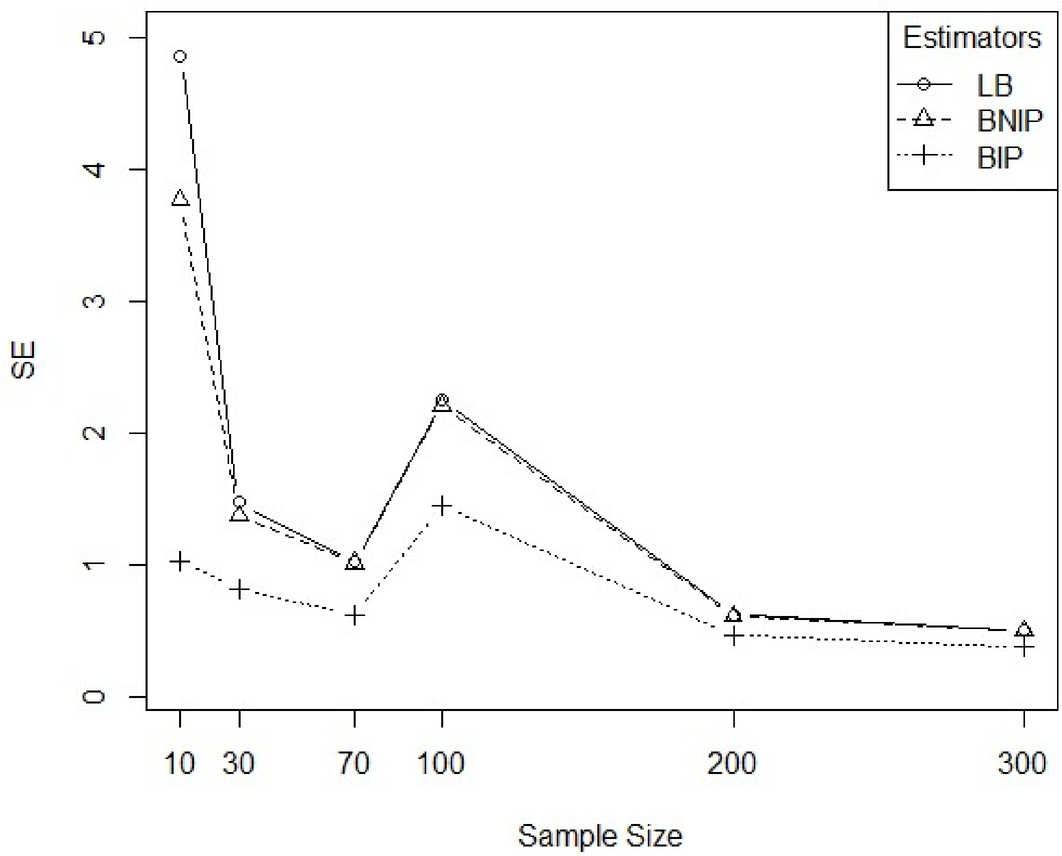

Figure 5.8: Plot of Standard Error for theta 3 of MPC (0.49)

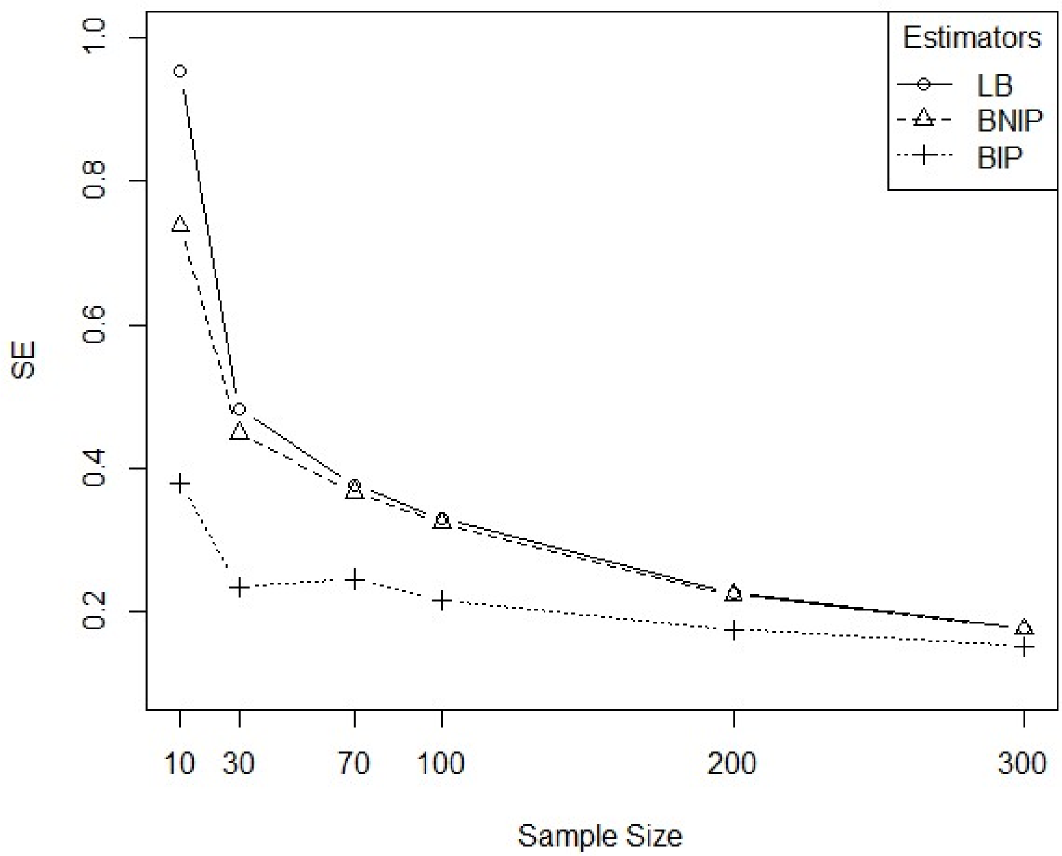

Figure 5.9: Plot of Standard Error for theta 0 of LPC (0.15)

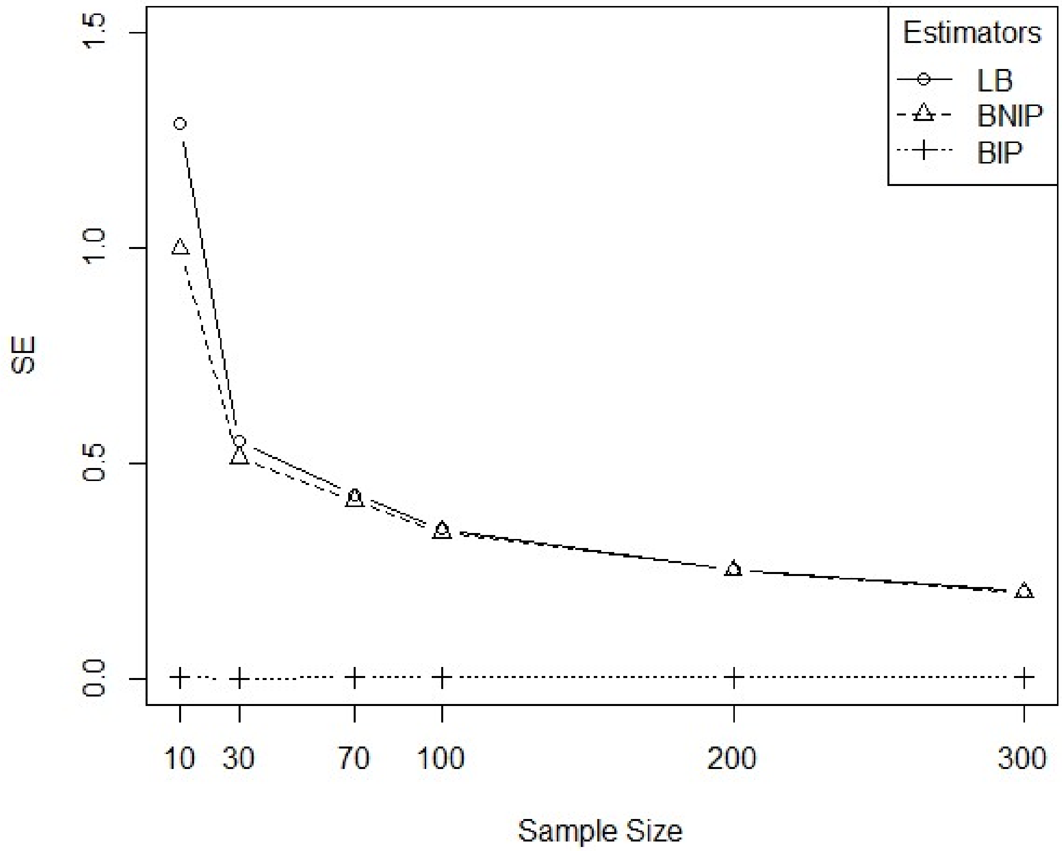

Figure 5.10: Plot of Standard Error for theta 1 of LPC (0.15)

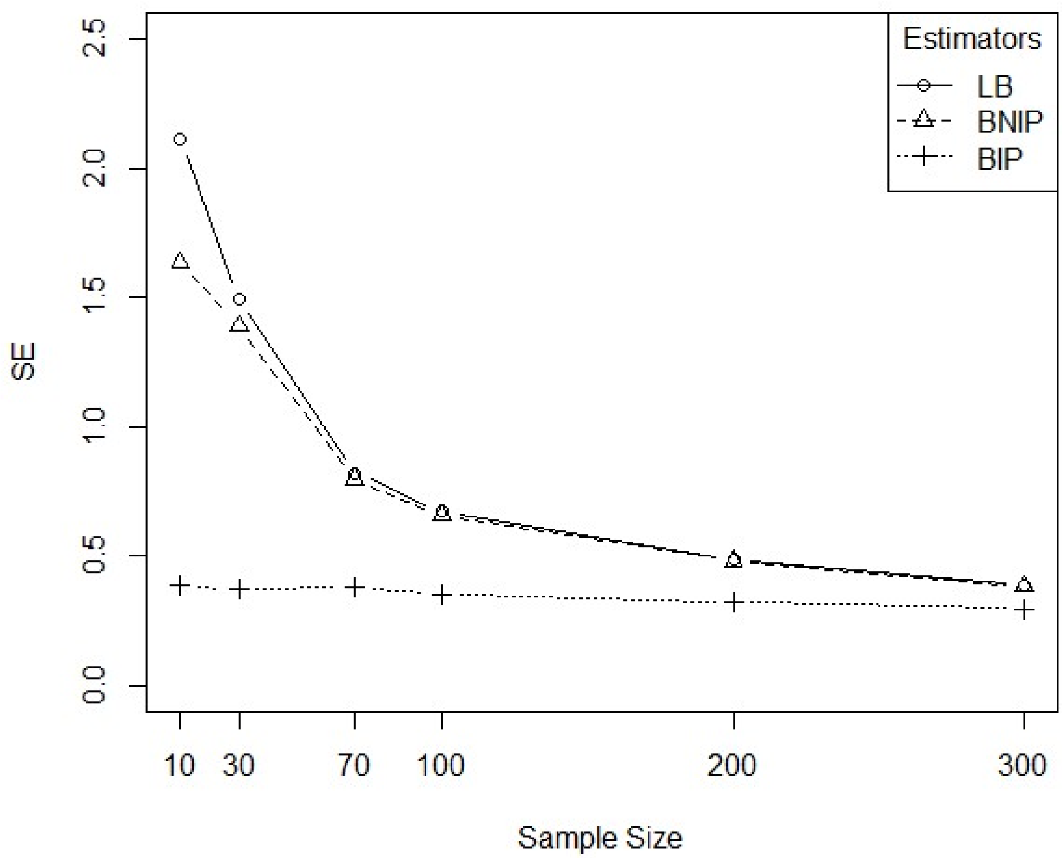

Figure 5.11: Plot of Standard Error for theta 2 of LPC (0.15)

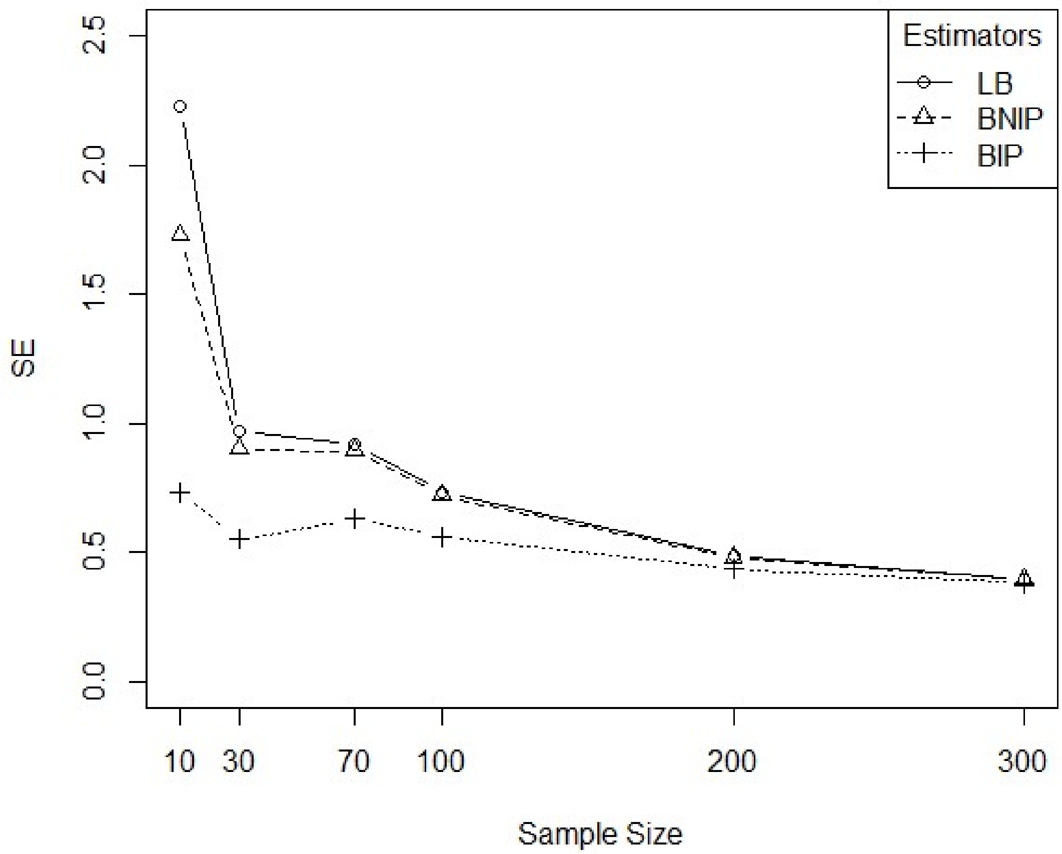

Figure 5.12: Plot of Standard Error for theta 3 of LPC (0.15)

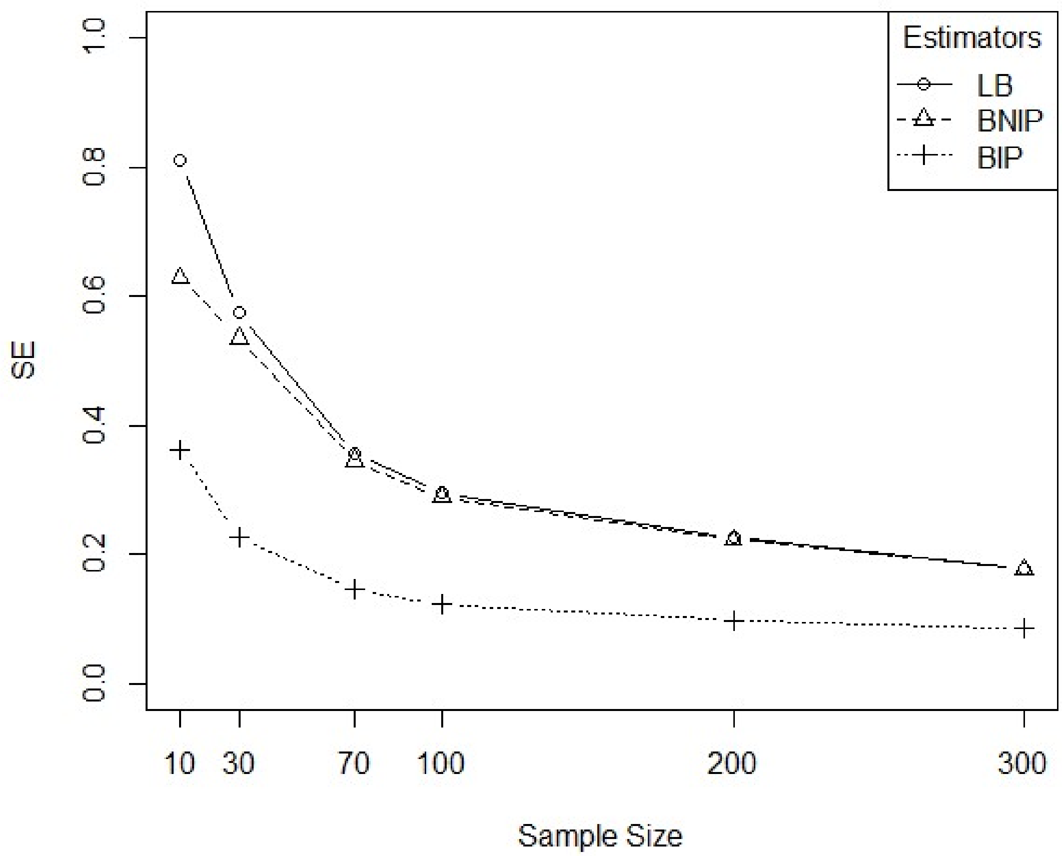

Figure 5.13: Plot of Standard Error for theta 0 of HNC (-0.95)

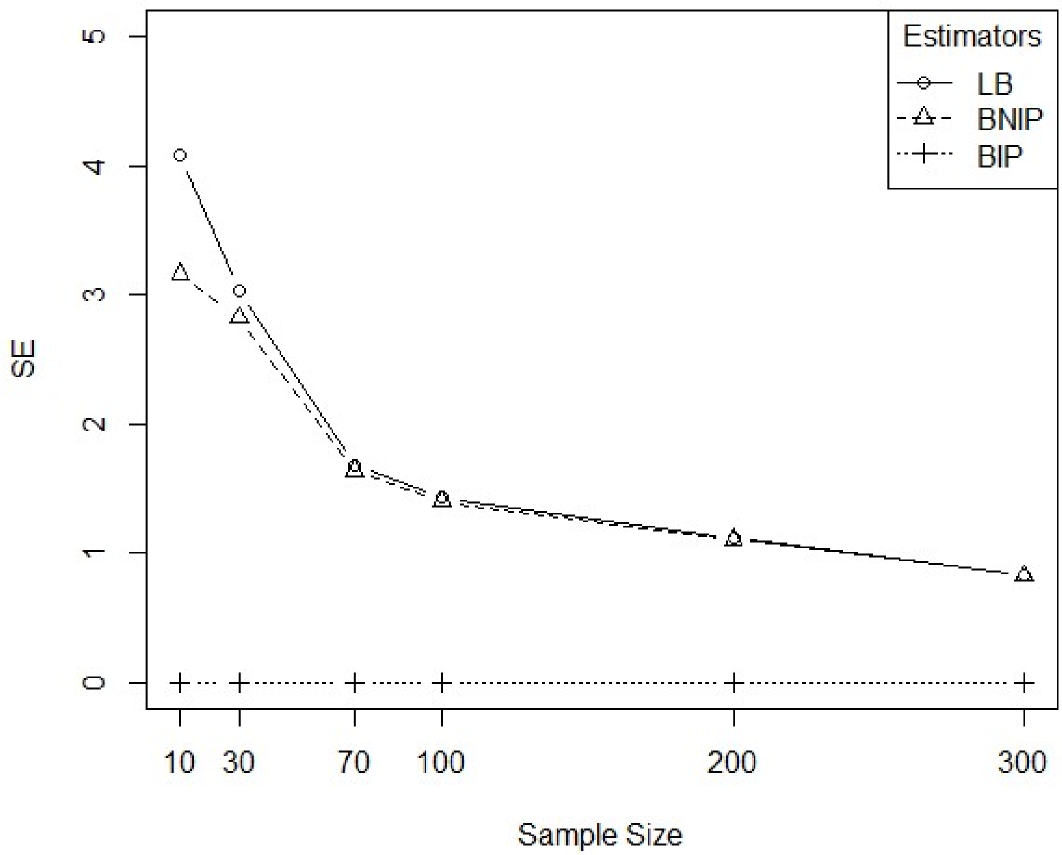

Figure 5.14: Plot of Standard Error for theta 1 of HNC (-0.95)

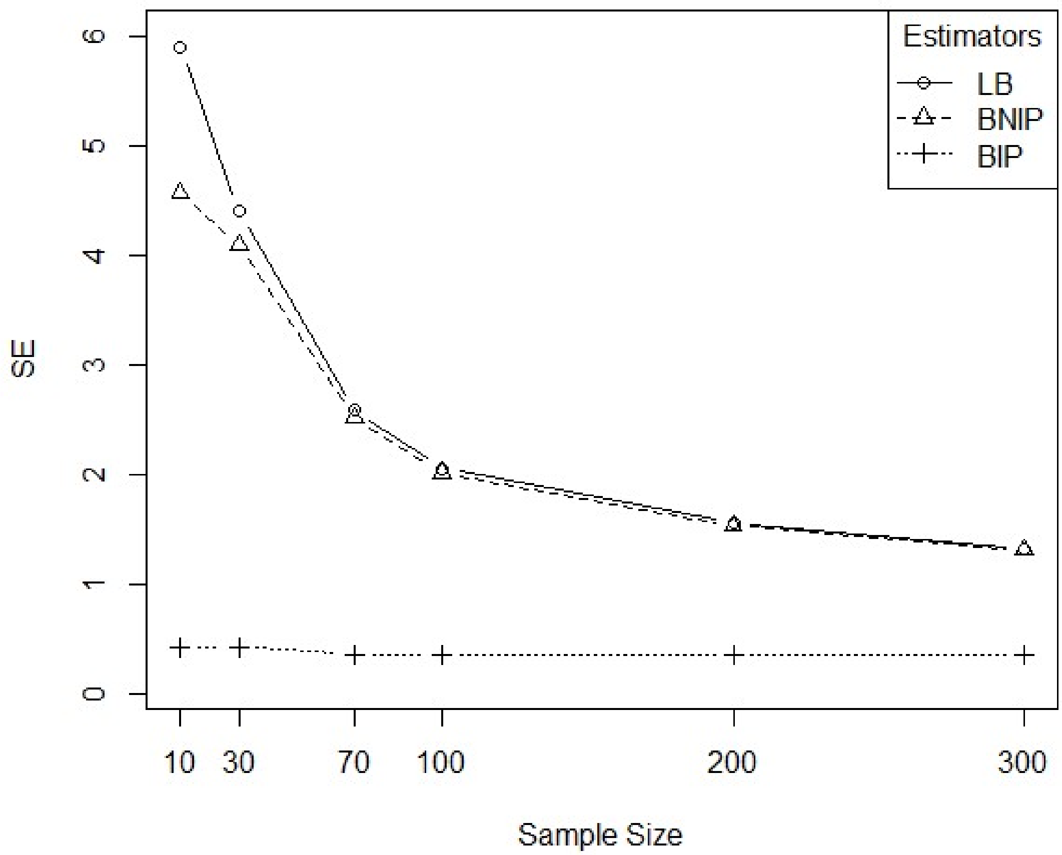

Figure 5.15: Plot of Standard Error for theta 2 of HNC (-0.95)

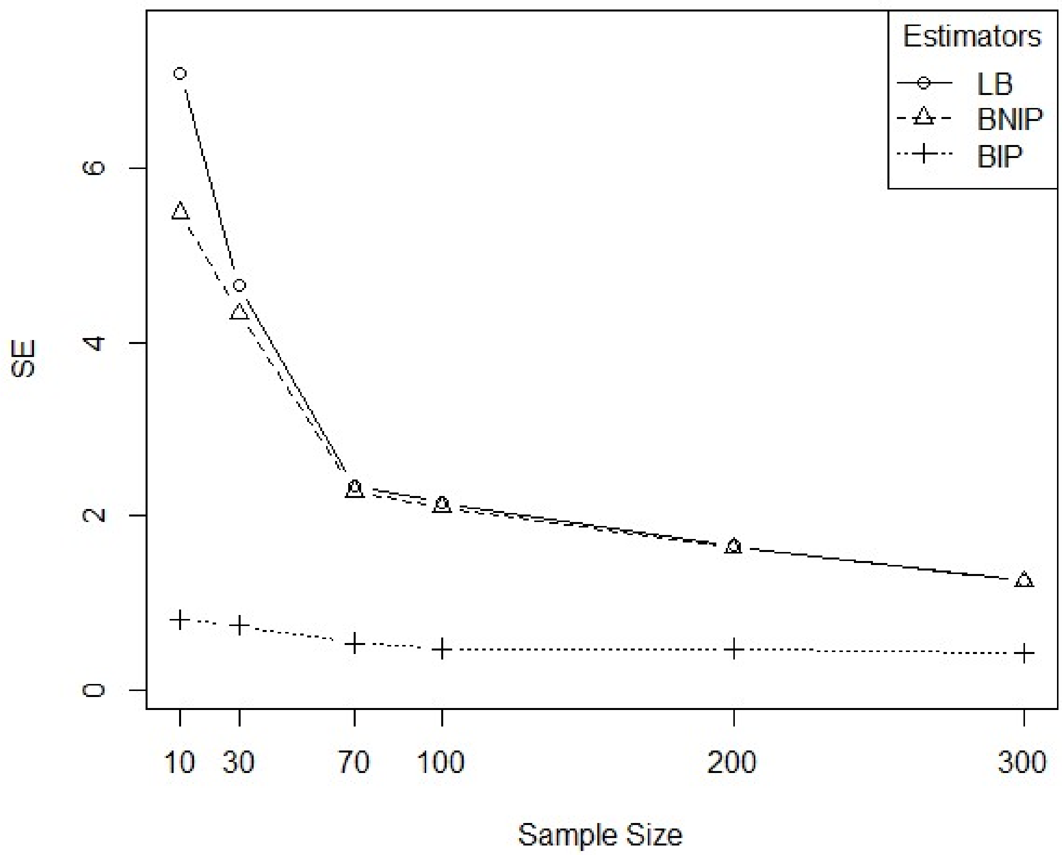

Figure 5.16: Plot of Standard Error for theta 3 of HNC (-0.95)

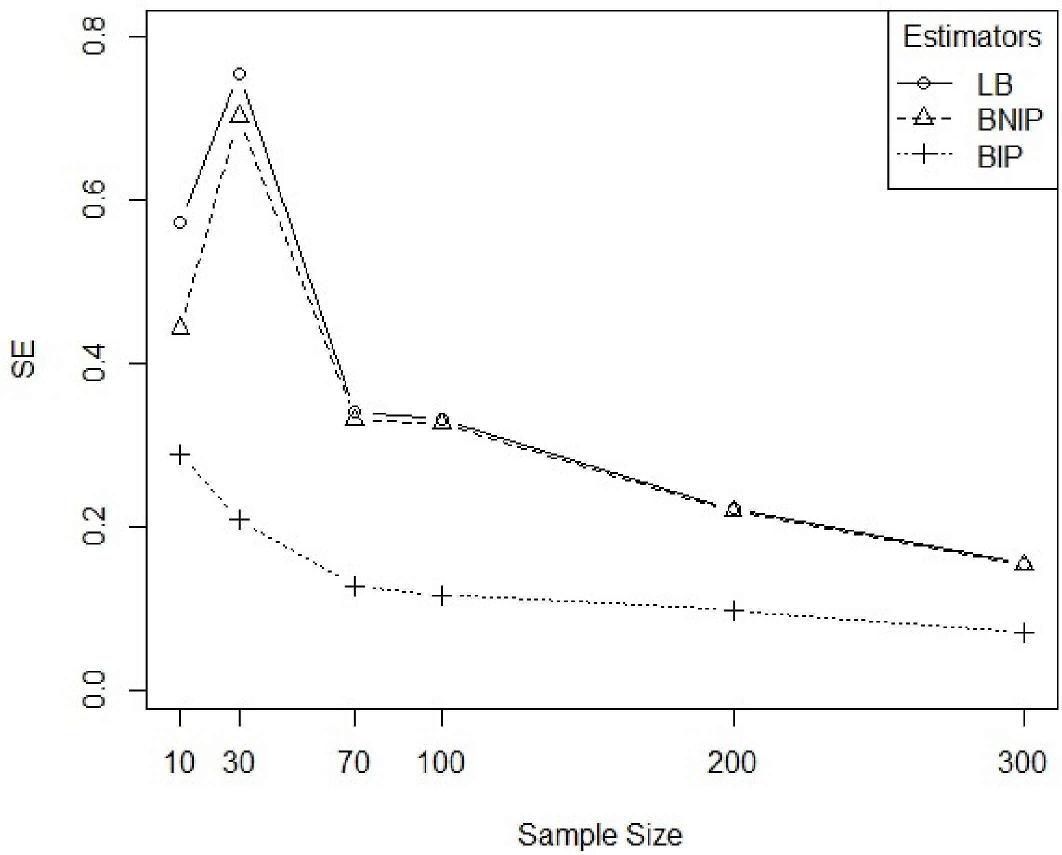

Figure 5.17: Plot of Standard Error for theta 0 of MNC (-0.49)

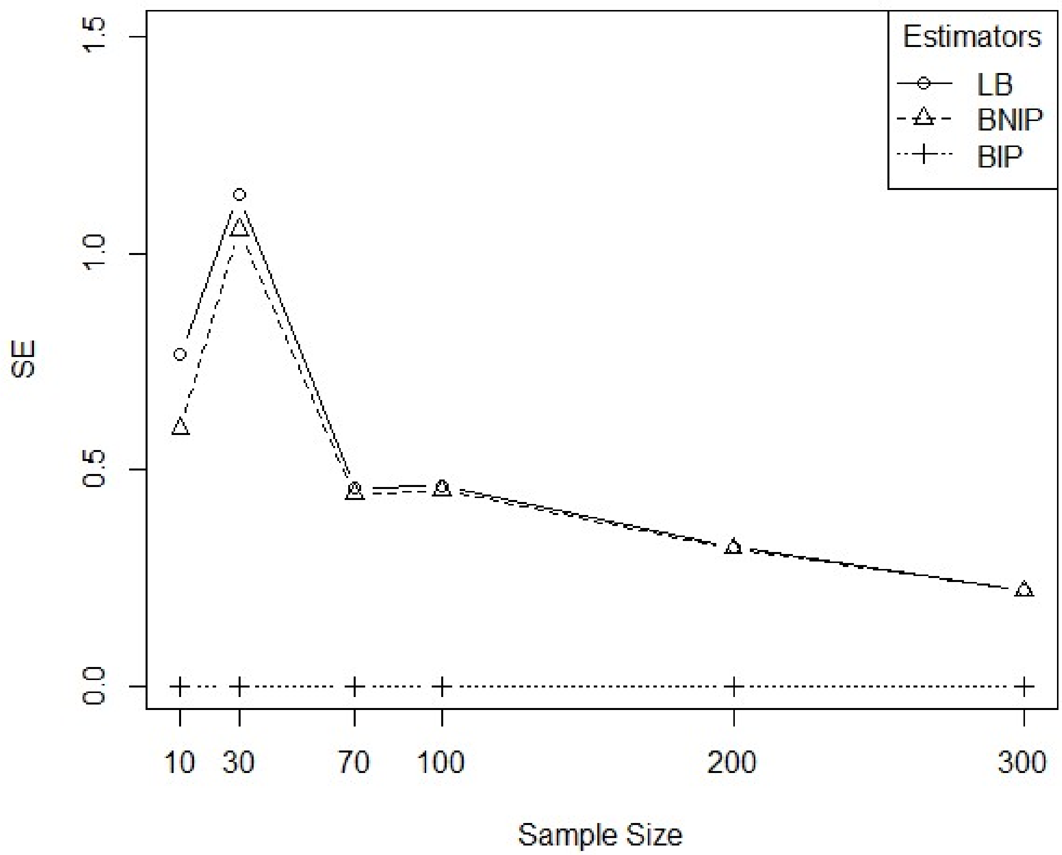

Figure 5.18: Plot of Standard Error for theta1 of MNC (-0.49)

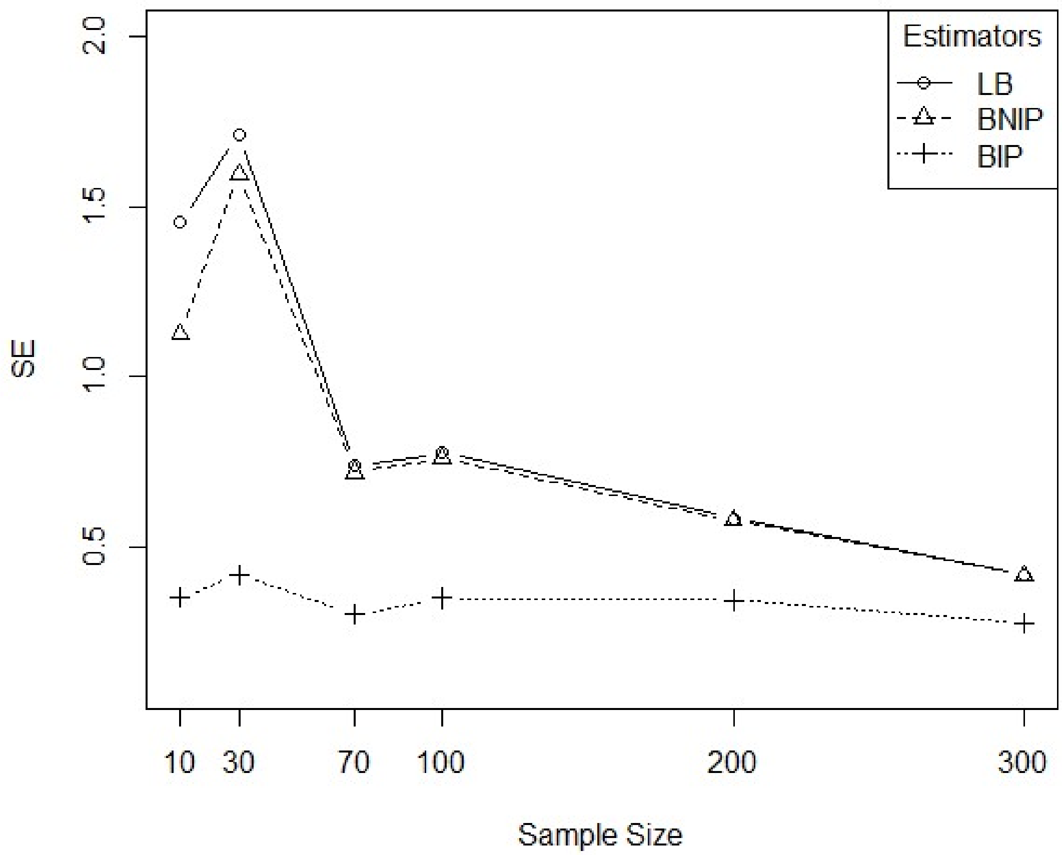

Figure 5.19: Plot of Standard Error for theta2 of MNC (-0.49)

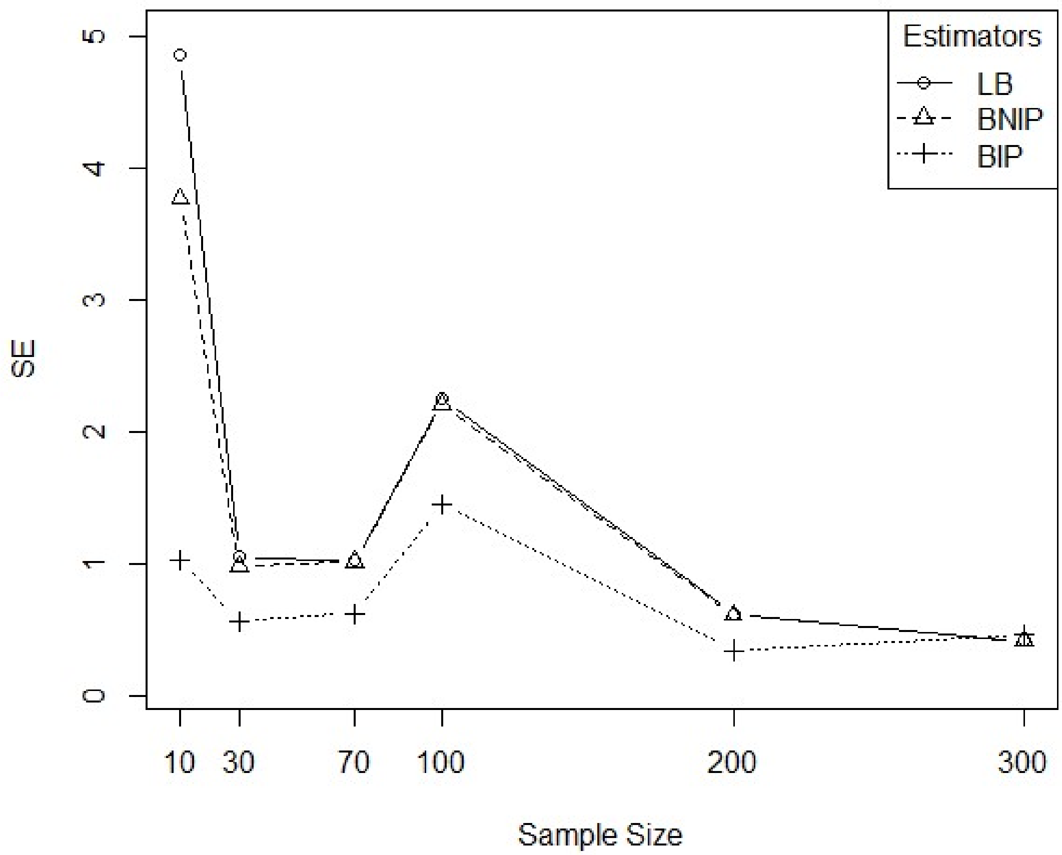

Figure 5.20: Plot of Standard Error for theta3 of MNC (-0.49)

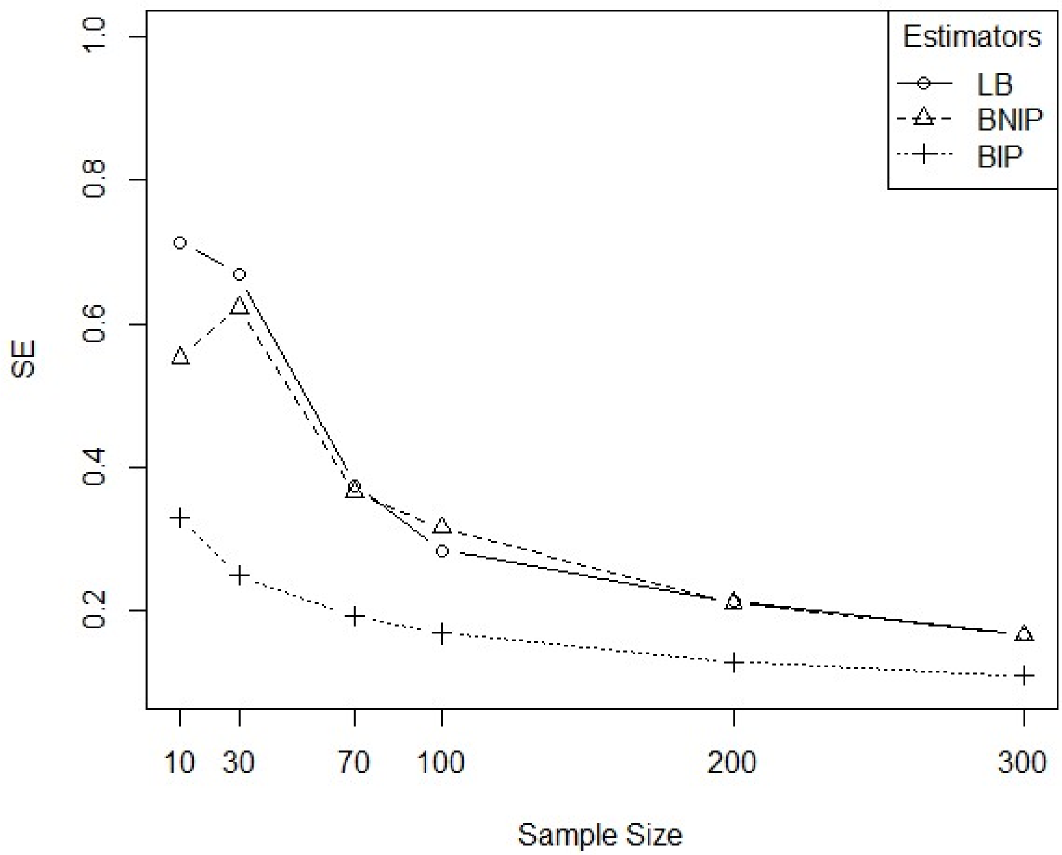

Figure 5.21: Plot of Standard Error for theta0 of LNC (-0.15)

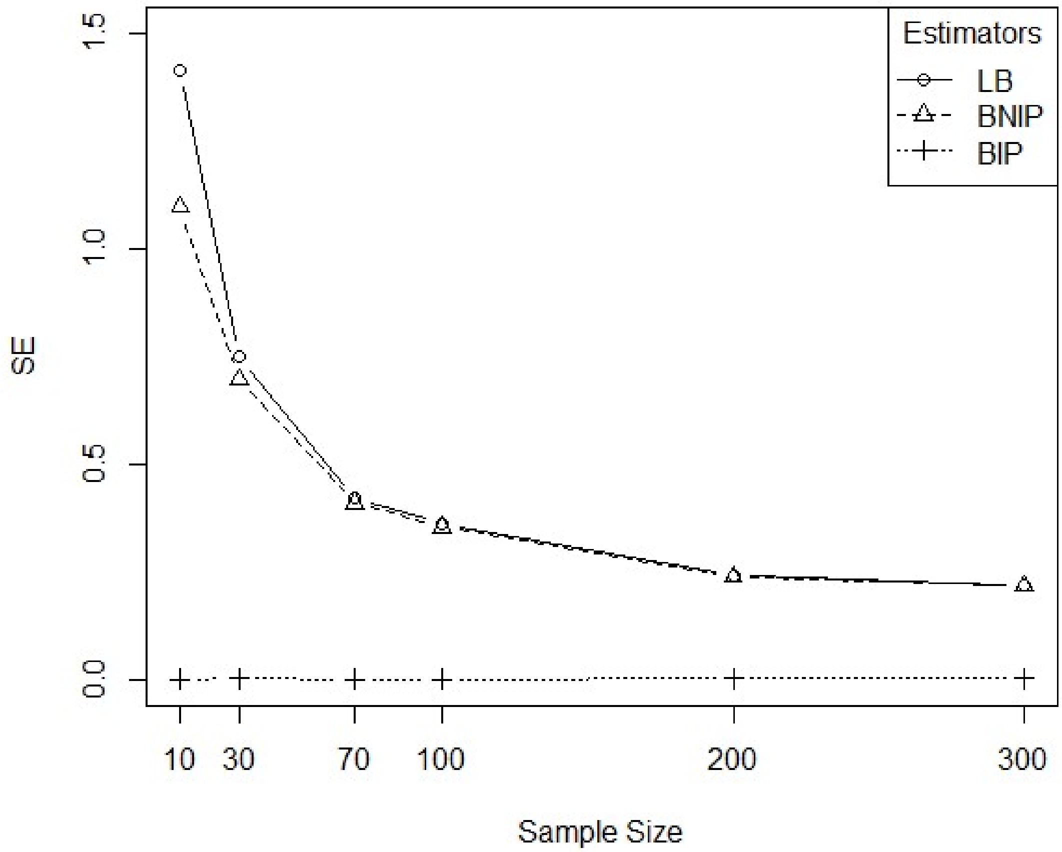

Figure 5.22: Plot of Standard Error for theta1 of LNC (-0.15)

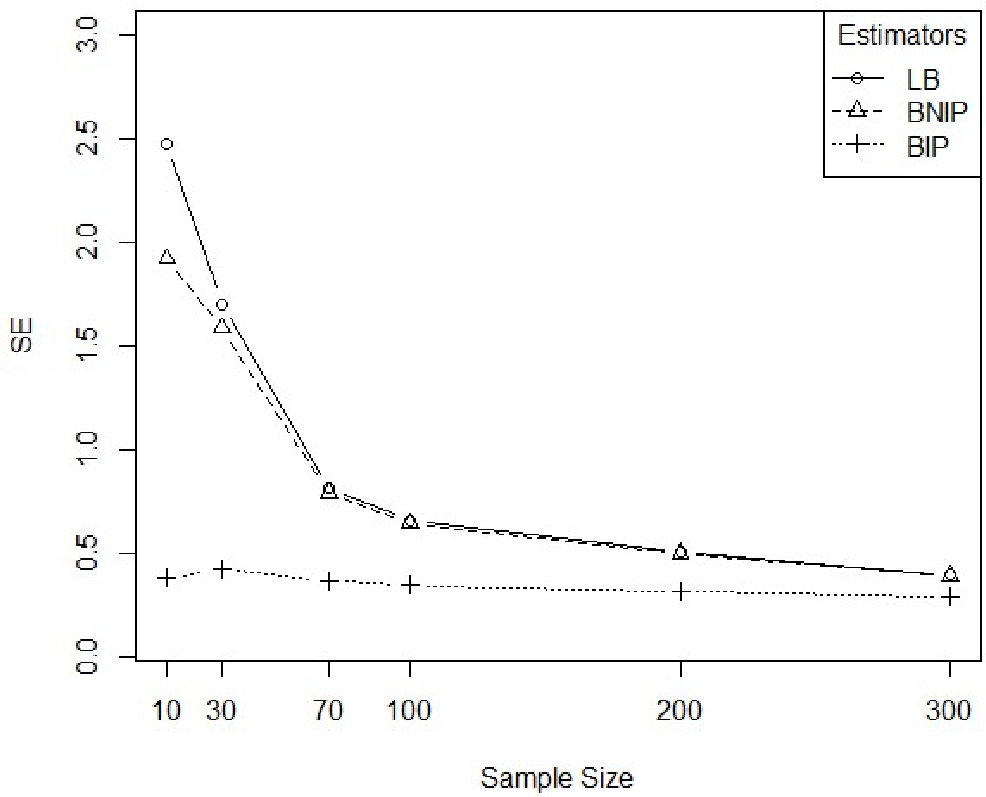

Figure 5.23: Plot of Standard Error for theta2 of LNC (-0.15)

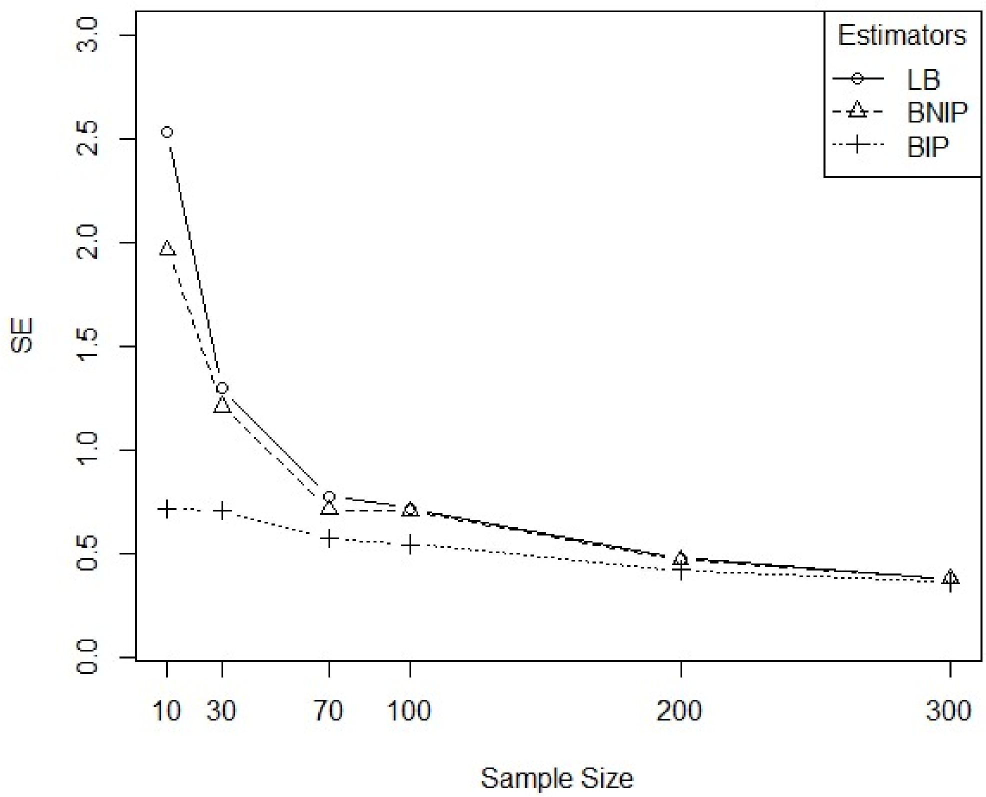

Figure 5.24: Plot of Standard Error for theta3 of LNC (-0.15)

The reliability of estimators can be visually seen in the Figures 5.1-5.24 about the SE, the smaller the SE, the better the model.

From the plots, as shown in Fig 5.1-5.24, LB has the highest SE followed by BNIP and BIP, it was also observed that as the sample size increases for HPC and HNC, the SE also decreases especially for LB and BNIP.

In Figure 5.5-5.8 and 5.17-5.20, the SE of the estimators for Moderate Positive Collinearity (MPC) and Moderate Negative Collinearity (MNC) shows an erratic pattern across the sample sizes.

The SE of all the estimators in figures 5.9-5.12 and 5.21-5.24 for Low Positive Collinearity (LPC) and Low Negative Collinearity (LNC) respectively are smaller than the HPC, HNC, MPC and MNC for all the sample sizes. However, SE of Bayesian estimators (BIP and BNIP) look better than the LB estimator having a smaller SE across all the sample sizes for the degrees of collinearity considered.

CHAPTER SIX

SUMMARY, CONCLUSION AND RECOMMENDATION

6.1 Summary

Multiollinearity is a violation of assumption of Linear Regression Model, and it is a situation where there is either an exact or approximately exact linear relationship among the regressors. Multicollinearity can make the regression coefficients indeterminate or to have a wrong sign, standard errors can tend to be large thereby reducing the precision of estimation.

Previous studies have proposed the use of classical method of estimation to overcome this problem but the classical inferences have shortcomings in that; the available information on parameters is ignored. Bayesian method combines the prior information on the parameters with the likelihood function to produce estimates. However, the use of out-of-sample information by the Bayesian approach to resolve this problem has not been fully explored in existing literature on the subject.

This study employed Bayesian technique to derive estimators to handle the problem of multicollinearity; Bayesian Non-informative Prior (BNIP) with a local uniform prior and Bayesian Informative Prior (BIP) with the natural conjugate prior, and their estimates with estimates of Likelihood Based (LB) in six cases of collinearity; High Positive Collinearity (HPC) ($0.50 \leq \mathrm{HPC} \leq 0.99$), Moderate Positive Collinearity (MPC) ($0.30 \leq \mathrm{MPC} \leq 0.49$), and Low Positive Collinearity (LPC) ($0.01 \leq \mathrm{LPC} \leq 0.29$), High Negative Collinearity (HNC) ($-0.99 \leq \mathrm{HNC} \leq -0.50$), Moderate Negative Collinearity (MNC) ($-0.49 \leq \mathrm{MNC} \leq -0.30$) and Low Negative Collinearity (LNC) ($-0.29 \leq \mathrm{LNC} \leq -0.01$) were compared using simulated data. For each case of collinearity, sample sizes were set at 10, 30, 70, 100, 200 and 300 while the posterior simulation for Bayesian Monte Carlo Integration (MCI) were replicated 1000, 10000 and 100000 times. In order to compare the performance of the estimators, the following criteria were used; mean, standard error and confidence/credible intervals.

6.2 Conclusion

Based on the results presented in Chapter five, the following conclusions were made:

Likelihood based (LB) estimator performed well in large samples (when N>30) for High Positive Collinearity (HPC) and High Negative Collinearity (HNC).

Likelihhood Based (LB) method has the same means with the Bayesian Non-Informative Prior (BNIP) method for all the cases of collinearity considered (i.e. High Positive Collinearity (HPC), Moderate Positive Collinearity (MPC), Low Positive Collinearity (LPC), High Negative Collinearity (HNC), Moderate Negative Collinearity (MNC) and Low Negative Collinearity (LNC)).

Bayesian estimators produced unbiased estimates when the prior is informative using the analytical and numerical solutions for Low Positive Collinearity (LPC) and Low Negative Collinearity (LNC) when the sample size is small.

Low Positive Colinearity (LPC) has the smallest standard error than other cases of collinearity considered (i.e. High Positive Collinearity (HPC), Moderate Positive Collinearity (MPC), High Negative Collinearity (HNC), Moderate Negative Collinearity (MNC) and Low Negative Collinearity (LNC)).

There is a remarkable asymptotic effect; sample size 70 appears to be a turning point by providing a better performance for Likelihood Based (LB) estimator at different cases of collinearity considered.

Likelihood Based (LB) estimator has a larger standard error and wider CI than Bayesian estimators (Bayesian Non-Informative Prior (BNIP) and Bayesian Informative (BIP)).

Likelihood Based (LB) and Bayesian Non-Informative Prior (BNIP) are almost similar when the collinearity is moderate, and low for positive and negative collinearity.

Bayesian Posterior simulation with 100000 replications outperformed other replications in the presence of multicollinearity.

Bayesian analytical solution with informative prior outperformed the Bayesian Posterior simulation method, using Monte Carlo Integration (MCI) for all the degrees of collinearity considered.

Bayesian estimators (Bayesian Non-Informative Prior (BNIP) and Bayesian Informative (BIP)) outperformed Likelihood Based (LB) method in the presence of multicollinearity.

Bayesian estimators (Bayesian Non-Informative Prior (BNIP) and Bayesian Informative (BIP)) are less sensitive to multicollinearity in the estimation of Normal Linear Regression Model.

6.3 Recommendation

Generally, from the results, Bayesian estimators outperformed the Likelihood Based method for all the cases of collinearity. Hence, the researchers are encouraged to use Bayesian estimators in the estimation of parameters of normal linear regression model when faced with the problem of multicollinearity while sufficient sample sizes should also be taken in the use of Likelihood based estimation method.

When performing a posterior simulation using Monte Carlo Integration (MCI) in the estimation of normal linear regression model, sufficient number of replications should be taken especially when encountered with the problem of multicollinearity in order to have accurate results.

6.4Research contributions

This work has contributed to knowledge through the following:

i. Introduction of Bayesian estimation method to the problem of multicollinearity.

ii. Provision of better Bayesian estimation procedure among the Bayesian estimators (Bayesian with Non-informative Prior and Bayesian Informative Prior) for different degrees of collinearity in the estimation of Normal Linear Regression Model.

iii. Providing the better method between the Bayesian analytical and Bayesian posterior simulation methods, using an informative prior in the estimation of parameters of Normal Linear Regression Model when there is multicollinearity

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APPENDIX I

MATLAB CODE FOR ANALYSIS

```
%program1 for Likelihood based, Bayesian Informative and Bayesian Non
informative priors when there is collineairty


%Data
n=size(Nmo100,1);
y=Nmo100(:,1);
xl=Nmo100(:,2:4)
x=[ones(n,1) xl];
k=4;

%Hyperparameters for natural conjugate prior
v0=4;
theta0=15*ones(k,1);
theta0(2,1)=10;
theta0(3,1)=5.5;
theta0(4,1)=2.5;

s02=1/1.5;
```

```
capv0=2.4*eye(k);
capv0(2,2)=6e-7;
capv0(3,3)=.15;
capv0(4,4)=.6;

capv0inv=inv(capv0);

%Call script which actually evaluates posterior analysis
Program2;



%Print out whatever you want
'Likelihood based results'
thetaols
s2
thetaolssd
v
thetahpdi95
thetahpdi99

'Hyperparameters for informative natural conjugate prior'
theta0
capv0
v0
s02

'Posterior results based on Informative Prior'
theta1
thetasd
probpos
thetahpdi95
thetahpdi99
hmean
hsd
Dse


%Hyperparameters for noninformative prior
v0=0;
capv0inv=0*eye(k);


%Call script which evaluates posterior analysis
ch3post;

%Print out whatever you want
'Posterior results based on Noninformative Prior'
theta1
thetasd
v0
v1s12
capv0inv
probpos
thetahpdi95
thetahpdi99
hmean
hsd
Dse
```

```
%The program that does the posterior analysis and calculate the
Likelihood Based, Non-informative and Informative

%Likelihood Based quantities
thetaols = inv(x'*x)*x'*y;
s2 = (y-x*thetaols)'*(y-x*thetaols)/(n-k);
thetaolscov = s2*inv(x'*x);
thetaolssd=zeros(k,1);
for i = 1:k
thetaolssd(i,1)=sqrt(thetaolscov(i,i));
end
v=n-k;

%Posterior hyperparameters for Normal-Gamma
xsquare=x'*x;
v1=v0+n;
capv1inv = capv0inv+ xsquare;
capv1=inv(capv1inv);
theta1 = capv1*(capv0inv*b0 + xsquare*thetaols);
if det(capv0inv)>0
    v1s12 = v0*s02 + v*s2 + (thetaols-b0)'*inv(capv0 +
inv(xsquare))*(thetaols-b0);
else
```

```
    v1s12 = v0*s02 + v*s2;
end
s12 = v1s12/v1;

thetacov = capv1*v1s12/(v1-2);
thetasd=zeros(k,1);
for i = 1:k
thetasd(i,1)=sqrt(thetacov(i,i));
end

%posterior probability that each element of theta is positive
%as well as 95 and 99 HPDIs for each element of theta

probpos=zeros(k,1);
thetahpdi95=zeros(k,2);
thetahpdi99=zeros(k,2);

%get quantiles of t for calculating HPDIs
invcdf95=tdis_inv(.975,v1);
invcdf99=tdis_inv(.995,v1);

for i = 1:k
    tnorm = -theta1(i,1)/sqrt(s12*capv1(i,i));
    probpos(i,1) = 1 - tdis_cdf(tnorm,v1);
    thetahpdi95(i,1) = theta1(i,1)-invcdf95*sqrt(s12*capv1(i,i));
    thetahpdi95(i,2) = theta1(i,1)+invcdf95*sqrt(s12*capv1(i,i));
    thetahpdi99(i,1) = theta1(i,1)-invcdf99*sqrt(s12*capv1(i,i));
    thetahpdi99(i,2) = theta1(i,1)+invcdf99*sqrt(s12*capv1(i,i));
end
for i = 1:k
Dse(i,1)=sqrt(s12*capv1(i,i));

end

%program for Posterior simulation using Monte Carlo Integration when
there is collinearity and sensitivity to increasing replications.


%Data
n=size(Nmo100,1);
y=Nmo100(:,1);
x=Nmo100(:,2:4);
x=[ones(n,1) x];
k=4;

%Hyperparameters for natural conjugate prior
v0=4;
theta0=15*ones(k,1);
theta 0(2,1)=10;
theta 0(3,1)=5.5;
theta 0(4,1)=2.5;

s02=1/1.5;
capv0=2.4*eye(k);
capv0(2,2)=6e-7;
capv0(3,3)=.15;
capv0(4,4)=.6;

capv0inv=inv(capv0);
```

```
%Ordinary least squares quantities
theta ols = inv(x'*x)*x'*y;
s2 = (y-x* theta ols)'*(y-x* theta ols)/(n-k);
v=n-k;

%Posterior hyperparameters for Normal-Gamma
xsquare=x'*x;
v1=v0+n;
capv1inv = capv0inv+ xsquare;
capv1=inv(capv1inv);
theta 1 = capv1*(capv0inv* theta 0 + xsquare* theta ols);
if det(capv0inv)>0
    v1s12 = v0*s02 + v*s1 + (theta ols-b0)'*inv(capv0 +
inv(xsquare))*(theta ols- theta 0);
else
    v1s12 = v0*s02 + v*s2;
end
s12 = v1s12/v1;

theta cov = capv1*v1s12/(v1-2);
thetasd=zeros(k,1);
for i = 1:k
theta sd(i,1)=sqrt(theta cov(i,i));
end

vscale = s12*capv1;
vchol=chol(vscale);
vchol=vchol';

%Now start Monte Carlo loop
%beta is t(b1,vscale,v1)
%For illustrative purpose we calculate only for theta(2)
theta 2mean=zeros(k,1);
theta 2square=zeros(k,1);

%Specify the number of replications
s=100000;

%tdis_rnd takes random draws from the multivariate t
%with mean zero and scale, V=I
%Hence we have to transform draws to get t(b1,bscale,v1)
for i = 1:s
%draw a t(0,1,v1) then transform to yield draw of beta
    theta draw= theta 1 + vchol*tdis_rnd(k,v1);
    theta 2mean= theta 2mean+ theta draw;
    theta 2square= theta 2square+ theta draw.^2;
end

theta 2mean= theta 2mean./s;
theta 2square= theta 2square./s;
theta 2var= theta 2square - theta 2mean.^2;
theta 2sd=sqr(theta 2var);


%posterior probability that each element of beta is positive
%as well as 95 and 99 HPDIs for each element of beta

probpos=zeros(k,1);
theta hpdi95=zeros(k,2);
```

```
theta hpdi99=zeros(k,2);

%get quantiles of t for calculating HPDIs
invcdf95=tdis_inv(.975,v1);
invcdf99=tdis_inv(.995,v1);

for i = 1:k
    tnorm = - theta 1(i,1)/sqrt(s12*capv1(i,i));
    probpos(i,1) = 1 - tdis_cdf(tnorm,v1);
    theta hpdi95(i,1) = theta 2mean(i,1)-
invcdf95*sqrt(s12*capv1(i,i));
    theta hpdi95(i,2) = theta
2mean(i,1)+invcdf95*sqrt(s12*capv1(i,i));
    theta hpdi99(i,1) = theta 2mean(i,1)-
invcdf99*sqrt(s12*capv1(i,i));
    theta hpdi99(i,2) = theta
2mean(i,1)+invcdf99*sqrt(s12*capv1(i,i));

end
%Print out whatever you want
'Hyperparameters for informative natural conjugate prior'
theta 0
capv0
v0
s02

'Posterior results based on Informative Prior'
theta 1
theta sd
s
theta 2mean
theta 2sd
nse
probpos
bhpdi95
bhpdi99
```

APPENDIX 2

Data set for sample size of 10 for high positive collinearity

Y	X_0	X_1	X_2	X_3
22.81752807	1	0.502405199	0.255004893	0.415827648
29.87958437	1	0.951488056	0.605534621	0.588613042
27.64102038	1	0.926320815	0.456799005	0.552386278
24.66092633	1	0.637852589	0.359515319	0.327931428
21.1577874	1	0.214433215	0.15021884	0.121946379
19.47349779	1	0.135152625	0.137302882	0.109082486
19.96084431	1	0.163307922	0.239016328	0.086688476
19.5727307	1	0.127562818	0.129516987	0.21617298

19.2092248	1	0.018513348	0.191113086	0.205232349
23.37408783	1	0.429523177	0.361826041	0.282196246

Data set for sample size of 10 for moderate positive collinearity

Y	X_0	X_1	X_2	X_3
17.75081469	1	0.030365676	0.343514995	0.16109384
28.66762205	1	0.964490665	0.530098214	0.464952474
25.47411902	1	0.934364763	0.314376414	0.456901277
26.77632226	1	0.684699135	0.568615547	0.3998622
22.79115291	1	0.332027274	0.137675369	0.101543223
24.84988608	1	0.594976025	0.324242591	0.192659085
24.40986648	1	0.696023346	0.379943386	0.323010095
25.51080479	1	0.829781724	0.525383552	0.575676256
26.23666213	1	0.855112314	0.472055423	0.296023645
21.29270787	1	0.193891306	0.256731054	0.326035185

Data set for sample size of 10 for low positive collinearity

Y	X_0	X_1	X_2	X_3
22.13427849	1	0.236692197	0.200680566	0.0468092
23.60815392	1	0.681777578	0.195789009	0.096544806
27.00125631	1	0.814875215	0.413026571	0.338954294
23.83331528	1	0.490913059	0.476763112	0.483132988
20.12962439	1	0.081513348	0.428830281	0.49508124
21.57535056	1	0.198127962	0.410095508	0.505574149
19.777154	1	0.098496365	0.410352893	0.155484997

23.80303005 1	0.492050074	0.480396208	0.209635077
18.41585028 1	0.076259246	0.082623183	0.398008845
23.85836073 1	0.639859278	0.389486072	0.075382879

Data set for sample size of 30 for high positive collinearity

Y	X_1	X_2	X_3
21.542	0.4287	0.2142	0.316
17.714	0.0053	0.0810	0.1926
16.998	0.0224	0.0889	0.020
23.758	0.4109	0.2233	0.2087
19.61	0.0832	0.1095	0.0514
20.025	0.2939	0.1525	0.1761
19.881	0.1348	0.1743	0.1812
23.44	0.4010	0.3048	0.3013
24.942	0.3519	0.3627	0.1900
17.326	0.0855	0.2063	0.101
22.869	0.3630	0.2669	0.2623
25.980	0.7180	0.4347	0.5169
23.668	0.529	0.4342	0.3170
29.378	0.8692	0.5838	0.5854
19.083	0.0989	0.2504	0.1072
20.986	0.3693	0.2520	0.1807
23.823	0.476	0.326	0.3765
19.782	0.0558	0.2294	0.0724
29.272	0.992	0.5696	0.4906
27.740	0.8681	0.5479	0.451
26.147	0.8223	0.4379	0.5057
20.598	0.3586	0.2080	0.283
29.228	0.7907	0.4878	0.3691
19.136	0.0682	0.1594	0.0608
23.856	0.4612	0.2160	0.3879
24.842	0.5889	0.4027	0.4605
23.424	0.3837	0.2	0.1823
20.233	0.4023	0.190	0.2234
21.615	0.3865	0.2235	0.2396
27.904	0.9556	0.5163	0.5707

Data set for sample size of 30 for moderate positiv collinearity

Y	X_1	X_2	X_3
24.461	0.5538	0.2557	0.2258
27.78	0.8262	0.5166	0.2501
25.276	0.5913	0.4859	0.2891
26.657	0.8529	0.3926	0.5982
25.436	0.6177	0.2610	0.3226
24.603	0.6791	0.2487	0.4336
27.333	0.9454	0.4805	0.4279

24.187	0.465	0.4592	0.228
23.453	0.4063	0.3909	0.2619
27.091	0.8815	0.4185	0.5860
26.022	0.7864	0.3034	0.3248
20.302	0.1848	0.3670	0.351
24.271	0.5818	0.4598	0.2165
17.711	0.027	0.0499	0.1938
28.701	0.9847	0.5620	0.6476
27.469	0.7660	0.6283	0.3135
25.422	0.5187	0.4794	0.2679
21.269	0.2880	0.390	0.0891
23.669	0.5814	0.4550	0.3463
27.764	0.9512	0.4500	0.3383
18.922	0.0687	0.0688	0.0925
19.754	0.1541	0.3152	0.2140
21.573	0.3061	0.1056	0.4531
24.660	0.6700	0.2639	0.3709
20.750	0.2922	0.283	0.4036
24.184	0.5023	0.2679	0.4069
27.689	0.7982	0.5886	0.5597
20.844	0.2242	0.3749	0.3792
21.501	0.2817	0.4526	0.3465
26.942	0.7030	0.4217	0.5863

Data set for sample size of 30 for low collinearity

Y	X_1	X_2	X_3
27.52	0.8073	0.2681	0.479
24.701	0.499	0.1847	0.4451
22.140	0.1770	0.4716	0.2029
25.827	0.8549	0.296	0.5219
19.940	0.0624	0.3905	0.4097
28.095	0.9300	0.1919	0.5485
21.623	0.3276	0.0820	0.1540
23.288	0.6615	0.2615	0.0857
28.730	0.9449	0.4981	0.4062
28.418	0.9609	0.2710	0.5165
27.523	0.8147	0.3795	0.4260
20.205	0.2376	0.1257	0.2612
28.45	0.8860	0.393	0.3295
23.362	0.6167	0.1742	0.0754
27.000	0.7631	0.3435	0.2625
26.795	0.9323	0.4524	0.3021
28.224	0.9853	0.2044	0.4331
20.240	0.230	0.1616	0.0933
23.287	0.2321	0.3883	0.4425

24.87	0.6600	0.34	0.0730
21.778	0.4481	0.4239	0.1132
20.271	0.0542	0.2359	0.0494
28.060	0.7212	0.4948	0.4346
23.87	0.2893	0.3871	0.4889
23.742	0.5164	0.284	0.0739
23.119	0.3701	0.2629	0.2693
23.46	0.5484	0.2427	0.2236
21.143	0.362	0.0826	0.0933
22.092	0.4196	0.236	0.5098
21.626	0.4626	0.2536	0.1023

Data set for sample size of 70 for high collinearity

Y	X_1	X_2	X_3
21.9	0.5046	0.3260	0.2446
22.229	0.4021	0.2565	0.2425
24.042	0.6936	0.4562	0.4552
27.689	0.9073	0.502	0.5027
23.028	0.4402	0.3562	0.241
24.854	0.6853	0.3642	0.4760
17.686	0.0522	0.233	0.0523
25.80	0.5939	0.3267	0.3395
18.775	0.0658	0.1736	0.1187
28.408	0.8996	0.5342	0.5491
23.090	0.5774	0.2750	0.4536
19.714	0.0040	0.1882	0.1544
16.421	0.0448	0.0933	0.0445
19.974	0.2932	0.1929	0.1319
20.954	0.2899	0.1874	0.238
23.892	0.5426	0.3018	0.3154
22.579	0.4351	0.2718	0.34
22.396	0.4429	0.2383	0.3526
22.169	0.4341	0.3362	0.3743
17.350	0.1828	0.1416	0.1613
19.620	0.1098	0.1331	0.1214
21.338	0.2498	0.323	0.1342
28.559	0.863	0.3978	0.4688
22.226	0.4095	0.3509	0.3737
28.993	0.9918	0.5146	0.585
19.483	0.0905	0.1571	0.1667
22.315	0.3536	0.2770	0.3366
23.093	0.4097	0.2580	0.3369
15.537	0.043	0.0530	0.0258
24.305	0.6049	0.448	0.4798
26.922	0.662	0.4316	0.3257
28.395	0.9074	0.4348	0.5521
28.059	0.8363	0.4561	0.40
26.936	0.6651	0.308	0.3087
20.345	0.3672	0.2647	0.2364
25.511	0.7743	0.3705	0.4237
21.449	0.4721	0.316	0.3945
22.388	0.4327	0.2428	0.3412
29.663	0.9604	0.5952	0.5248
30.953	0.9776	0.6124	0.4837
20.426	0.146	0.0944	0.1096
24.89	0.5964	0.3477	0.2941

20.591	0.3965	0.2526	0.1901
23.165	0.4258	0.383	0.2706
28.063	0.911	0.4148	0.5829
22.537	0.3622	0.2002	0.2965
27.970	0.8238	0.4828	0.5383
26.535	0.723	0.4653	0.5175
23.859	0.5494	0.2760	0.4334
18.822	0.0575	0.1286	0.2179
22.334	0.4020	0.2213	0.3702
27.480	0.9588	0.5316	0.5798
29.109	0.9458	0.4529	0.5520
19.546	0.2884	0.2852	0.2958
28.6	0.9050	0.4557	0.5690
19.434	0.2108	0.184	0.1545
24.802	0.4827	0.4137	0.2808
27.292	0.7504	0.3785	0.3833
24.686	0.6757	0.4028	0.3538
23.16	0.4963	0.2316	0.2715
19.765	0.3175	0.2611	0.2201
22.318	0.2672	0.2863	0.1728
19.923	0.1585	0.1649	0.2368
25.776	0.8013	0.3628	0.5161
23.272	0.4894	0.3540	0.4168
22.842	0.4766	0.3959	0.3436
26.894	0.8854	0.6131	0.5153
20.624	0.3210	0.192	0.1447
18.579	0.1275	0.0962	0.0863
21.806	0.4636	0.305	0.3236

Data set for sample size of 70 for moderate positive collinearity

Y	X_1	X_2	X_3
23.399	0.3644	0.4148	0.1544
25.549	0.7004	0.3628	0.5253
27.101	0.7972	0.4984	0.2618
24.475	0.5406	0.318	0.1997
25.73	0.8514	0.2814	0.2759
23.360	0.483	0.4805	0.3312
16.861	0.030	0.0988	0.3735
25.49	0.3959	0.4318	0.3866
19.132	0.0964	0.4150	0.3922
24.486	0.5821	0.2283	0.2607
24.145	0.4771	0.4039	0.272
24.337	0.3616	0.1567	0.4069
20.772	0.0529	0.3651	0.2660

23.13	0.4142	0.3034	0.4735
20.431	0.0650	0.1980	0.3137
19.597	0.285	0.1673	0.2562
30.288	0.950	0.5539	0.681
21.166	0.3935	0.2359	0.1653
20.958	0.1261	0.230	0.0898
21.866	0.3414	0.3098	0.1025
22.912	0.6553	0.242	0.571
27.042	0.7390	0.6157	0.2708
22.704	0.481	0.4568	0.3358
24.627	0.527	0.3554	0.5333
27.02	0.9015	0.4593	0.5155
25.974	0.6199	0.3494	0.5384
24.452	0.4980	0.2641	0.4809
21.063	0.3310	0.3583	0.2300
21.350	0.2710	0.4797	0.1489
21.61	0.2188	0.2215	0.2792
23.104	0.3679	0.5074	0.426
25.113	0.4884	0.4454	0.5268
22.241	0.4032	0.4615	0.2570
18.086	0.0947	0.0807	0.3603
19.10	0.3082	0.1647	0.2869
20.416	0.1732	0.3643	0.1926
19.138	0.2247	0.1445	0.3851
27.508	0.9094	0.3667	0.3357
25.327	0.4818	0.3508	0.4589
21.529	0.3330	0.4205	0.1871
26.80	0.7982	0.3647	0.465
19.831	0.0759	0.0354	0.1493
29.550	0.9831	0.5697	0.2998
27.923	0.7262	0.3986	0.3788
23.442	0.3551	0.4058	0.5064
27.883	0.9473	0.3965	0.3878
23.782	0.4759	0.4689	0.321
24.349	0.6608	0.3278	0.4668
23.461	0.4081	0.4382	0.1278
18.809	0.0614	0.1042	0.1936
27.264	0.9210	0.6309	0.547
20.473	0.2363	0.2838	0.0768
26.763	0.8652	0.4723	0.4808
21.929	0.3817	0.1432	0.2368
17.353	0.0649	0.2654	0.0353
24.895	0.6721	0.4170	0.4767
20.732	0.3310	0.3453	0.324
18.087	0.0466	0.1958	0.2629
25.14	0.6555	0.5187	0.202

21.758	0.3271	0.4789	0.1302
25.808	0.650	0.5009	0.3864
18.018	0.0360	0.1600	0.2899
19.300	0.1082	0.4143	0.095
22.191	0.4476	0.1953	0.2649
21.104	0.3197	0.1869	0.2035
26.735	0.6548	0.5509	0.5636
25.57	0.6637	0.3642	0.5674
18.773	0.2288	0.1541	0.2619
20.244	0.0650	0.3419	0.3196
26.371	0.7372	0.6029	0.2857

Data set for sample size of 70 for low collinearity

Y	X_1	X_2	X_3
27.17	0.878	0.4522	0.2679
21.059	0.3306	0.2418	0.2094
28.502	0.9889	0.5602	0.5828
22.29	0.5380	0.1498	0.0921
26.089	0.5980	0.4691	0.2946
19.412	0.086	0.4170	0.0771
26.162	0.7846	0.0939	0.489
28.271	0.9594	0.405	0.2709
21.60	0.3342	0.3754	0.364
25.92	0.6673	0.4867	0.4387
25.680	0.8154	0.3336	0.3034
26.710	0.8876	0.1835	0.2558
27.558	0.7629	0.4188	0.1443
24.417	0.6588	0.3831	0.1380
21.679	0.2734	0.0595	0.0574
22.428	0.2782	0.3816	0.3857
21.858	0.151	0.3803	0.3955
24.940	0.6595	0.5399	0.0750
25.677	0.862	0.2171	0.1510
22.644	0.5815	0.1912	0.5228
29.614	0.9971	0.3881	0.4802
26.192	0.9701	0.115	0.2120
27.465	0.9354	0.3488	0.3071
20.027	0.1010	0.1446	0.3159
25.692	0.9193	0.2779	0.3776
21.327	0.2032	0.128	0.0666
27.430	0.8239	0.4531	0.2534
21.017	0.7261	0.2508	0.2029
19.793	0.2429	0.1376	0.3793
25.844	0.6676	0.5107	0.2077
20.514	0.1392	0.1899	0.090

25.3	0.7179	0.3429	0.1755
17.910	0.0244	0.1230	0.3600
23.903	0.5624	0.3928	0.1853
24.630	0.4511	0.4078	0.2182
20.309	0.322	0.0780	0.313
23.817	0.4190	0.2441	0.5298
20.888	0.3728	0.0422	0.3170
26.23	0.6555	0.3819	0.2745
25.235	0.5511	0.4716	0.3537
18.716	0.1496	0.0738	0.2316
20.312	0.2529	0.2905	0.272
25.800	0.9180	0.2391	0.3540
23.493	0.4879	0.1155	0.3765
27.714	0.9958	0.1440	0.3581
22.792	0.5669	0.207	0.1593
24.721	0.7544	0.2565	0.0882
23.643	0.6584	0.2451	0.2942
21.941	0.1449	0.4441	0.0883
23.230	0.3312	0.2322	0.415
18.994	0.0444	0.4897	0.0798
19.682	0.2110	0.3226	0.1428
19.68	0.030	0.2503	0.4306
23.598	0.6502	0.3467	0.4389
25.165	0.5247	0.4631	0.4766
26.988	0.815	0.1841	0.1502
18.811	0.0446	0.3987	0.1343
22.922	0.2007	0.4752	0.3590
26.742	0.9423	0.3524	0.5001
22.165	0.3699	0.2665	0.0470
26.524	0.6685	0.4594	0.0753
24.8	0.5755	0.4723	0.1483
29.513	0.9914	0.5759	0.5306
26.441	0.6608	0.2637	0.5139
22.492	0.4402	0.1272	0.4784
23.578	0.5535	0.5196	0.4870
17.823	0.1824	0.1791	0.2487
21.171	0.2449	0.3072	0.0533
26.460	0.7615	0.2286	0.3412
23.440	0.5027	0.4095	0.0725

Data set for sample size of 100 for high collinearity

Y	X_1	X_2	X_3
27.075	0.8899	0.5231	0.3712
22.978	0.4146	0.330	0.3205
24.289	0.4240	0.3492	0.4218
27.051	0.7125	0.4354	0.5148
23.149	0.5178	0.300	0.4999
24.608	0.5372	0.300	0.4216
18.955	0.020	0.0987	0.3005
29.620	0.9030	0.4891	0.6574
18.363	0.0844	0.0489	0.1425
25.313	0.6210	0.4779	0.462
21.865	0.2738	0.3287	0.2584
19.268	0.0992	0.1398	0.2937
27.929	0.9794	0.4693	0.4636
22.234	0.270	0.1709	0.3987
28.350	0.9822	0.4693	0.430
21.950	0.3222	0.1575	0.2115
17.827	0.0364	0.1149	0.1170
21.395	0.2077	0.2495	0.1830
21.093	0.1600	0.2432	0.2820
29.067	0.9446	0.6073	0.5198
29.621	0.9254	0.4852	0.5966
26.607	0.8262	0.4191	0.5986
22.522	0.4327	0.3062	0.2596
19.384	0.221	0.2430	0.1875
28.758	0.9368	0.6004	0.3927
17.536	0.0224	0.1564	0.1433
24.142	0.646	0.4459	0.3133
18.907	0.0090	0.1948	0.2888
21.297	0.1118	0.2654	0.3372
25.226	0.56	0.3640	0.3966
17.90	0.1849	0.2063	0.3302
20.939	0.2725	0.1917	0.2447
21.277	0.2324	0.2819	0.1203
29.023	0.9525	0.5270	0.4713
21.461	0.1737	0.1970	0.2705
29.513	0.8502	0.5120	0.6245
29.993	0.9758	0.6551	0.4889
28.804	0.83	0.565	0.3422
24.03	0.5933	0.4007	0.3072
16.901	0.1093	0.066	0.1932
20.908	0.2931	0.303	0.3769
20.288	0.4016	0.3417	0.3269

26.644	0.74	0.4398	0.3068
29.068	0.9997	0.5520	0.4044
28.560	0.9256	0.615	0.5768
18.991	0.0376	0.2234	0.0913
25.616	0.639	0.3865	0.3463
29.203	0.9769	0.4607	0.6884
19.724	0.207	0.0953	0.3348
22.536	0.6301	0.2864	0.5343
27.517	0.8401	0.4016	0.353
20.430	0.3173	0.2809	0.2218
24.421	0.6206	0.3407	0.3874
26.913	0.7480	0.5205	0.3007
24.726	0.3651	0.3599	0.424
22.716	0.2240	0.3045	0.2344
19.834	0.1734	0.2888	0.1583
21.17	0.4541	0.3637	0.2012
26.595	0.6351	0.4623	0.5220
30.055	0.98	0.577	0.6368
26.151	0.8218	0.4646	0.5572
23.134	0.2879	0.2113	0.121
19.526	0.2365	0.1824	0.343
20.97	0.2730	0.2912	0.1679
22.307	0.4264	0.3151	0.2118
25.838	0.5263	0.395	0.4611
19.113	0.0988	0.2376	0.1541
22.410	0.3856	0.1917	0.3030
25.594	0.5706	0.3570	0.314
29.678	0.8795	0.4249	0.5908
18.968	0.0135	0.096	0.1794
17.692	0.0984	0.142	0.056
21.530	0.3749	0.1785	0.2417
22.305	0.1932	0.2034	0.312
28.059	0.8638	0.5601	0.3658
27.459	0.9279	0.4548	0.5995
25.513	0.5633	0.3569	0.4476
24.169	0.4950	0.3347	0.2497
25.725	0.569	0.3243	0.4470
26.755	0.8636	0.47	0.5162
19.190	0.1753	0.2053	0.1362
25.873	0.7758	0.4778	0.3463
18.378	0.142	0.0679	0.3070
24.825	0.7124	0.3562	0.2934
19.7	0.0996	0.1861	0.2372
18.998	0.085	0.1636	0.3325
22.903	0.4114	0.2879	0.4607
21.09	0.209	0.2526	0.085

19.500	0.1892	0.1495	0.3680
28.346	0.9091	0.4297	0.6209
23.001	0.4778	0.3994	0.3900
23.903	0.6451	0.4653	0.3918
19.54	0.1195	0.1734	0.0675
21.286	0.3346	0.3108	0.2175
18.846	0.2063	0.1241	0.1804
29.809	0.8723	0.5836	0.5096
24.866	0.4582	0.4233	0.4139
23.572	0.4505	0.2950	0.2175
22.858	0.5209	0.3045	0.3298
19.565	0.1223	0.0917	0.3065

Data set for sample size of 100 for moderate positiv
collinearity

Y	X_1	X_2	X_3
	0.8140	0.2133	0.3321
24.356	0.6192	0.2483	0.2347
18.738	0.2545	0.1906	0.1947
18.436	0.0252	0.1045	0.3027
23.492	0.4796	0.3207	0.1441
26.196	0.7751	0.3848	0.4358
26.06	0.7780	0.3737	0.421
24.350	0.5008	0.3683	0.2151
25.980	0.7748	0.2203	0.6063
21.182	0.2511	0.4490	0.1922
24.773	0.4940	0.3952	0.301
23.223	0.5278	0.4554	0.3206
20.650	0.0490	0.1361	0.2811
29.953	0.9837	0.4746	0.5506
22.857	0.3727	0.5176	0.2089
27.042	0.8763	0.4925	0.2436
24.905	0.6658	0.2955	0.5519
19.506	0.1989	0.1516	0.086
23.982	0.4915	0.3337	0.5059
17.757	0.099	0.1761	0.0282
20.158	0.0544	0.3714	0.0272
27.768	0.9582	0.3400	0.4454
27.893	0.9800	0.5173	0.4487
22.415	0.4520	0.1518	0.2164
24.848	0.5791	0.4181	0.1768
21.667	0.1996	0.3607	0.0739
27.577	0.9526	0.3896	0.2782
20.855	0.0044	0.3805	0.2374
24.094	0.5081	0.3219	0.2469

25.514	0.771	0.3614	0.2310
21.794	0.2364	0.196	0.4179
23.895	0.6066	0.260	0.2280
27.784	0.7524	0.5966	0.45
23.327	0.5247	0.2028	0.5457
23.179	0.4758	0.3953	0.4877
18.234	0.0075	0.2232	0.0663
19.773	0.219	0.1326	0.1819
21.062	0.1564	0.4026	0.2122
25.52	0.53	0.3611	0.468
24.511	0.693	0.2260	0.3965
26.953	0.8019	0.4084	0.5325
17.021	0.1086	0.0596	0.2702
17.794	0.041	0.1484	0.1538
21.916	0.3741	0.4080	0.2075
24.554	0.3752	0.3745	0.2753
24.971	0.8065	0.3949	0.4369
24.489	0.6554	0.4550	0.2712
20.238	0.3089	0.2013	0.1083
28.586	0.9252	0.4247	0.5851
29.298	0.9208	0.4104	0.5217
21.007	0.2788	0.1208	0.3173
25.378	0.6027	0.3312	0.1587
20.091	0.339	0.1511	0.2312
20.081	0.2797	0.2439	0.1313
26.804	0.9090	0.4763	0.431
27.100	0.8307	0.5851	0.4653
21.207	0.0331	0.3989	0.2311
23.571	0.3623	0.1793	0.3068
24.395	0.6234	0.4871	0.2531
25.351	0.5030	0.3920	0.4475
21.973	0.3736	0.1148	0.3008
26.182	0.7720	0.2868	0.3618
22.495	0.2121	0.1760	0.4598
23.097	0.4682	0.1919	0.2647
26.029	0.6443	0.4860	0.166
24.264	0.8537	0.2733	0.2787
22.817	0.4774	0.1658	0.5312
18.369	0.0549	0.144	0.2697
24.625	0.5065	0.1655	0.2250
19.258	0.2289	0.2568	0.1828
18.687	0.1559	0.0615	0.2034
19.118	0.0035	0.0163	0.3058
19.09	0.1103	0.0618	0.0311
19.246	0.0622	0.138	0.1364
21.990	0.3631	0.2635	0.3034

25.522	0.7329	0.5521	0.1953
19.640	0.367	0.1959	0.3091
21.5	0.3784	0.1594	0.2537
21.660	0.3183	0.2958	0.3673
2'	0.938	0.3211	0.4168
18.781	0.0151	0.0432	0.3232
24.794	0.6733	0.5070	0.2135
22.06	0.3634	0.165	0.4870
20.81	0.2368	0.1385	0.3767
20.052	0.2082	0.1436	0.1557
20.97	0.1492	0.1958	0.4171
22.990	0.4245	0.5095	0.1527
21.70	0.624	0.2227	0.1835
25.277	0.6329	0.5357	0.3511
20.042	0.0268	0.2139	0.283
21.971	0.5245	0.1915	0.3725
26.150	0.7911	0.2087	0.5895
25.763	0.6433	0.510	0.4650
28.818	0.9574	0.5794	0.6295
21.702	0.3239	0.151	0.1583
20.299	0.0824	0.0392	0.3433
26.451	0.9115	0.3522	0.4413
20.956	0.202	0.4049	0.2577
24.106	0.5891	0.5316	0.1943
27.309	0.7047	0.4290	0.541

Data set for sample size of 100 for low collinearity

Y	X_1	X_2	X_3
27.937	0.9852	0.311	0.4019
24.565	0.69	0.2961	0.1118
22.917	0.4610	0.4584	0.1683
22.829	0.6381	0.1408	0.1042
23.354	0.3823	0.4164	0.4246
21.515	0.3265	0.501	0.1577
18.016	0.138	0.102	0.1047
19.92	0.1638	0.130	0.4503
20.368	0.3640	0.067	0.4064
19.858	0.046	0.48	0.0602
20.361	0.0870	0.4615	0.1760
22.296	0.5333	0.2192	0.433
21.803	0.4355	0.3252	0.3533
25.565	0.8555	0.3446	0.5302
21.553	0.3100	0.2656	0.2065
21.52	0.1728	0.3538	0.4489
25.184	0.6195	0.3076	0.2921

28.907	0.9305	0.4296	0.1952
22.476	0.4810	0.065	0.3267
28.680	0.9622	0.5698	0.3492
22.591	0.4433	0.2839	0.187
23.922	0.6693	0.1871	0.2085
24.607	0.6978	0.1400	0.1578
21.781	0.3538	0.1980	0.3933
19.371	0.0096	0.1834	0.25
21.452	0.0935	0.3725	0.3663
19.850	0.0872	0.3048	0.4779
24.439	0.5976	0.3774	0.1442
22.607	0.7337	0.0807	0.074
18.113	0.1520	0.325	0.1306
22.837	0.4048	0.1270	0.429
23.755	0.2394	0.4245	0.2961
24.147	0.6907	0.2584	0.1087
23.06	0.4742	0.1176	0.5005
18.957	0.2045	0.1502	0.2421
26.74	0.8135	0.2042	0.4087
20.934	0.2893	0.2528	0.3117
24.692	0.9752	0.2156	0.2858
19.968	0.1877	0.2703	0.3593
23.54	0.3883	0.4162	0.2439
26.068	0.6681	0.4436	0.4285
19.779	0.3244	0.1650	0.0640
24.230	0.4271	0.5222	0.4570
21.645	0.3487	0.1123	0.1356
27.018	0.8961	0.2174	0.4693
27.176	0.9055	0.129	0.5319
22.364	0.3650	0.2765	0.1970
27.303	0.8707	0.4974	0.1323
21.983	0.338	0.1998	0.4095
19.647	0.0725	0.1670	0.3075
25.951	0.6116	0.5055	0.4065
30.733	0.9071	0.5016	0.5643
24.739	0.6874	0.0986	0.2016
17.231	0.0190	0.3517	0.3256
25.664	0.7108	0.2946	0.2859
23.70	0.6153	0.3990	0.425
24.192	0.593	0.492	0.4897
23.643	0.5296	0.3653	0.3707
26.43	0.8181	0.5067	0.4527
22.372	0.3826	0.1258	0.3966
21.389	0.4529	0.083	0.2398
21.297	0.3441	0.1740	0.4929
20.143	0.2938	0.1044	0.3474

21.433	0.0708	0.2477	0.4387
26.013	0.8069	0.3203	0.4752
26.7	0.6974	0.5364	0.5498
21.003	0.2308	0.1967	0.1418
20.769	0.27	0.152	0.2401
22.636	0.4153	0.4072	0.4314
19.336	0.2457	0.1351	0.2053
20.658	0.2830	0.2647	0.2828
21.595	0.4132	0.0978	0.3950
25.395	0.895	0.2126	0.2085
25.915	0.8120	0.3208	0.1972
25.792	0.8399	0.3489	0.3407
22.	0.4499	0.35	0.0778
23.486	0.4417	0.3174	0.1212
26.169	0.879	0.288	0.1996
21.585	0.1633	0.1250	0.4920
19.834	0.0780	0.1064	0.4269
24.383	0.3847	0.5249	0.3580
24.464	0.6262	0.490	0.4754
21.065	0.1671	0.4059	0.4280
18.667	0.2186	0.0824	0.1551
27.246	0.9698	0.5370	0.1429
18.488	0.0486	0.1989	0.4869
20.972	0.2473	0.2250	0.0483
27.393	0.7894	0.4860	0.5558
28.561	0.9992	0.2389	0.3031
25.008	0.4886	0.4360	0.0808
25.507	0.7253	0.341	0.3489
25.345	0.8444	0.1221	0.3610
24.703	0.2455	0.5017	0.3020
22.498	0.3899	0.0544	0.4597
25.386	0.7835	0.1274	0.121
23.104	0.4114	0.2083	0.1527
24.86	0.6000	0.4560	0.1317
18.735	0.0180	0.0726	0.1006
24.256	0.2083	0.4906	0.2001
22.400	0.4639	0.099	0.4645

Data set for sample size of 200 for high positive collinearity

28.262	0.9448	0.4991	0.4582
19.431	0.3163	0.1530	0.3159
22.447	0.4192	0.3510	0.252
20.926	0.252	0.2772	0.2364
19.408	0.1816	0.1519	0.2815
26.64	0.8106	0.5593	0.4054
26.862	0.6170	0.4909	0.3152
29.131	0.9045	0.5515	0.4294
17.402	0.0182	0.0105	0.1295
29.364	0.9767	0.4793	0.5212
24.427	0.5474	0.2639	0.2797
28.6	0.8837	0.446	0.5898
23.181	0.4751	0.3526	0.3379
28.560	0.8482	0.5830	0.3982
23.242	0.479	0.4329	0.3755
17.775	0.0756	0.1480	0.1639
18.700	0.0572	0.1973	0.0900
27.062	0.5993	0.3921	0.4853
25.353	0.6385	0.4588	0.4306
29.879	0.9658	0.5425	0.5832
30.629	0.9010	0.4118	0.6188
30.695	0.9777	0.649	0.641
20.856	0.4316	0.329	0.2342
21.508	0.2757	0.1716	0.2159
28.962	0.8894	0.4481	0.6087
26.339	0.8367	0.4592	0.390
26.931	0.8027	0.5689	0.5044
27.332	0.6854	0.4684	0.4120
25.525	0.5628	0.4012	0.4169
25.384	0.726	0.3514	0.3597
26.727	0.5666	0.3506	0.2638
25.264	0.677	0.4059	0.4770
26.636	0.7332	0.4240	0.4278
29.537	0.9536	0.6273	0.4676
26.723	0.7952	0.5592	0.3892
28.034	0.8497	0.4896	0.5079
26.151	0.7936	0.4848	0.5050
27.893	0.8287	0.5740	0.4453
27.169	0.8481	0.5289	0.4013
20.017	0.0071	0.1352	0.1396
18.518	0.1006	0.2586	0.0752
23.89	0.4391	0.2707	0.2300

24.802	0.545	0.3458	0.2687
27.588	0.8670	0.4717	0.5398
25.013	0.5208	0.4134	0.3001
23.676	0.5266	0.2782	0.3957
19.86	0.1592	0.1904	0.1852
24.81	0.6145	0.415	0.4337
23.013	0.4769	0.2700	0.3987
20.962	0.2397	0.2808	0.2153
25.68	0.7354	0.5079	0.3861
26.666	0.7030	0.5056	0.5285
18.809	0.1903	0.1696	0.2347
17.912	0.0427	0.1748	0.2230
23.064	0.3909	0.3820	0.2266
24.312	0.5911	0.2826	0.3748
26.670	0.8059	0.5026	0.4154
27.968	0.8439	0.5361	0.4981
28.795	0.8440	0.4519	0.4007
25.728	0.5183	0.4496	0.4126
22.882	0.6048	0.2748	0.4246
20.758	0.3635	0.2196	0.2520
24.921	0.4901	0.3286	0.2919
20.545	0.2277	0.1175	0.1941
27.443	0.8094	0.5238	0.4007
21.614	0.3043	0.2515	0.2229
21.012	0.2959	0.331	0.1606
29.307	0.967	0.6306	0.444
26.400	0.7716	0.4669	0.3824
20.246	0.0811	0.1856	0.0425
25.824	0.6866	0.484	0.4143
23.973	0.4037	0.3337	0.3562
28.872	0.9069	0.572	0.4630
25.75	0.8571	0.5224	0.4595
22.412	0.4510	0.2316	0.2357
28.304	0.9883	0.5974	0.4557
24.100	0.5858	0.3840	0.3068
27.269	0.8715	0.4555	0.5312
19.251	0.0574	0.2280	0.118
17.673	0.1650	0.0779	0.0820
23.768	0.4128	0.3835	0.2865
19.242	0.0687	0.2094	0.1395
21.304	0.2663	0.3103	0.3280
26.873	0.7473	0.4463	0.4704
18.02	0.1003	0.2381	0.0709
20.489	0.1713	0.240	0.2211
16.854	9.41E-	0.0754	0.08
22.889	0.4267	0.3795	0.258

18.792	0.2184	0.1899	0.1427
21.817	0.4026	0.220	0.2626
20.408	0.2076	0.2643	0.1443
19.847	0.1598	0.1046	0.0906
27.563	0.724	0.3605	0.5105
20.460	0.1311	0.1756	0.2617
25.614	0.669	0.4150	0.3843
17.42	0.024	0.1119	0.1606
29.077	0.9618	0.4817	0.6017
27.097	0.7429	0.4322	0.5474
17.96	0.0450	0.0301	0.0505
23.479	0.5255	0.3054	0.4241
19.721	0.2866	0.3043	0.2905
21.474	0.4224	0.2203	0.1901
21.483	0.2899	0.3164	0.2757
23.091	0.4036	0.3403	0.2806
24.259	0.5636	0.3601	0.4072
30.483	0.9917	0.5642	0.4762
22.152	0.3346	0.1982	0.2004
20.678	0.1258	0.2127	0.2055
20.072	0.2393	0.1152	0.202
19.497	0.2494	0.2432	0.2656
28.751	0.927	0.434	0.6051
29.64	0.8265	0.4449	0.576
24.032	0.4092	0.3752	0.2747
18.901	0.1249	0.1853	0.0610
29.696	0.9856	0.5656	0.6348
20.720	0.2124	0.280	0.1656
20.503	0.3148	0.15	0.3131
21.005	0.4282	0.2035	0.1998
21.742	0.4927	0.2360	0.271
22.0	0.4511	0.2923	0.2460
25.812	0.6783	0.3542	0.3447
21.261	0.3070	0.3307	0.2775
20.751	0.2078	0.1419	0.1888
26.498	0.8294	0.3863	0.472
27.047	0.8187	0.4228	0.5582
20.824	0.2563	0.2543	0.1932
24.530	0.6876	0.323	0.3367
19.924	0.3409	0.1835	0.1772
23.820	0.4661	0.2780	0.342
24.039	0.44	0.2405	0.335
29.54	0.899	0.5304	0.5289
19.813	0.1927	0.2212	0.1894
25.069	0.4194	0.3500	0.2872
29.94	0.9969	0.6510	0.6185

18.364	0.0173	0.0627	0.0207
25.372	0.6354	0.2993	0.3077
19.091	0.1607	0.0894	0.259
23.360	0.5658	0.3788	0.3536
20.642	0.3044	0.2922	0.2779
19.960	0.2039	0.279	0.1122
19.803	0.2680	0.1679	0.1721
28.423	0.8270	0.5832	0.5473
19.668	0.1084	0.1627	0.1067
17.286	0.1511	0.1817	0.2499
21.047	0.3295	0.2699	0.2047
27.087	0.7721	0.5191	0.4085
26.058	0.6957	0.4677	0.4385
19.246	0.0002	0.2122	0.044
24.34	0.6094	0.4036	0.4052
25.759	0.6867	0.4735	0.362
27.692	0.8054	0.3665	0.4527
18.824	0.2167	0.1334	0.0991
22.141	0.347	0.3533	0.3638
18.55	0.042	0.078	0.0686
18.401	0.0277	0.0549	0.09
19.271	0.1408	0.2511	0.2470
26.431	0.7586	0.5369	0.390
20.441	0.1450	0.1682	0.1264
22.347	0.2932	0.3180	0.3303
20.931	0.1562	0.2408	0.0710
19.33	0.0486	0.0881	0.2228
19.757	0.0563	0.2338	0.0407
21.223	0.0893	0.203	0.234
27.189	0.838	0.4660	0.5680
22.225	0.2540	0.2700	0.2712
21.643	0.3587	0.1888	0.2616
23.112	0.7211	0.3280	0.5187
27.368	0.9068	0.4906	0.4660
21.357	0.3870	0.193	0.3791
19.743	0.3355	0.1644	0.3039
29.186	0.8153	0.4832	0.4866
27.836	0.9654	0.5106	0.549
27.07	0.9109	0.4215	0.5733
29.681	0.9027	0.519	0.4767
26.095	0.8607	0.4790	0.4443
26.834	0.5087	0.4091	0.3706
22.38	0.3309	0.215	0.2708
29.536	0.7809	0.5564	0.4551
17.495	0.051	0.0231	0.1830
19.180	0.1122	0.1505	0.1762

19.879	0.1608	0.2316	0.1838
19.261	0.0100	0.2199	0.0512
25.783	0.5882	0.3290	0.4430
27.900	0.8517	0.4396	0.4971
27.946	0.8942	0.59	0.5129
25.058	0.7294	0.4028	0.4230
25.093	0.6806	0.3191	0.513
30.323	0.994	0.6364	0.5018
26.706	0.6503	0.4690	0.4422
25.926	0.6319	0.3464	0.4565
19.810	0.0870	0.1698	0.1749
17.483	0.0557	0.0391	0.0345
28.115	0.663	0.4282	0.4776
27.678	0.7630	0.3725	0.4951
18.508	0.0626	0.1231	0.220
24.423	0.607	0.4303	0.4531
27.327	0.9093	0.4960	0.4543
16.714	0.0355	0.0883	0.1061
27.716	0.9020	0.5630	0.4116
21.164	0.4719	0.2565	0.3877

Data set for sample size of 200 for moderate positiv collinearity

Y	X_1	X_2	X_3
23.562	0.4153	0.439	0.3249
18.545	0.211	0.2743	0.1912
18.614	0.0041	0.2757	0.0885
29.886	0.9532	0.6115	0.5129
26.434	0.7912	0.3030	0.5411
21.488	0.0706	0.3738	0.296
28.089	0.9438	0.3281	0.3276
28.729	0.938	0.3254	0.3963
21.638	0.3278	0.4447	0.461
23.163	0.4494	0.2859	0.3714
22.903	0.1931	0.2255	0.4268
23.434	0.3915	0.406	0.3551
25.338	0.613	0.3069	0.4721
23.023	0.4684	0.4058	0.3497
21.492	0.3958	0.1460	0.3063
24.547	0.6785	0.232	0.2607
30.413	0.9259	0.5249	0.5561
22.161	0.2001	0.3017	0.3842
26.610	0.8916	0.3396	0.3919
21.102	0.1630	0.3016	0.382
23.043	0.3954	0.3513	0.1645

23.685	0.3713	0.4434	0.4146
26.028	0.4882	0.4929	0.1813
23.604	0.5150	0.5227	0.3167
23.593	0.5006	0.1746	0.3341
26.643	0.8631	0.4965	0.4449
20.717	0.1438	0.3851	0.2059
21.157	0.4348	0.1979	0.4086
23.737	0.3845	0.3174	0.1787
23.449	0.4839	0.2958	0.4056
26.664	0.850	0.334	0.3788
21.914	0.3373	0.223	0.1160
20.966	0.2123	0.2637	0.1164
20.89	0.3166	0.1696	0.2683
21.265	0.3897	0.1418	0.4228
23.203	0.4140	0.1938	0.469
26.218	0.9139	0.396	0.4490
21.873	0.2433	0.3942	0.2178
18.615	0.1029	0.1436	0.0376
26.286	0.807	0.4845	0.6214
17.51	0.1020	0.2193	0.1199
20.097	0.1364	0.2434	0.1906
23.250	0.4338	0.2327	0.4919
25.185	0.7069	0.4302	0.2204
27.474	0.7445	0.3221	0.3071
22.803	0.3977	0.4760	0.2693
22.351	0.3243	0.4502	0.112
23.098	0.43	0.4247	0.3726
22.449	0.3171	0.278	0.4755
22.896	0.6930	0.2298	0.2156
27.27	0.775	0.5438	0.443
22.221	0.3923	0.3508	0.4335
18.309	0.0953	0.1850	0.0929
18.744	0.1153	0.062	0.1943
21.395	0.3404	0.1050	0.3771
20.2	0.3008	0.4354	0.1734
24.798	0.6800	0.29	0.4321
25.388	0.6554	0.4151	0.221
24.978	0.6409	0.4641	0.3322
23.49	0.7152	0.3627	0.2305
20.393	0.1524	0.1642	0.3926
21.138	0.0059	0.3682	0.2357
26.651	0.8898	0.5960	0.3933
24.956	0.4420	0.3659	0.3420
25.476	0.7608	0.3445	0.2691
20.02	0.1017	0.1993	0.0583
19.307	0.1062	0.3444	0.1013

27.742	0.8605	0.474	0.3798
20.32	0.139	0.0689	0.3804
29.53	0.8864	0.5652	0.3752
19.964	0.212	0.2173	0.3820
22.729	0.4406	0.2491	0.3396
25.133	0.5987	0.2469	0.2918
23.149	0.4010	0.3041	0.1519
19.867	0.2561	0.1365	0.211
22.839	0.4861	0.1884	0.2716
27.376	0.7864	0.5061	0.5401
27.778	0.8823	0.4752	0.4318
25.035	0.4559	0.4933	0.3024
29.932	0.9049	0.5224	0.6266
19.105	0.1242	0.0481	0.1981
21.216	0.2302	0.2306	0.372
20.14	0.3410	0.1741	0.1676
23.354	0.4571	0.4056	0.291
27.712	0.8084	0.6213	0.3258
23.264	0.4596	0.4551	0.3740
23.894	0.7437	0.3645	0.3884
19.104	0.0468	0.1197	0.0763
21.647	0.1566	0.4224	0.0780
25.096	0.69	0.4430	0.5425
25.38	0.645	0.2214	0.4629
19.993	0.3176	0.2308	0.3445
22.815	0.3395	0.3424	0.449
26.329	0.6359	0.4582	0.4725
26.107	0.6999	0.5475	0.4065
29.122	0.7927	0.616	0.3851
27.363	0.8632	0.4459	0.6516
22.469	0.3201	0.438	0.4144
25.224	0.5407	0.2860	0.5170
29.650	0.900	0.6423	0.5534
21.768	0.2727	0.1869	0.2877
18.136	0.03	0.0105	0.3028
27.650	0.8670	0.453	0.5158
18.615	0.0558	0.0344	0.3866
19.568	0.0313	0.334	0.3845
22.656	0.6102	0.2159	0.4426
23.276	0.331	0.2539	0.4186
23.896	0.3380	0.4186	0.1870
25.059	0.5682	0.2273	0.3702
28.542	0.9385	0.5258	0.5840
29.234	0.973	0.5469	0.3845
22.733	0.3210	0.3887	0.3637
21.984	0.1852	0.2572	0.1175

19.132	0.1159	0.429	0.3820
20.196	0.2836	0.0974	0.0873
26.754	0.8313	0.3	0.6473
27.231	0.8599	0.3565	0.5838
26.822	0.9497	0.4751	0.3007
28.039	0.8962	0.5577	0.4762
27.250	0.5368	0.5455	0.4257
18.306	0.1537	0.0922	0.2775
24.516	0.5243	0.5491	0.3013
30.176	0.9794	0.5663	0.516
22.072	0.3247	0.2474	0.3170
18.247	0.0476	0.3342	0.0766
24.959	0.6170	0.4835	0.2324
25.672	0.8177	0.2581	0.2769
19.687	0.0518	0.2790	0.1933
21.172	0.1388	0.3487	0.249
22.633	0.3788	0.1646	0.3835
26.549	0.8530	0.6552	0.4397
19.395	0.1624	0.2348	0.1368
22.427	0.3770	0.3022	0.3180
24.14	0.3982	0.4685	0.2653
18.283	0.0938	0.1019	0.407
22.411	0.3187	0.281	0.40
20.97	0.3824	0.3038	0.2574
20.504	0.3063	0.3279	0.3094
22.038	0.4065	0.3009	0.3908
22.688	0.514	0.2986	0.1623
28.31	0.9837	0.4897	0.5939
26.836	0.7419	0.523	0.3809
22.792	0.3642	0.1591	0.4062
19.748	0.2262	0.1548	0.2431
27.332	0.5994	0.5720	0.4604
20.000	0.1168	0.3633	0.3248
27.52	0.6223	0.530	0.365
19.459	0.3419	0.1327	0.3929
22.225	0.3678	0.254	0.3257
21.800	0.1775	0.4186	0.4290
26.868	0.8305	0.4915	0.5014
24.683	0.6309	0.374	0.2640
25.970	0.8411	0.2663	0.5057
20.475	0.2227	0.2313	0.2186
27.70	0.9760	0.3747	0.5299
21.204	0.1730	0.438	0.3228
19.105	0.0291	0.2191	0.0305
22.173	0.4091	0.188	0.4902
25.316	0.5483	0.5614	0.5280

18.964	0.2473	0.3330	0.136
24.844	0.8160	0.2645	0.4413
23.950	0.4585	0.4956	0.294
26.491	0.6082	0.5766	0.2963
26.959	0.7223	0.5889	0.3105
23.157	0.6319	0.2011	0.3244
19.609	0.1096	0.3469	0.1913
18.056	0.0242	0.1146	0.2866
19.820	0.1259	0.1497	0.3364
23.452	0.7142	0.4110	0.2312
22.573	0.4513	0.2615	0.1636
22.451	0.6217	0.2467	0.2534
18.63	0.1269	0.1040	0.0607
21.807	0.3380	0.4135	0.3678
20.865	0.3279	0.4078	0.2978
21.135	0.3059	0.4736	0.1011
18.674	0.0955	0.0548	0.155
21.009	0.0458	0.3327	0.293
27.017	0.6345	0.4595	0.4102
19.918	0.1016	0.2894	0.0909
21.745	0.2776	0.2737	0.4079
24.764	0.6589	0.4912	0.5967
20.054	0.2596	0.2934	0.334
18.694	0.1194	0.1145	0.0376
28.374	0.7531	0.6089	0.2780
24.41	0.4717	0.4448	0.3465
24.041	0.5979	0.5373	0.3672
22.138	0.4556	0.5035	0.254
19.130	0.1249	0.1949	0.1549
20.021	0.1801	0.2899	0.1943
29.137	0.9904	0.5931	0.6759
29.141	0.9232	0.5707	0.4396
25.626	0.616	0.4726	0.3547
26.210	0.784	0.3962	0.2949
28.211	0.9817	0.5485	0.4939
17.424	0.1734	0.1303	0.0869
23.52	0.5615	0.2781	0.1870
19.67	0.0353	0.1616	0.3140
25.759	0.6040	0.1877	0.5167
22.052	0.4115	0.4722	0.149
25.488	0.6058	0.3191	0.5524

Data set for a sample size of 200 for low collinearit

Y	X_1	X_2	X_3
24.359	0.5983	0.4276	0.2494

23.148	0.4564	0.3316	0.1869
25.880	0.7908	0.3974	0.3395
24.406	0.5472	0.5161	0.2162
28.718	0.9332	0.4813	0.5749
27.159	0.8467	0.5359	0.3095
26.744	0.7776	0.4402	0.4468
28.055	0.9075	0.4163	0.5
24.152	0.3335	0.3967	0.2914
25.043	0.4957	0.4630	0.4120
28.648	0.8150	0.5317	0.3105
23.099	0.5198	0.2809	0.4599
28.39	0.9180	0.4957	0.580
21.1	0.3881	0.0953	0.0509
23.146	0.4223	0.391	0.1764
29.096	0.9484	0.4304	0.2371
25.80	0.6989	0.4244	0.3587
25.68	0.6296	0.4207	0.2085
19.936	0.3983	0.1380	0.4277
27.213	0.734	0.5242	0.3496
25.237	0.7525	0.3316	0.4216
18.869	0.1401	0.1704	0.2907
22.863	0.4459	0.1218	0.1922
18.222	0.0400	0.0589	0.4631
18.974	0.1451	0.0537	0.160
19.543	0.3813	0.0939	0.4563
23.5	0.5023	0.356	0.3806
27.309	0.7842	0.4116	0.5041
19.387	0.0789	0.1970	0.0791
20.208	0.1885	0.1838	0.1993
25.748	0.708	0.3598	0.5180
19.220	0.0552	0.2382	0.0256
25.420	0.6981	0.3134	0.42
26.113	0.8502	0.3466	0.2549
19.32	0.2783	0.0342	0.2479
22.267	0.2951	0.469	0.3571
28.181	0.88	0.5482	0.2708
28.189	0.8963	0.4438	0.5739
25.163	0.8136	0.1978	0.2658
16.554	0.0029	0.0052	0.0177
22.410	0.4442	0.1844	0.4838
25.016	0.6633	0.3573	0.3561
19.757	0.1033	0.3429	0.2357
21.772	0.432	0.2743	0.0738
18.818	0.0566	0.4036	0.2412
19.952	0.1970	0.4205	0.1118
19.469	0.0767	0.3856	0.193

21.832	0.2808	0.191	0.1911
18.135	0.0211	0.3735	0.1416
22.432	0.4278	0.3252	0.4583
25.960	0.7573	0.4435	0.2875
29.744	0.9960	0.4639	0.47
23.060	0.3073	0.0778	0.5194
22.219	0.3973	0.080	0.4647
20.383	0.0389	0.3781	0.3562
23.866	0.5026	0.3627	0.2547
25.282	0.6354	0.264	0.3085
21.537	0.3127	0.2017	0.1302
24.45	0.6299	0.2517	0.4669
23.583	0.6450	0.1043	0.5270
26.27	0.6721	0.3573	0.4621
27.205	0.6119	0.4243	0.3921
19.988	0.003	0.2972	0.1667
18.6	0.0059	0.3206	0.2406
23.242	0.3426	0.1979	0.4736
22.46	0.4470	0.152	0.1671
26.998	0.8508	0.2865	0.5650
23.272	0.8456	0.1666	0.1024
27.234	0.7439	0.300	0.3097
20.891	0.2416	0.2776	0.4284
23.376	0.1844	0.4848	0.2189
25.366	0.6893	0.3791	0.4346
26.924	0.8449	0.3958	0.2213
27.40	0.9758	0.2013	0.2394
20.899	0.1640	0.3612	0.2594
24.65	0.7192	0.2427	0.538
25.682	0.8230	0.3101	0.2015
21.356	0.3709	0.0708	0.4373
21.588	0.198	0.4759	0.205
19.004	0.0514	0.009	0.2498
25.330	0.6212	0.3938	0.0731
27.988	0.9909	0.3523	0.2471
20.835	0.3327	0.1208	0.2083
27.803	0.9185	0.5790	0.5729
22.049	0.3499	0.1072	0.1755
25.113	0.6753	0.5523	0.2817
18.04	0.0099	0.1573	0.4100
26.539	0.852	0.2890	0.3082
28.702	0.9938	0.4054	0.1317
21.475	0.1181	0.4616	0.4983
21.844	0.4066	0.0894	0.3331
19.093	0.0363	0.2539	0.1578
21.666	0.1987	0.2379	0.4663

20.580	0.2641	0.1289	0.0987
21.581	0.4593	0.0767	0.2705
28.684	0.9562	0.468	0.4711
25.290	0.6377	0.4511	0.1854
26.482	0.8732	0.2876	0.4269
27.391	0.8236	0.5252	0.3801
20.046	0.312	0.1324	0.0997
24.297	0.3014	0.4890	0.3120
22.989	0.4418	0.0524	0.5291
25.432	0.8846	0.1729	0.4278
24.541	0.7503	0.1862	0.440
26.364	0.6298	0.2857	0.0743
22.30	0.4449	0.4076	0.3673
24.151	0.8555	0.2832	0.1518
22.489	0.5618	0.2462	0.3343
26.579	0.7729	0.3292	0.1525
25.259	0.7499	0.3249	0.3314
28.929	0.9907	0.2220	0.4171
26.609	0.8741	0.3043	0.4411
25.107	0.4942	0.4440	0.4255
21.612	0.1817	0.4590	0.3656
22.469	0.4074	0.0870	0.4973
21.837	0.1513	0.4949	0.1335
25.463	0.660	0.43	0.1331
24.803	0.5041	0.3108	0.0577
22.667	0.5506	0.3080	0.4527
20.773	0.2684	0.3007	0.1156
23.047	0.6107	0.0928	0.5405
28.514	0.8504	0.5424	0.2895
20.940	0.221	0.4857	0.2644
25.547	0.5243	0.0924	0.3721
20.406	0.1659	0.2482	0.2159
21.805	0.6084	0.093	0.5243
21.419	0.1810	0.2517	0.2439
23.156	0.3450	0.3090	0.2651
26.180	0.8433	0.1088	0.3191
28.256	0.962	0.243	0.2741
20.706	0.3991	0.2646	0.2268
21.718	0.3054	0.1651	0.2013
23.511	0.3645	0.4383	0.432
20.869	0.1075	0.2888	0.4289
25.366	0.9584	0.1640	0.2619
28.54	0.9309	0.3188	0.3323
21.386	0.4269	0.1276	0.4383
23.908	0.8598	0.121	0.2290
20.88	0.202	0.1456	0.141

28.332	0.9033	0.1899	0.53
27.012	0.9495	0.1531	0.4262
24.470	0.8896	0.1441	0.2409
21.047	0.1597	0.1739	0.0580
23.18	0.4821	0.3057	0.4990
23.132	0.3559	0.4159	0.1076
28.525	0.8781	0.4207	0.3393
27.035	0.9918	0.1127	0.5094
24.409	0.7502	0.4300	0.1566
23.849	0.5033	0.3051	0.4660
24.25	0.4963	0.4016	0.1517
24.768	0.8708	0.1748	0.3129
24.793	0.8477	0.1236	0.4389
26.541	0.9400	0.139	0.5420
22.16	0.432	0.3803	0.3401
25.795	0.7825	0.4405	0.5671
22.295	0.2542	0.3992	0.2869
24.787	0.5968	0.2228	0.124
25.653	0.912	0.1706	0.3256
29.1	0.9797	0.4421	0.4151
23.647	0.5344	0.3962	0.055
24.872	0.7285	0.1046	0.380
20.651	0.2558	0.0430	0.4196
17.499	0.0191	0.1418	0.1584
26.126	0.8264	0.2045	0.0826
23.968	0.7154	0.0887	0.3356
24.036	0.7485	0.2154	0.1975
24.516	0.5978	0.2062	0.4257
23.551	0.5184	0.2679	0.36
20.531	0.240	0.0777	0.354
19.896	0.0472	0.3149	0.3713
25.405	0.6611	0.4720	0.2241
26.66	0.9929	0.3048	0.3228
17.779	0.1159	0.1098	0.0656
20.289	0.1199	0.2200	0.3765
22.68	0.5821	0.3383	0.1441
26.393	0.6540	0.5345	0.5195
22.045	0.5794	0.0844	0.2661
21.144	0.4143	0.1127	0.1476
26.3	0.6277	0.115	0.3979
20.903	0.1241	0.2643	0.3883
22.683	0.2090	0.5071	0.3899
22.888	0.2650	0.4968	0.3034
23.927	0.5616	0.4603	0.058
20.497	0.2135	0.1884	0.2698
25.779	0.6343	0.421	0.3559

28.972	0.881	0.366	0.5437
28.341	0.7029	0.4847	0.3395
20.728	0.5344	0.1539	0.1092
18.571	0.0289	0.0495	0.3046
26.788	0.8355	0.2818	0.5098
22.628	0.4670	0.4047	0.2080
27.911	0.8469	0.5561	0.0871
24.45	0.6689	0.4572	0.2351
23.066	0.432	0.1540	0.091
22.551	0.3695	0.4162	0.0993
23.752	0.7952	0.2165	0.1650
23.896	0.7785	0.1092	0.204
25.777	0.5320	0.538	0.3454
20.020	0.1545	0.235	0.3621
24.991	0.4718	0.1316	0.501

Data set for sample size of 300 for high positive collineari

Y	X_1	X_2	X_3
26.561	0.965	0.4794	0.6134
27.901	0.8182	0.5785	0.4674
29.367	0.8092	0.5555	0.4805
19.593	0.1785	0.2625	0.2354
28.886	0.9233	0.463	0.5400
30.189	0.9805	0.5695	0.6189
19.962	0.0776	0.1206	0.053
16.398	0.1176	0.1103	0.2517
25.311	0.6937	0.4731	0.42
28.081	0.9392	0.4665	0.4983
30.40	0.9832	0.621	0.54
23.382	0.5265	0.2895	0.3079
28.132	0.9622	0.4426	0.5808
20.766	0.2433	0.1932	0.1932
25.392	0.6793	0.4519	0.471
18.312	0.061	0.1097	0.2104
24.957	0.5632	0.4101	0.3376
23.13	0.5629	0.2968	0.2748
21.236	0.3593	0.3247	0.3085
25.098	0.6018	0.4801	0.3896
29.185	0.9945	0.5637	0.5016
25.94	0.7812	0.5098	0.3985
25.040	0.6644	0.5129	0.3445
19.008	0.1530	0.1260	0.2577
23.693	0.5500	0.2901	0.3225
19.247	0.195	0.1109	0.2893
19.857	0.2197	0.1418	0.2574
20.889	0.1941	0.1108	0.2504
30.25	0.9090	0.6207	0.624
16.333	0.070	0.1003	0.0836
26.902	0.8051	0.5471	0.4747
26.52	0.8133	0.3695	0.4246
19.953	0.1783	0.1632	0.2178
23.23	0.4208	0.3911	0.3114
19.87	0.2341	0.1405	0.1875
21.077	0.3542	0.2792	0.3219
17.279	0.0314	0.0455	0.0210
22.418	0.3688	0.3551	0.2688
20.324	0.2568	0.121	0.309
24.199	0.6042	0.2943	0.2968
19.419	0.1393	0.089	0.1077
20.914	0.2346	0.3059	0.1357
16.614	0.0964	0.0495	0.1276

22.78	0.5772	0.3618	0.3050
28.81	0.9417	0.5118	0.4636
25.94	0.6619	0.4001	0.3660
23.093	0.4607	0.3014	0.2634
26.731	0.6459	0.4830	0.3043
23.177	0.4999	0.3675	0.3043
20.144	0.1599	0.2007	0.1030
24.203	0.5999	0.4547	0.333
22.700	0.6176	0.3092	0.4290
28.126	0.7482	0.4757	0.3619
18.645	0.0764	0.2071	0.2302
17.869	0.0326	0.1889	0.0478
18.712	0.13	0.1979	0.1133
20.032	0.2349	0.1634	0.2024
17.324	0.047	0.1131	0.0417
27.333	0.9052	0.4723	0.5770
28.733	0.8886	0.4864	0.5038
24.825	0.6287	0.484	0.3821
20.389	0.2060	0.2120	0.2559
21.663	0.4194	0.2225	0.2074
24.639	0.4523	0.3267	0.2231
28.569	0.8164	0.4966	0.375
27.459	0.8189	0.5307	0.4131
19.035	0.1405	0.2214	0.0962
23.140	0.433	0.3602	0.2830
24.565	0.6742	0.3746	0.4472
22.629	0.2830	0.3241	0.1339
25.550	0.6774	0.4643	0.3103
25.328	0.4452	0.3882	0.35
28.278	0.9707	0.6509	0.4477
26.727	0.6948	0.4222	0.5033
24.23	0.3367	0.3601	0.2095
20.142	0.1745	0.1798	0.1048
23.628	0.591	0.4752	0.41
18.053	0.0326	0.0183	0.0926
27.715	0.9076	0.4753	0.5291
20.755	0.2163	0.1829	0.0996
28.724	0.9499	0.5517	0.5678
25.334	0.6798	0.3595	0.4475
21.18	0.2463	0.1297	0.3152
24.742	0.4294	0.3914	0.4070
20.318	0.2708	0.1489	0.2808
27.234	0.7170	0.5306	0.5223
25.419	0.7364	0.3393	0.5098
24.820	0.6857	0.3770	0.4257
23.947	0.3465	0.1958	0.1622

28.816	0.9686	0.4501	0.510
28.882	0.923	0.470	0.5280
19.434	0.028	0.1946	0.116
24.214	0.5027	0.4157	0.3784
24.831	0.628	0.3843	0.383
21.315	0.2410	0.3079	0.174
29.232	0.8244	0.5218	0.4783
29.350	0.9043	0.5773	0.603
23.614	0.6246	0.3075	0.3679
21.460	0.4403	0.2051	0.2860
25.953	0.6708	0.4108	0.3555
30.447	0.9805	0.6107	0.5767
17.842	0.0505	0.0263	0.141
20.578	0.1576	0.1811	0.1758
20.015	0.2770	0.2260	0.2918
24.338	0.4396	0.3880	0.3429
23.785	0.527	0.2719	0.2443
25.572	0.6818	0.3897	0.4403
27.332	0.8752	0.5670	0.4895
22.865	0.4344	0.2860	0.3582
22.008	0.374	0.3626	0.1934
28.966	0.9182	0.5743	0.6074
27.571	0.9842	0.507	0.6508
20.373	0.1625	0.2046	0.216
21.562	0.3786	0.3133	0.2911
17.231	0.0856	0.2461	0.0848
17.247	0.0780	0.1590	0.1492
17.896	0.021	0.0978	0.1884
25.468	0.7763	0.386	0.4771
28.50	0.8455	0.4902	0.5406
27.874	0.9444	0.5414	0.6345
19.486	0.2198	0.2290	0.1821
21.703	0.191	0.2693	0.211
21.573	0.2487	0.3161	0.1146
23.360	0.4382	0.3890	0.3710
24.314	0.5702	0.3618	0.3839
27.695	0.8548	0.459	0.5848
28.049	0.9782	0.4673	0.5463
19.949	0.1079	0.0820	0.0957
24.293	0.4885	0.2890	0.3028
22.572	0.4608	0.3690	0.2480
27.492	0.9330	0.4243	0.5230
17.432	0.0073	0.075	0.1935
26.611	0.6197	0.4340	0.3532
19.908	0.1334	0.1811	0.086
19.485	0.1784	0.2766	0.2021

28.180	0.8174	0.5779	0.44
22.469	0.4284	0.2230	0.2234
18.060	0.0383	0.0976	0.1237
18.796	0.0874	0.1205	0.1072
19.743	0.1559	0.1644	0.0854
23.835	0.4026	0.2957	0.3949
20.047	0.2415	0.125	0.2081
20.715	0.1726	0.1902	0.2952
18.212	0.0758	0.1162	0.0588
28.276	0.8691	0.4961	0.5155
24.376	0.6595	0.3328	0.4974
25.770	0.6525	0.3315	0.4235
17.045	0.0040	0.0018	0.1586
18.186	0.0304	0.0383	0.0934
26.123	0.6676	0.4679	0.3303
21.085	0.1832	0.1344	0.248
24.464	0.4579	0.3557	0.2851
22.927	0.4560	0.2072	0.3561
23.780	0.4864	0.3444	0.2969
24.255	0.5670	0.3769	0.4020
29.331	0.9501	0.6076	0.546
26.129	0.7062	0.3611	0.4367
20.260	0.1965	0.1645	0.2632
24.974	0.469	0.3300	0.4256
25.574	0.7266	0.401	0.4297
25.979	0.5551	0.3807	0.3700
28.111	0.9858	0.5904	0.5365
24.802	0.6582	0.343	0.361
22.459	0.4474	0.2539	0.3511
19.691	0.1872	0.2295	0.2014
25.473	0.8324	0.4635	0.4429
32.459	0.9905	0.6281	0.5695
24.681	0.5970	0.4465	0.4079
24.151	0.5521	0.4231	0.3123
22.833	0.5003	0.3486	0.3088
18.832	0.0875	0.0902	0.1658
28.353	0.8564	0.5523	0.43
22.488	0.3765	0.378	0.2655
18.667	0.0741	0.1559	0.0799
27.569	0.7850	0.5291	0.422
22.427	0.34	0.2445	0.2735
24.858	0.5589	0.4157	0.443
18.66	0.0563	0.0988	0.0372
18.857	0.1522	0.1236	0.197
19.298	0.2763	0.1412	0.1810
22.196	0.3397	0.2527	0.2293

17.770	0.0228	0.0428	0.1737
19.416	0.081	0.1422	0.2094
22.005	0.4460	0.2085	0.227
26.465	0.7659	0.4916	0.4834
25.138	0.6818	0.374	0.4384
21.89	0.2716	0.1705	0.1466
19.632	0.2746	0.2449	0.1838
23.034	0.508	0.4006	0.28
25.380	0.7154	0.4224	0.5062
28.814	0.9867	0.563	0.5449
26.411	0.922	0.4979	0.4401
27.735	0.9470	0.4949	0.5009
15.790	0.0090	0.0060	0.0724
20.722	0.2404	0.2056	0.1985
23.951	0.4568	0.2391	0.2946
27.52	0.9050	0.5920	0.6029
28.262	0.8441	0.5304	0.5607
21.315	0.400	0.2295	0.2532
29.126	0.9226	0.6061	0.5130
25.212	0.5444	0.3013	0.4004
20.3	0.3988	0.2446	0.3083
26.449	0.7950	0.5369	0.4168
20.059	0.1536	0.138	0.1862
21.541	0.4935	0.2604	0.2516
21.866	0.3522	0.3164	0.3439
26.988	0.7921	0.5608	0.4345
27.353	0.7532	0.5059	0.4813
27.590	0.9109	0.4554	0.5339
27.329	0.6455	0.3478	0.417
19.928	0.2679	0.1709	0.2026
28.586	0.9086	0.4511	0.459
22.055	0.4561	0.2768	0.308
26.161	0.7608	0.3592	0.5357
26.910	0.7374	0.453	0.3688
20.383	0.1975	0.3016	0.0956
27.83	0.8547	0.595	0.5004
23.684	0.5289	0.4545	0.2680
22.472	0.3990	0.2286	0.1943
21.076	0.4967	0.3003	0.2314
25.174	0.6363	0.4359	0.3491
25.1	0.7792	0.4274	0.4216
18.822	0.0515	0.0539	0.0654
23.023	0.3278	0.2644	0.2813
18.660	0.1561	0.1146	0.1336
23.685	0.6395	0.3760	0.4473
17.35	0.0040	0.0995	0.133

23.703	0.5700	0.2818	0.2626
21.431	0.2380	0.312	0.128
28.093	0.9019	0.4930	0.5980
28.677	0.8925	0.4616	0.5959
28.702	0.8947	0.611	0.5964
19.060	0.1622	0.1081	0.2281
27.234	0.7503	0.4197	0.4764
27.21	0.6973	0.4450	0.4032
19.287	0.1641	0.0966	0.2730
24.062	0.5844	0.2932	0.4617
23.067	0.4120	0.3442	0.3056
28.860	0.8979	0.5525	0.557
25.544	0.5408	0.244	0.4211
22.926	0.367	0.211	0.2780
23.537	0.4537	0.2680	0.3076
23.660	0.5732	0.3766	0.3281
25.446	0.6789	0.3338	0.3639
18.313	0.1955	0.2876	0.2969
22.042	0.2595	0.309	0.318
27.929	0.934	0.5879	0.5064
24.503	0.5871	0.4286	0.3468
26.011	0.6465	0.3263	0.4795
18.084	0.0996	0.1043	0.0710
28.378	0.9274	0.6110	0.4623
26.353	0.8589	0.4246	0.4037
25.060	0.6689	0.499	0.4617
18.809	0.1411	0.2335	0.0788
28.061	0.8366	0.4689	0.3832
20.469	0.2273	0.2070	0.1239
21.851	0.3082	0.3553	0.2079
18.178	0.0498	0.0387	0.0990
30.763	0.9854	0.6511	0.5388
28.850	0.9329	0.537	0.5928
25.816	0.7468	0.4157	0.3902
26.708	0.8275	0.5039	0.3995
28.363	0.7899	0.5715	0.5305
20.140	0.1027	0.1836	0.0681
25.647	0.5877	0.2899	0.4070
18.244	0.082	0.2006	0.2448
18.572	0.1938	0.2183	0.109
29.034	0.8713	0.4424	0.4037
28.873	0.9239	0.5943	0.587
19.204	0.2603	0.2693	0.1730
23.499	0.5628	0.4698	0.2686
17.267	0.0065	0.1775	0.0126
26.549	0.694	0.3342	0.4530

26.052	0.6971	0.463	0.4233
26.850	0.7649	0.5356	0.4568
18.469	0.1200	0.0927	0.2690
25.331	0.7140	0.3252	0.5212
23.624	0.4413	0.3423	0.3136
25.489	0.8174	0.5392	0.5192
20.292	0.22	0.2464	0.162
27.401	0.8723	0.5082	0.5150
20.638	0.1232	0.1079	0.1546
26.48	0.7796	0.5381	0.4604
23.469	0.491	0.4100	0.2750
19.58	0.0597	0.1263	0.0425
26.034	0.6329	0.4052	0.3530
18.58	0.0820	0.0573	0.1319
19.326	0.0276	0.1651	0.0621
25.420	0.6148	0.4671	0.3820
23.888	0.537	0.2562	0.2764
26.175	0.6144	0.4517	0.3796
25.882	0.6926	0.409	0.4736
24.262	0.5644	0.4377	0.3678
27.4	0.9128	0.4153	0.5612
25.162	0.6633	0.3326	0.3212
25.30	0.6177	0.3326	0.4914
23.963	0.477	0.3748	0.3218
21.291	0.2770	0.1	0.2718
22.176	0.4418	0.4012	0.3878
19.198	0.1460	0.2271	0.23

Data set for sample size of 300 for moderate collinearity

Y	X_1	X_2	X_3
29.979	0.906	0.6653	0.2933
19.298	0.1250	0.2253	0.1615

19.866	0.1574	0.2524	0.3218
19.58	0.1755	0.2442	0.3645
27.582	0.9199	0.5549	0.287
23.523	0.5960	0.194	0.2433
24.866	0.5420	0.2858	0.4397
20.882	0.3151	0.3078	0.4464
24.652	0.5837	0.4713	0.1827
24.664	0.7081	0.3271	0.3160
23.672	0.4501	0.277	0.3124
18.71	0.1612	0.441	0.1659
19.793	0.091	0.2190	0.3372
26.921	0.8584	0.4602	0.5420
29.525	0.8855	0.4564	0.3277
25.649	0.5440	0.3941	0.3547
22.714	0.4277	0.2220	0.1909
21.705	0.211	0.4434	0.2278
29.66	0.9768	0.2943	0.4829
22.971	0.4972	0.441	0.1892
26.669	0.8378	0.2544	0.6239
24.929	0.7509	0.329	0.6031
23.372	0.3751	0.5011	0.3160
25.01	0.7378	0.3664	0.5353
24.837	0.600	0.4474	0.5350
23.655	0.5114	0.3442	0.1610
19.53	0.2156	0.3935	0.11
29.36	0.8937	0.5747	0.563
19.332	0.0576	0.3345	0.0635
19.619	0.1266	0.239	0.3418
28.261	0.8461	0.4746	0.4419
20.485	0.0523	0.3785	0.3880
21.314	0.3779	0.3019	0.4763
20.202	0.1255	0.2348	0.1144
26.097	0.8007	0.3279	0.337
19.761	0.0730	0.3496	0.1610
22.353	0.3698	0.3344	0.4815
23.505	0.4967	0.3498	0.3759
25.009	0.6715	0.269	0.4484
26.806	0.6928	0.5465	0.406
22.800	0.4441	0.282	0.3045
26.356	0.7046	0.3065	0.3784
22.903	0.3416	0.3074	0.3228
26.152	0.6348	0.4489	0.5559
21.224	0.075	0.3053	0.2264
29.35	0.9915	0.5833	0.6675
27.458	0.6918	0.4959	0.5795
20.407	0.0073	0.3984	0.1243

18.952	0.0836	0.0649	0.3439
20.250	0.0704	0.3236	0.390
24.639	0.5158	0.4931	0.3154
26.281	0.7865	0.3329	0.6333
23.29	0.2887	0.3094	0.2225
24.113	0.3877	0.4353	0.4371
21.15	0.1108	0.2017	0.0718
22.954	0.5374	0.1948	0.2510
18.46	0.0192	0.1319	0.2378
22.965	0.404	0.5131	0.2433
25.246	0.5570	0.3313	0.3635
24.048	0.5380	0.2664	0.1800
28.669	0.9778	0.4490	0.5211
19.913	0.1499	0.1193	0.2797
26.825	0.8577	0.6083	0.6000
17.958	0.0838	0.0555	0.290
19.668	0.1288	0.4005	0.0951
23.476	0.6198	0.3795	0.2049
27.609	0.6684	0.3887	0.5119
23.839	0.455	0.273	0.4969
18.984	0.0971	0.1923	0.0377
30.124	0.836	0.3268	0.5545
22.335	0.4220	0.3360	0.274
27.915	0.9723	0.6564	0.3541
20.017	0.1222	0.1984	0.1177
20.042	0.2468	0.1011	0.4612
28.005	0.7242	0.3681	0.5776
23.877	0.4989	0.4033	0.3547
22.521	0.4011	0.1260	0.4167
17.695	0.1292	0.1927	0.1975
20.686	0.3071	0.2773	0.1893
24.935	0.4580	0.1919	0.5121
18.831	0.203	0.0834	0.1740
21.986	0.2648	0.3654	0.0846
26.773	0.9036	0.2904	0.5083
24.65	0.6120	0.3574	0.2238
22.155	0.3172	0.1653	0.3152
23.764	0.4865	0.3068	0.3335
19.781	0.0310	0.1735	0.1845
26.658	0.8380	0.3979	0.6199
24.807	0.5000	0.3129	0.2997
22.040	0.2809	0.1792	0.428
23.546	0.3785	0.4586	0.399
21.161	0.432	0.2797	0.2249
18.453	0.0561	0.1294	0.1958
26.012	0.6971	0.3404	0.3761

25.853	0.5535	0.499	0.2731
20.366	0.2928	0.2650	0.3364
18.573	0.1251	0.3485	0.2445
23.83	0.5984	0.2708	0.393
27.106	0.6171	0.519	0.4713
30.47	0.9800	0.6070	0.6372
20.312	0.0285	0.3652	0.0196
26.617	0.6327	0.5040	0.4430
22.803	0.3802	0.1809	0.1874
25.545	0.6227	0.5004	0.5760
22.947	0.6149	0.2943	0.2770
19.057	0.1827	0.2048	0.0793
29.407	0.9019	0.6693	0.3442
24.868	0.636	0.3627	0.3899
25.342	0.5855	0.5597	0.2014
20.984	0.1941	0.3791	0.3587
22.762	0.4298	0.1411	0.1946
24.309	0.6242	0.4980	0.5336
26.419	0.7312	0.4129	0.5015
26.347	0.753	0.5379	0.3192
26.247	0.7412	0.3456	0.4716
25.917	0.758	0.3767	0.3984
16.769	0.0040	0.0736	0.0051
18.653	0.0410	0.2397	0.3826
22.349	0.2552	0.3515	0.1244
24.478	0.5596	0.4842	0.2154
20.145	0.0765	0.3269	0.3494
23.879	0.5236	0.2824	0.5449
28.83	0.9240	0.535	0.6597
26.976	0.79	0.2857	0.3039
21.619	0.2920	0.2233	0.2456
26.493	0.7205	0.5224	0.3602
21.802	0.3730	0.3888	0.3934
18.244	0.0062	0.3054	0.2074
27.066	0.6465	0.4314	0.5268
24.24	0.5875	0.4358	0.2261
20.531	0.0056	0.3033	0.4016
27.97	0.8629	0.558	0.5360
20.588	0.1541	0.1416	0.3198
27.229	0.7702	0.592	0.2652
28.092	0.9188	0.4803	0.6614
18.5	0.0366	0.1755	0.0227
20.785	0.2826	0.303	0.2818
21.267	0.1505	0.4128	0.1854
22.050	0.433	0.1642	0.2004
25.37	0.575	0.4322	0.5066

29.304	0.8646	0.6447	0.6295
19.516	0.1814	0.2827	0.1801
21.783	0.3337	0.3219	0.1274
17.54	0.0606	0.2458	0.1162
23.032	0.381	0.2122	0.2652
24.525	0.7148	0.2333	0.2205
23.2	0.4418	0.2896	0.5035
24.561	0.4208	0.3953	0.3957
27.341	0.8421	0.5497	0.424
24.145	0.5531	0.2500	0.2397
28.370	0.866	0.6307	0.4022
20.729	0.1899	0.302	0.231
22.645	0.1606	0.3237	0.4186
26.073	0.7802	0.4630	0.5330
22.806	0.4350	0.485	0.3092
22.514	0.5422	0.1629	0.2182
22.535	0.4662	0.4049	0.4007
17.429	0.220	0.1342	0.1403
25.113	0.7755	0.6186	0.380
23.301	0.5879	0.4024	0.4093
26.119	0.6669	0.3039	0.4184
25.920	0.8395	0.3883	0.5893
27.444	0.8294	0.4923	0.4427
22.508	0.4176	0.3553	0.1845
25.187	0.4936	0.3325	0.4933
21.960	0.1150	0.1609	0.3656
24.616	0.6373	0.5836	0.2046
20.851	0.3504	0.1302	0.4564
22.689	0.4516	0.2231	0.5297
19.194	0.0091	0.3120	0.0586
22.480	0.4909	0.3070	0.2385
23.977	0.5755	0.3332	0.2812
20.374	0.1412	0.2864	0.2054
19.240	0.0582	0.3532	0.3158
26.835	0.9580	0.4068	0.6857
23.162	0.5007	0.3346	0.3387
25.667	0.7238	0.4125	0.4638
26.59	0.8037	0.628	0.3495
24.271	0.5140	0.3318	0.3617
25.342	0.6784	0.2971	0.3149
25.314	0.5963	0.4560	0.4040
26.046	0.7076	0.6008	0.5721
27.093	0.9390	0.5048	0.5347
30.935	0.9648	0.6085	0.3699
23.69	0.4515	0.4879	0.2669
26.414	0.702	0.3809	0.4096

28.731	0.8555	0.6314	0.3499
29.198	0.8996	0.6008	0.4003
16.775	0.1400	0.1854	0.4373
25.267	0.7015	0.4902	0.247
23.869	0.7034	0.2	0.4353
29.003	0.9877	0.4412	0.424
21.465	0.2188	0.3626	0.4592
25.2	0.7244	0.3740	0.2267
17.495	0.0082	0.1290	0.1184
22.604	0.3798	0.4107	0.3663
26.179	0.7404	0.5330	0.4434
22.0	0.2040	0.4282	0.3283
23.821	0.5062	0.3936	0.5218
26.868	0.7556	0.5777	0.3960
25.722	0.6257	0.5857	0.1894
19.630	0.0803	0.2903	0.1946
20.87	0.3177	0.264	0.3698
18.825	0.150	0.3495	0.3402
22.184	0.1879	0.4173	0.3567
29.11	0.8716	0.6135	0.5709
24.083	0.6027	0.5552	0.520
25.993	0.705	0.3870	0.4423
29.337	0.9597	0.3891	0.6541
22.295	0.6143	0.2398	0.3848
21.379	0.2855	0.1738	0.3189
22.657	0.4097	0.2435	0.3036
20.538	0.1666	0.1230	0.1332
23.084	0.4792	0.172	0.4792
18.029	0.0395	0.2776	0.0791
18.655	0.1245	0.36	0.286
18.997	0.147	0.2761	0.2154
28.037	0.9613	0.332	0.3322
18.256	0.1686	0.0981	0.3245
20.302	0.3711	0.2304	0.1401
22.673	0.4973	0.1498	0.381
27.041	0.6823	0.3126	0.5483
24.426	0.745	0.4454	0.3962
25.995	0.9324	0.4052	0.5399
26.054	0.6976	0.4883	0.4729
22.065	0.392	0.1648	0.3176
19.661	0.146	0.1353	0.1951
23.901	0.4715	0.4220	0.5131
17.89	0.0360	0.1880	0.0676
21.747	0.2372	0.4331	0.3161
18.842	0.039	0.1349	0.0643
18.889	0.1001	0.3937	0.1607

17.596	0.1296	0.1780	0.1857
27.090	0.6751	0.5116	0.2908
20.845	0.2520	0.2289	0.2529
24.99	0.5231	0.4559	0.1625
22.252	0.3588	0.2876	0.3552
21.042	0.0618	0.4028	0.2880
22.202	0.627	0.1949	0.4614
23.578	0.5185	0.2754	0.2863
26.772	0.8961	0.3112	0.5470
22.659	0.4504	0.1701	0.3127
23.775	0.450	0.3611	0.2537
29.309	0.8975	0.6108	0.6598
22.230	0.3232	0.1336	0.360
24.681	0.659	0.3817	0.4727
17.574	0.0867	0.0713	0.2715
19.509	0.0126	0.3768	0.06
18.770	0.019	0.1044	0.306
25.360	0.7594	0.2746	0.336
22.525	0.5093	0.5206	0.3855
22.047	0.2989	0.3307	0.3008
26.425	0.6635	0.5633	0.5738
25.005	0.6869	0.4678	0.420
20.982	0.2114	0.2668	0.174
26.125	0.6258	0.5698	0.1950
22.1	0.5806	0.2488	0.2023
17.254	0.068	0.095	0.3829
21.223	0.3014	0.1142	0.3843
27.214	0.8193	0.2740	0.5495
17.478	0.0261	0.023	0.2520
22.021	0.3610	0.1870	0.3481
22.374	0.4105	0.2076	0.3610
25.97	0.8630	0.4896	0.5818
25.149	0.618	0.4401	0.2488
26.329	0.8959	0.3034	0.4195
28.253	0.904	0.4972	0.6415
22.29	0.6033	0.245	0.320
20.104	0.1347	0.0511	0.2117
21.537	0.0925	0.1851	0.4180
20.980	0.2969	0.2437	0.3623
28.317	0.7803	0.5988	0.3917
22.259	0.4245	0.3979	0.1551
27.374	0.879	0.4762	0.5328
19.901	0.0574	0.1051	0.0678
21.11	0.0620	0.2142	0.4128
24.551	0.6977	0.2635	0.2841
20.959	0.345	0.2407	0.1413

21.793	0.316	0.4447	0.2444
27.120	0.8465	0.3819	0.5436
20.243	0.0935	0.2966	0.3387
27.976	0.8600	0.4353	0.2989
20.120	0.2423	0.4209	0.1933
22.732	0.2256	0.3582	0.3071
25.956	0.8666	0.4167	0.4008
27.207	0.9025	0.5310	0.3870
22.848	0.329	0.4000	0.4386
27.102	0.5910	0.4687	0.3983
24.835	0.5561	0.2740	0.5502
26.270	0.6769	0.406	0.4362
24.663	0.5987	0.4218	0.288
19.405	0.0075	0.3096	0.173
28.183	0.8809	0.4501	0.4419
22.161	0.1662	0.4221	0.265
22.772	0.3812	0.2752	0.3741
23.052	0.4766	0.5159	0.2655
28.735	0.9121	0.5369	0.3673
23.488	0.6180	0.3803	0.3012
20.861	0.1786	0.4269	0.1105
26.06	0.8486	0.4436	0.5869

Data set for sample size of 300 for low collinearity

Y	X_1	X_2	X_3
25.570	0.4880	0.5045	0.4552
25.78	0.9059	0.2539	0.2951
18.353	0.1099	0.0780	0.2672
23.07	0.3358	0.4732	0.1777
20.714	0.1908	0.5034	0.2715
19.808	0.1718	0.4090	0.2329
22.716	0.5586	0.1514	0.338
22.677	0.3820	0.3619	0.4462

21.047	0.3408	0.2337	0.1583
21.30	0.5756	0.2895	0.3388
25.729	0.9115	0.2310	0.2155
21.498	0.314	0.3001	0.2506
24.196	0.5141	0.266	0.2909
26.962	0.7775	0.4689	0.306
24.829	0.7523	0.2836	0.5047
22.364	0.3679	0.2191	0.117
21.432	0.4861	0.0521	0.1236
24.264	0.5982	0.3412	0.2937
22.146	0.5041	0.2158	0.5064
24.457	0.5963	0.4119	0.1409
28.22	0.9398	0.5732	0.2944
21.24	0.2830	0.2291	0.3067
27.83	0.8793	0.5447	0.130
22.266	0.3004	0.3471	0.24
22.600	0.3746	0.112	0.4113
25.536	0.8405	0.3205	0.2899
25.725	0.6574	0.206	0.4759
21.000	0.2476	0.2103	0.4611
21.73	0.3285	0.1531	0.2128
26.443	0.8441	0.4383	0.2262
21.3	0.2190	0.2430	0.1768
21.082	0.4083	0.1196	0.4893
28.361	0.9280	0.5513	0.3130
25.357	0.8504	0.3247	0.2615
20.45	0.4454	0.0687	0.3516
20.202	0.3818	0.0704	0.2685
24.091	0.6379	0.3575	0.3301
17.958	0.1224	0.1342	0.3116
20.752	0.308	0.180	0.444
21.9	0.1332	0.3580	0.2925
22.77	0.5037	0.0629	0.237
20.544	0.2228	0.4432	0.3595
21.94	0.3152	0.3623	0.1879
23.903	0.4145	0.222	0.1694
17.275	0.1082	0.0385	0.2423
18.501	0.140	0.2498	0.0978
23.852	0.4177	0.4516	0.4734
23.285	0.5733	0.1552	0.2240
26.307	0.8143	0.2499	0.4069
22.797	0.4904	0.3891	0.458
20.877	0.284	0.4049	0.2393
27.53	0.8615	0.3775	0.4751
24.377	0.5491	0.3722	0.1745
21.568	0.5440	0.2054	0.1717

27.162	0.8872	0.4053	0.3606
22.800	0.533	0.1162	0.0978
23.834	0.7373	0.0840	0.294
25.877	0.5875	0.2924	0.3998
20.911	0.4069	0.1047	0.0758
27.327	0.8857	0.400	0.2042
25.760	0.7792	0.5170	0.2659
19.284	0.1676	0.4164	0.0172
20.757	0.1048	0.4157	0.2494
24.224	0.4535	0.3936	0.1947
22.246	0.5499	0.0595	0.3998
25.79	0.8469	0.2257	0.4694
26.898	0.874	0.3575	0.5699
27.976	0.7273	0.5352	0.1376
19.867	0.2184	0.0654	0.3108
19.651	0.4192	0.1382	0.2248
24.271	0.915	0.1266	0.4526
25.388	0.6354	0.3509	0.3916
24.345	0.4772	0.3524	0.4454
19.319	0.2775	0.1767	0.2016
25.673	0.8311	0.3612	0.4236
19.190	0.0650	0.1987	0.1478
29.263	0.9574	0.5802	0.5055
28.310	0.9444	0.3880	0.1724
26.241	0.6721	0.5465	0.0731
19.855	0.0360	0.4326	0.3761
25.338	0.7938	0.2547	0.3366
28.983	0.9131	0.267	0.2825
24.6	0.7442	0.4697	0.1590
25.130	0.5819	0.283	0.4833
25.505	0.6368	0.4843	0.1508
18.406	0.0829	0.4203	0.0346
23.489	0.671	0.2876	0.2174
20.45	0.4971	0.1170	0.0555
25.177	0.6951	0.3563	0.3916
25.567	0.7996	0.3833	0.5131
20.521	0.2958	0.4475	0.0699
24.350	0.8171	0.4635	0.1974
25.444	0.95	0.206	0.1866
26.597	0.6969	0.1500	0.3146
22.421	0.376	0.4515	0.2194
23.295	0.4917	0.3914	0.3779
26.070	0.946	0.1673	0.2129
23.299	0.5388	0.0966	0.3044
21.147	0.2855	0.2836	0.3895
22.591	0.5458	0.1139	0.2693

26.401	0.848	0.5462	0.5685
21.568	0.2169	0.0559	0.4376
25.487	0.7642	0.3859	0.3597
21.291	0.2727	0.3879	0.2515
24.736	0.5141	0.488	0.2085
24.340	0.8363	0.1168	0.2392
20.165	0.2231	0.0844	0.054
21.091	0.3222	0.0533	0.4735
20.359	0.319	0.0503	0.1153
26.91	0.6568	0.4535	0.5408
20.057	0.000	0.3025	0.0526
21.746	0.4759	0.0524	0.5173
27.057	0.870	0.2690	0.497
21.461	0.139	0.3811	0.1220
21.423	0.3738	0.3043	0.4596
21.306	0.3039	0.3918	0.0484
19.61	0.006	0.4260	0.4864
25.129	0.7449	0.277	0.0858
22.530	0.3690	0.469	0.0857
19.89	0.1050	0.3119	0.1676
24.395	0.3532	0.5222	0.435
20.382	0.3876	0.2145	0.4027
26.812	0.8924	0.3969	0.1707
21.069	0.1427	0.4021	0.4012
19.068	0.1381	0.2948	0.2136
21.809	0.5176	0.0731	0.1063
26.543	0.9137	0.4640	0.2759
23.842	0.6624	0.3434	0.0677
22.359	0.2435	0.3407	0.2338
24.205	0.3834	0.2972	0.1127
24.72	0.7507	0.3274	0.2278
26.554	0.8533	0.4015	0.5460
25.374	0.7053	0.2093	0.3655
26.717	0.9318	0.2517	0.2186
25.591	0.9389	0.395	0.2404
28.20	0.9564	0.3817	0.2462
24.662	0.815	0.4521	0.3352
18.828	0.0415	0.1163	0.3478
18.948	0.215	0.031	0.0425
17.402	0.0333	0.4850	0.1594
27.383	0.8144	0.4175	0.2682
18.538	0.0989	0.0741	0.2508
23.933	0.5832	0.3305	0.1804
22.610	0.3755	0.4205	0.240
19.228	0.1271	0.4445	0.3010
22.301	0.4383	0.192	0.4045

20.380	0.2478	0.0865	0.0360
20.217	0.179	0.3953	0.2971
27.085	0.843	0.2802	0.0859
20.505	0.0372	0.1244	0.350
21.740	0.2822	0.290	0.396
18.723	0.2245	0.2168	0.1082
24.522	0.6513	0.3404	0.1619
18.889	0.1006	0.3702	0.0599
23.359	0.5872	0.4054	0.242
20.363	0.1654	0.3946	0.2000
27.685	0.8283	0.1347	0.1556
19.805	0.0311	0.0878	0.2366
20.328	0.1202	0.0974	0.1608
26.808	0.5826	0.3244	0.2380
19.843	0.1154	0.1632	0.4814
25.42	0.5751	0.4222	0.433
18.647	0.2187	0.1009	0.3559
24.010	0.5483	0.2684	0.5079
20.32	0.1614	0.3885	0.2692
19.787	0.039	0.1905	0.2428
22.217	0.0575	0.4928	0.3866
18.138	0.0661	0.1890	0.1107
27.715	0.8628	0.3981	0.2782
23.965	0.6340	0.3196	0.1858
21.620	0.607	0.1736	0.4389
25.437	0.6034	0.345	0.4748
22.358	0.4554	0.0484	0.4829
22.308	0.4011	0.4133	0.246
21.697	0.4957	0.29	0.2820
19.59	0.1453	0.2041	0.4547
23.142	0.7438	0.0855	0.0778
16.865	0.0343	0.0469	0.4897
25.705	0.8682	0.1862	0.473
19.30	0.0016	0.3736	0.2084
26.95	0.6198	0.3877	0.4326
22.556	0.2572	0.34	0.4074
27.700	0.9242	0.2332	0.1995
29.317	0.8890	0.5569	0.5245
23.333	0.3999	0.0614	0.3886
23.159	0.6542	0.245	0.3358
23.157	0.5872	0.195	0.1444
20.161	0.1389	0.2650	0.0175
17.644	0.049	0.1917	0.0896
24.174	0.4985	0.4114	0.3619
17.573	0.0302	0.0685	0.0101
22.169	0.5291	0.1807	0.2901

22.907	0.6396	0.1646	0.0701
24.209	0.3990	0.4443	0.3462
18.882	0.0477	0.1122	0.3703
25.898	0.9249	0.3141	0.3857
19.500	0.1565	0.1467	0.2623
22.701	0.4414	0.165	0.4808
24.509	0.6970	0.1365	0.3955
21.222	0.2381	0.5001	0.3951
20.764	0.0902	0.4572	0.0662
22.823	0.2312	0.4983	0.453
26.700	0.6797	0.309	0.1965
18.75	0.0304	0.321	0.0554
22.417	0.6069	0.131	0.2502
21.816	0.1164	0.4076	0.0177
27.054	0.9243	0.4373	0.5590
25.743	0.6605	0.3117	0.3421
22.958	0.5185	0.3216	0.5065
26.661	0.6932	0.3914	0.4867
22.115	0.3059	0.4703	0.3931
25.294	0.7518	0.2331	0.1347
25.50	0.8973	0.2333	0.1334
24.687	0.4665	0.4152	0.2692
24.243	0.672	0.2973	0.1905
26.135	0.6524	0.4190	0.5145
29.829	0.9648	0.5718	0.2109
21.508	0.2634	0.3029	0.4767
23.988	0.37	0.4182	0.5016
19.652	0.1181	0.3710	0.0917
20.653	0.1004	0.2622	0.0603
25.05	0.8224	0.3339	0.26
18.709	0.1165	0.0768	0.246
20.664	0.2524	0.0355	0.3810
24.638	0.8449	0.2251	0.5493
24.678	0.5125	0.3653	0.4592
24.687	0.7926	0.1904	0.1795
26.137	0.8880	0.1734	0.3104
16.033	0.017	0.1191	0.3919
20.910	0.1015	0.2767	0.2060
25.050	0.6846	0.3662	0.3421
19.303	0.2043	0.2438	0.1715
22.950	0.6692	0.2009	0.092
22.104	0.4412	0.0858	0.1755
27.641	0.8685	0.5331	0.223
24.162	0.5547	0.4700	0.1619
19.877	0.0628	0.2752	0.0650
25.21	0.7582	0.4297	0.3353

23.857	0.6636	0.1001	0.4280
28.420	0.9251	0.3495	0.3496
22.112	0.3653	0.3283	0.0535
26.820	0.8089	0.5421	0.2251
20.764	0.1246	0.3302	0.4978
22.276	0.2931	0.4750	0.2816
29.039	0.9558	0.5117	0.2727
25.211	0.7357	0.120	0.5125
19.628	0.0989	0.1874	0.3798
24.333	0.7622	0.1872	0.3707
18.358	0.1125	0.0656	0.2630
21.720	0.2177	0.3022	0.3235
22.26	0.2738	0.1554	0.3928
27.692	0.8056	0.5419	0.3691
19.779	0.0886	0.0785	0.4331
22.078	0.3376	0.4304	0.3912
19.29	0.1260	0.0327	0.3215
24.394	0.7945	0.1926	0.3169
22.083	0.1124	0.4409	0.4125
21.12	0.3146	0.0342	0.3553
27.020	0.7594	0.4910	0.2193
20.559	0.2120	0.3938	0.2161
24.149	0.6499	0.1126	0.3405
26.773	0.9171	0.2800	0.2397
26.223	0.8389	0.3551	0.2361
25.682	0.851	0.1297	0.355
23.470	0.551	0.120	0.5227
21.71	0.2765	0.4750	0.0878
22.012	0.5661	0.1085	0.1080
21.759	0.2767	0.4722	0.0540
24.123	0.5441	0.1080	0.206
28.007	0.7764	0.5556	0.0971
24.920	0.772	0.2465	0.1577
21.199	0.3752	0.1101	0.141
25.550	0.8068	0.2884	0.5431
22.493	0.3060	0.2425	0.088
27.92	0.9730	0.1279	0.2345
18.927	0.055	0.1752	0.4856
26.944	0.6685	0.2170	0.3726
23.84	0.6209	0.1543	0.2208
27.571	0.8084	0.4457	0.3124
25.954	0.64	0.4893	0.072
22.197	0.373	0.1756	0.1262
25.720	0.8905	0.0918	0.3545
24.343	0.8677	0.3089	0.2342
21.091	0.2773	0.1078	0.2272

18.424	0.1470	0.1404	0.249
22.483	0.3952	0.3801	0.4127
26.340	0.9788	0.268	0.5800
20.639	0.3554	0.091	0.1668
19.736	0.1112	0.339	0.4344
23.58	0.4080	0.4503	0.3305
18.338	0.1910	0.0588	0.2478
21.0	0.1444	0.1932	0.3061
25.079	0.8987	0.2658	0.2607
26.701	0.7745	0.3082	0.4280
27.747	0.9508	0.4300	0.2766
26.043	0.6830	0.4287	0.1918
23.773	0.5523	0.1686	0.3310
22.973	0.6569	0.2876	0.5290
25.852	0.8345	0.2577	0.3957
24.138	0.5661	0.3136	0.1696